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# NOTES

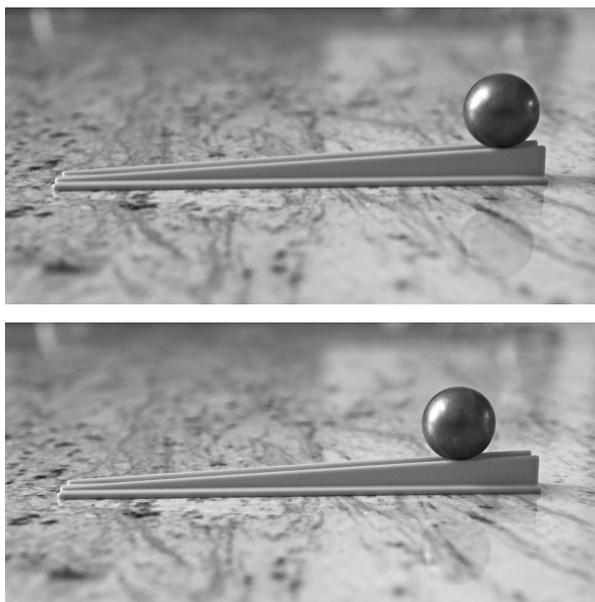
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## The Geometry of the Snail Ball

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An amusing device called the snail ball can be found at some puzzle shops (e.g., [1], which has a video of the device). The ball rolls down an inclined ramp, but very, very slowly. It rolls just a few millimeters at a time, with long periods during which the ball just stands motionless. On a six-inch ramp tilted at an angle of  $4^\circ$ , the ball can take six minutes or more to descend. This is indeed a snail-like pace of an inch a minute: a typical snail race has fastest times of about 6 inches per minute. The device is shown in FIGURE 1.



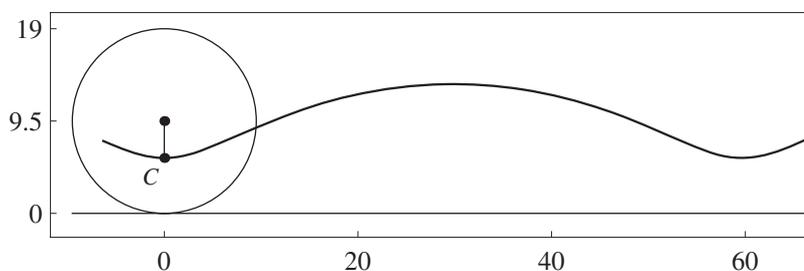
**Figure 1** Two views of the snail ball; the second was taken 30 seconds after the first.

The mystery can be explained by some familiar concepts: the center of gravity and the curtate cycloid curve. Recall that a point on a rolling wheel of radius  $r$  traces out the curve  $r(t - a \sin t, 1 - a \cos t)$  where  $a$  is the proportion of the radius that the tracing point is away from the rim. When  $a < r$  this is called a *curtate cycloid*, but we will just use the term *cycloid* here for brevity.

The snail ball consists of a spherical metal shell (diameter 19 mm) hiding a smaller core (a solid ball of diameter 12 mm) in its interior. There is also a viscous liquid in

the interior (perhaps glycerol), which delays the ability of gravity to lower the core to the bottom position. By making the simplifying assumption that the viscosity acts very strongly to delay motion, we can use some simple geometry to explain the effect. (The unit of viscosity is the *poise*, for Poiseuille; water has poise 0.894 at 25°C while glycerol has 1500 poise at the same temperature—roughly the same as corn syrup.) As a bonus we can figure out the center of gravity of the snail ball without breaking it open and weighing the components.

