B1 Suppose that \( P(x) = a_1 x + a_2 x^2 + \cdots + a_n x^n \) is a polynomial with integer coefficients, with \( a_1 \) odd. Suppose that \( e^{P(x)} = b_0 + b_1 x + b_2 x^2 + \cdots \) for all \( x \). Prove that \( b_k \) is nonzero for all \( k \geq 0 \).

B2 Let \( \times \) represent the cross product in \( \mathbb{R}^3 \). For what positive integers \( n \) does there exist a set \( S \subset \mathbb{R}^3 \) with exactly \( n \) elements such that
\[
S = \{ v \times w : v, w \in S \}.
\]

B3 Assign to each positive real number a color, either red or blue. Let \( D \) be the set of all distances \( d > 0 \) such that there are two points of the same color at distance \( d \) apart. Recolor the positive reals so that the numbers in \( D \) are red and the numbers not in \( D \) are blue. If we iterate this recoloring process, will we always end up with all the numbers red after a finite number of steps?

B4 Find all integers \( n \) with \( n \geq 4 \) for which there exists a sequence of distinct real numbers \( x_1, \ldots, x_n \) such that each of the sets
\[
\{x_1, x_2, x_3\}, \{x_2, x_3, x_4\}, \ldots, \{x_{n-2}, x_{n-1}, x_n\}, \{x_{n-1}, x_n, x_1\}, \text{ and } \{x_n, x_1, x_2\}
\]
forms a 3-term arithmetic progression when arranged in increasing order.

B5 For \( 0 \leq p \leq 1/2 \), let \( X_1, X_2, \ldots \) be independent random variables such that
\[
X_i = \begin{cases} 
1 & \text{with probability } p, \\
-1 & \text{with probability } p, \\
0 & \text{with probability } 1 - 2p,
\end{cases}
\]
for all \( i \geq 1 \). Given a positive integer \( n \) and integers \( b, a_1, \ldots, a_n \), let \( P(b, a_1, \ldots, a_n) \) denote the probability that \( a_1 X_1 + \cdots + a_n X_n = b \). For which values of \( p \) is it the case that
\[
P(0, a_1, \ldots, a_n) \geq P(b, a_1, \ldots, a_n)
\]
for all positive integers \( n \) and all integers \( b, a_1, \ldots, a_n \)?

B6 Find all continuous functions \( f : \mathbb{R}^+ \to \mathbb{R}^+ \) such that
\[
f(xf(y)) + f(yf(x)) = 1 + f(x + y)
\]
for all \( x, y > 0 \).