1. Define the following terms:

   Eigenvalue:

   Eigenvector:

   Characteristic Polynomial:

2. For each of the following matrices (and one other of your own construction with all elements not equal to zero):

a. $\begin{bmatrix} 3 & 2 \\ 1 & -3 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$ c. $\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$ d. $\begin{bmatrix} -1 & 2 \\ -2 & 4 \end{bmatrix}$ e. $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ f. $\begin{bmatrix} -3 & 0 & 5 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$

answer the following questions (if possible):

i. Using **Eigenizer**, determine how many times the vector **x** and A**x** are colinear? When the vector **x** and A**x** are colinear, what is the significance of the value of the eigenvalues in relation to the direction of an eigenvector of A, **x**, and its image **T(x)**?

ii. How many eigenvalues does A have? Are they real or nonexistent? Are they distinct or multiple? How many are positive? negative? zero? Is A invertible? Explain.

iii. Obtain a rough estimate of an eigenvector for each eigenvalue and then reveal the rest of the eigenvalue equation by pressing one of the “Show lamda # equation” buttons. Is the eigenvalue equation a truth within reasonable error? Move the vector **x** so it is no longer an eigenvector, is the new equation a truth? Is the set of eigenvectors linearly independent? Explain.

iv. Compute by hand or by using **MATLAB**, the characteristic polynomial (v=**poly**(A) gives the coefficients of the characteristic polynomial of matrix A, starting with the highest-degree term.), exact eigenvalues (Use the command *k* = **roots**(v) to obtain the roots of the characteristic polynomial of A or use **eig(**A) to accomplish this in **MATLAB**) and exact eigenvectors (use **nulbasis**(A-*k(1))*****eye**(2) and **nulbasis**(A-*k(2))*****eye**(2), etc for larger matrices) can be used to accomplish this in **MATLAB** where *k(i)* is a particular eigenvalue of A) and compare your results to your estimates .

v. If a matrix produced non-existent eigenvalues what did you determine about the eigenvalues from (iv) when you computed them by hand or via **MATLAB**.

a. $\begin{bmatrix} 3 & 2 \\ 1 & -3 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$

c. $\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$

d. $\begin{bmatrix} -1 & 2 \\ -2 & 4 \end{bmatrix}$

e. $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

f. $\begin{bmatrix} -3 & 0 & 5 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$

g.

3. For the matrices:

a. $\begin{bmatrix} 4 & 13 & -5 & -29 & -17 \\ -3 & -11 & 0 & 32 & 24 \\ 0 & -3 & 7 & 3 & -3 \\ -2 & -5 & -5 & 18 & 17 \\ -10 & -2 & -5 & -10 & -11 \end{bmatrix}$ b. $\begin{bmatrix} 7 & 6 & 24 & -2 & 14 \\ 6 & 5 & 18 & 0 & 12 \\ -8 & -6 & -25 & 2 & -14 \\ -12 & -8 & -36 & 3 & -20 \\ 6 & 4 & 18 & -2 & 9 \end{bmatrix}$

use **MATLAB** to find the characteristic polynomial, eigenvalues, and eigenvectors using the commands discussed in (2) and then compare your results when you use the **MATLAB** command: [P D]=**eig(**A). Explain what this new command provides you and how to interpret it.

4. In your own words, explain what an eigenvalue and its corresponding eigenvector has to do with a matrix A.

Bonus: Based upon your experience with (d) in question 2 and additional exploration, state a conjecture about the circumstances under which 0 an eigenvalue of a matrix A and then prove the conjecture. (Hint: your conjecture should have something to do with the invertibility or non-invertibility of A).