This research examines the property of constant vector curvature (cvc(e)) on three-dimensional model spaces with a positive definite inner product. It has been proven that constant vector curvature is well defined in three-dimensions \([2]\). The main result of this research is that all three-dimensional model spaces with positive definite inner products have cvc(e) for some \(e\).

**BACKGROUND**

Definition: \(M = (V, \langle \cdot, \cdot \rangle, R)\) is called a model space if \(V\) is a vector space, \(\langle \cdot, \cdot \rangle\) is an inner product, and \(R\) is a 4-tensor with the same symmetries as the Riemann curvature tensor. Let \(M = (V, \langle \cdot, \cdot \rangle, R)\) be a model space. \(M\) is said to have:

- Constant Sectional Curvature \(e (\text{cvc}(e))\) if all non-degenerate 2-planes have section curvature \(e\).
- Constant Vector Curvature \(e (\text{cvc}(e))\) if for all \(v \neq 0 \in V\) there exists \(w \in V\) such that the section curvature of the 2-plane spanned by \(v\) and \(w\) is \(e\).
- Extremal constant vector curvature \(e\) if \(M\) has cvc(e) and \(e\) is a bound on all sectional curvatures.

If \(\phi\) is a symmetric bilinear form, then define the algebraic curvature tensor

\[
R_\phi(x, y, z, w) = \phi(x, w)\phi(y, z) - \phi(x, z)\phi(y, w).
\]

Let \(v, w \in V\) and \(\{e_1, ..., e_n\}\) be an orthonormal basis for \(V\). The Ricci tensor is a symmetric bilinear form \(\rho\) defined in the following way:

\[
\rho(v, w) = \sum_{i=1}^{n} \phi(e_i, v)\phi(e_i, w).
\]

It is always possible to find a basis for \(V\) such that the eigenvalues of the Ricci tensor are diagonalized and that \(R_{1221} \geq R_{1331} \geq R_{2233}\). In this event all other curvature tensors are 0. For every curvature tensor in three-dimensions there is either exactly one symmetric form such that \(R = R_\phi\) or two symmetric forms such that \(R = R_\phi + R_{\bar{\phi}}\). \([1]\)

**INTRODUCTION**

This flowchart can be used to classify all of the types of constant curvature in three-dimensions based on the properties of a model space’s curvature tensor for model spaces with positive definite inner products.

- \([R_{ijji}]\) refers to the number of distinct values of the three nonzero curvature entries.
- In the \(R = \pm R_\phi\) case \(e = R_{1331}\) and thus refers to the only possible cvc value for a particular model space.
- In the \(R = R_\phi + R_{\bar{\phi}}\) case \(-\lambda_1 = R_{1331}\).

**REFERENCES**


**CVC(0) EXAMPLE**

Consider a curvature tensor with \(R_{1221} = 7, R_{1331} = 4, \) and \(R_{2233} = 3\). The values of this model space’s Ricci tensor can be found below:

\[
\rho_{11} = R_{1221} + R_{1331} = 11 \\
\rho_{22} = R_{1221} + R_{2233} = 10 \\
\rho_{33} = R_{1331} + R_{2233} = 7
\]

In this case \(R = \pm R_\phi\). Also, since \(\text{Rank}(\phi) = 3\) and \(||R_{ijji}|| = 3\) this model space has cvc(e) where \(e = 4\).

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