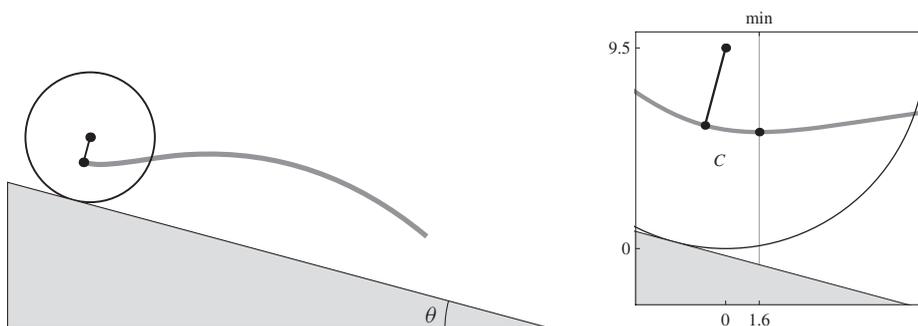
If a wheel has a fixed center of gravity (call it  $C$ ) located below its center, then it cannot be rolled very far. The center of gravity cannot, in the ideal case where its initial speed is infinitesimally small, rise above its starting point (the sum of potential and kinetic energy cannot rise and the kinetic energy is 0 at the start). Of course a small initial speed generates some momentum, and this raises  $C$  a little before the slope of the curtate cycloid (FIGURE 2) forces it back down to its original level; the effect of friction thus returns it to its starting state via a damped oscillation.



**Figure 2** When the center of gravity is not at the geometrical center of a wheel, rolling is inhibited because of the local minimum of the curtate cycloid.

But suppose the ball is on a ramp, tilted downward at angle  $\theta$  (FIGURE 3; where the angle is an artificially steep  $15^\circ$ ). The cycloidal curve is tilted as well and its local minimum is shifted a bit to the right. Thus the ball wants to roll down a short distance. If the center of gravity were fixed, then the ball would simply settle into this new position. But the viscous fluid allows the core to slowly return to the bottom of the shell. When that happens the device is back to its initial state and again moves forward a very little bit.

This model also explains the surprising back-and-forth motion of the snail ball. For as it falls into the new minimum its momentum carries it up the other side before it falls backward and settles into its new location.



**Figure 3** The tilted cycloid has its minimum at a level slightly lower than the initial center of gravity, as shown in the magnified view.

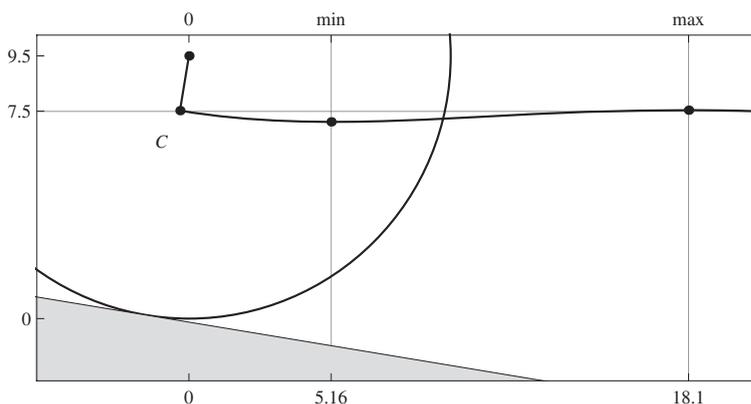
This geometrical view allows us to determine the center of gravity without knowing the exact details of what is hidden inside the shell. For if the ramp is very steep, the ball will just roll down without ever stopping: the center of gravity will rise and fall over the bumps of the cycloid but the viewer will just see this is a variation in the speed, which never becomes 0. Thus we can, by tilting the ramp, determine the steepest angle that causes the ball to rest during its journey; this resting time is what allows the core to return to the bottom and the motion to restart.

