3. a) Red
   i) Blue
   Red

2)
   Blue
   Red
   Blue
   Red
   Blue
   Red

3.)
   Blue
   Red
   Blue
   Red
   Blue
   Red
   Blue
   Red
   Blue
   Red
   Blue
   Red
   Blue

4.) chromatic number is 2 for a cycle with an even number of vertices.

b.)
   1.) Blue
   Red
   Green

2.) Blue
   Red
   Green
   Blue
   Red
   Green
   Blue

chromatic # = 2

chromatic # = 2

chromatic # = 2

chromatic # = 3

chromatic # = 3
3.) b)

3.)

chromatic # = 3

4.) chromatic number is 3 for a cycle with an odd number of vertices.

5.) chromatic number would have to be at least three.

Exercise Set 4.6

2)

chromatic # 2

4.)

chromatic # 4

6.)

chromatic # 3
Exercise Set 4.6

10.) a.)


b.)

c.)

11.) a.)


b.)

c.)
Exercise Set 5.1

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Indegree</th>
<th>Outdegree</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>b</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>c</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>d</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>e</td>
<td>3</td>
<td>0</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>vertex</th>
<th>Indegree</th>
<th>Outdegree</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>q</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>r</td>
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<td>1</td>
</tr>
<tr>
<td>s</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>t</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>u</td>
<td>1</td>
<td>1</td>
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</table>

4. is weakly connected, there are more than two vertices for which the indegree does not equal the outdegree. Does not have an Euler Path nor Euler Circuit.

5. is weakly connected, there are more than two vertices for which the indegree does not equal the outdegree. Does not have an Euler Path nor Euler Circuit.
Exercise Set 5.1

10.)

<table>
<thead>
<tr>
<th>vertex</th>
<th>indegree</th>
<th>outdegree</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>b</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>c</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>d</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>e</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

In degree equals the out degree for each vertex, so graph has an Euler Circuit.

\[(a, d), (d, e), (e, b), (b, a), (a, e), (e, d), (d, b), (b, c), (c, b), (b, d), (d, c), (c, d), (d, a)\]

14.)

b.) All are winners because each has a hamilton path that begins with themselves.

c.) No, we also know this because the graph has simple circuits, and by theorem 4 there cannot be a unique winner.
Exercise Set 5.1

14.) d.) There can only be a unique ranking when there is a unique Hamilton path.

Exercise Set 5.2

1.) yes
2.) No, has a cycle
3.) No, not connected
4.) yes
5.) yes
6.) No, not connected

<table>
<thead>
<tr>
<th></th>
<th>vertices</th>
<th>edges</th>
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<tbody>
<tr>
<td>#1</td>
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<td>4</td>
</tr>
<tr>
<td>#4</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>#5</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

confirms theorem (6)

10.)
   a.) Asst. Superintendent; Instruction
       Superintendent
   b.) Asst. Superintendent; Instruction
   c.) Director; Budget
       Director; Transportation
   d.) Superintendent
19. a.) By the definition of a cycle, the simple circuit is a cycle.
   b.) Since \( y_1 = v_j \), only one vertex is repeated where it is the initial and terminal. Therefore \( \mathcal{C} \) is a cycle.
   c.) There are only a finite number of vertices.
   d.) There are no repeated vertices in the simple circuit produced by the process other than the initial and terminal vertices.

20. a.) There is a unique simple path between each pair of vertices.
   b.) Because both are equal to \( y \), because both are equal to \( x \).
   c.) There is supposed to be two unique paths. \( t \geq 1 \) because \( u_1 = u_i = x \).
   d.) The path has to end at \( y \).
   e.) Because \( w_1 \) is only vertex repeated, and no edge has been repeated.
   f.) Because there is a simple circuit where the initial and terminal vertex is the only repeated vertex then there is a cycle. However, a tree is a connected graph with no cycles. So, contradiction.