To start the experiment we place the ball on a flat surface so that the core can settle (as in FIGURE 2) and then we tilt the ramp and ball as shown in FIGURE 3 and immediately release the ball. From this we learn that, for the particular snail ball shown in FIGURE 1, the critical angle is about  $9.5^\circ$ . What this means is that the first local maximum of the tilted cycloid is at the same vertical level as the center of gravity  $C$ . We can use calculus to find this maximum. The  $y$ -coordinate of the cycloid after clockwise rotation through  $\theta$  is  $r(1 - a \cos(t + \theta) - t \sin \theta)$ , where  $r$  is the radius of the ball,  $\theta$  is the tilt angle, and  $a$  is the proportion of the total radius that locates the center of gravity below the center of the ball. The derivative with respect to  $t$  is  $r(a \sin(t + \theta) - \sin \theta)$ , which we can set to 0 and solve. Making the correct choice of solution, one finds that the maximum corresponds to  $t = \pi - \arcsin((\sin \theta)/a) - \theta$  and the corresponding local maximum of the height is

$$r \left( 1 - (\pi - \theta) \sin \theta + \arcsin[(\sin \theta)/a] \sin \theta + \sqrt{a^2 - \sin^2 \theta} \right).$$

So now we can determine the center of gravity for the snail ball by setting  $r = 9.5$  and  $\theta$  to be the critical angle of  $9.5^\circ$ . Numerical root-finding then gives the final answer: the value of  $a$  so that the maximum just given coincides with  $r(1 - a \cos \theta)$ , the height of the center of gravity in its initial position. This occurs at  $a \approx 0.2113$ , meaning that  $C$  lies at height  $r(1 - a \cos 9.5^\circ) \approx 7.5$ . A visual check (see FIGURE 4, which shows the tilted cycloid's tangency to the horizontal from  $C$ ) shows that we have successfully located it. If the center of gravity were any higher, the ball would roll over the invisible bump and continue down the ramp without pausing.

As typically happens, the real world device is a little more complicated than the simplest mathematical model. The friction between the ball and the ramp plays a role: as the core returns to its low point, the motion will not restart until the gravity force is enough to overcome the frictional force. This helps explain why the core must end up quite near the bottom for the restart to occur, and perhaps explains why the device does not just move slowly down in some equilibrium state.



**Figure 4** When the center of gravity is at the exact same level as the next local maximum of the tilted cycloid, the ramp is at its critical steepness.



**Figure 5** One can make a transparent snail ball using a jar, some corn syrup, and a moderately heavy metal cylinder.

One can easily build a simple transparent version of the device. Just partially fill a glass jar with corn syrup and then insert a heavy metal cylinder. I used a 3.5-inch diameter jar with a 1.5-inch diameter cylindrical piece of aluminum to act as the weight (FIGURE 5). One can then see the effect of friction by wrapping elastic bands around the jar and also varying the ramp surface. A more detailed study using advanced techniques from fluid dynamics is available in [2].

## REFERENCES

1. Grand Illusions, [http://www.grand-illusions.com/acatalog/Snail\\_Ball.html](http://www.grand-illusions.com/acatalog/Snail_Ball.html).
2. N. J. Balmforth, J. W. M. Bush, D. Vener, and W. R. Young, Dissipative descent: Rocking and rolling down an incline, *Journal of Fluid Mechanics* 590 (2007) 295–318. doi:10.1017/S0022112007008051

**Summary** The snail ball is a device that rolls down an inclined plane, but very slowly, repeatedly coming to a stop and staying motionless for several seconds. The interior of the ball is hollow, with a smaller solid ball inside it, surrounded by a very viscous fluid. We show how to model the stop-and-start motion by analyzing the cycloidal curve that would correspond to the motion of the center of gravity as the ball rolls down an inclined plane.

## Another Morsel of Honsberger

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In both of his interesting books [5] and [6], Ross Honsberger presents and proves the following property:

In FIGURE 1, as  $A$  moves along the circular arc  $\widehat{BC}$ ,  $AB + AC$  attains its maximum when  $A$  is the midpoint  $M$ . (1)

In [5, Problem 9, pp. 16–17], he describes (1) as *intuitively obvious* and gives a proof. Actually, the last paragraph there can be thought of as another proof. In [6, pp. 21–24], he gives two more proofs. He attributes the first to I. van Yzeren and describes it as *full*