Seeing and Understanding Data

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1 Introduction

Visual displays of data are commonly used today in media reports online or in print. For example, data visualizations are sometimes used as a marketing tool to convince people to purchase a certain product, or they are displayed in articles or magazines as a way to graphically display data to emphasize a certain point. In general, it is hard to imagine the majority of disciplines of science and mathematics not using data visualizations. However, before standard data visualization techniques were developed (and accepted by the community), mathematicians and scientists very rarely used graphical displays or pictures to represent empirical data.

This project has four main parts. The first section introduces some of the earliest data visualizations, which were novel constructions in their time. Next, we consider works of Michael Florent van Langren (1598–1675) and William Playfair (1759–1823) that contain the first known uses of statistical representations of data, some of which are still used today. The third section focuses on the work of Florence Nightingale (1820–1910) and Charles Joseph Minard (1781–1870) and their ability to construct data displays that made an argument or told a story. Finally, in the last section we consider works of Edward Tufte (1942–) and Hans Rosling (1948–2017) that had a major impact on the current field of data visualizations and how to best create them using computer software.

2 "Ancient" Visualization

One of the earliest data visualizations in printed form is a times-series graph (Figure 1) from the late 10th or early 11th century that shows changes in the orbital positions of seven "planets"[1] over time and space. It appears in an appendix to a commentary on a work of Cicero (106–43 BCE) that reviews the physics and astronomy of the day. Take a few minutes to look at the graph below, then discuss with a partner the following questions.

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[1] At the time this graph was drawn, the term *planets* was used to describe heavenly bodies that seemed to "wander" (moving relative to the background of stars) and could be seen with unaided eyes. Therefore, the sun and moon would be called planets.

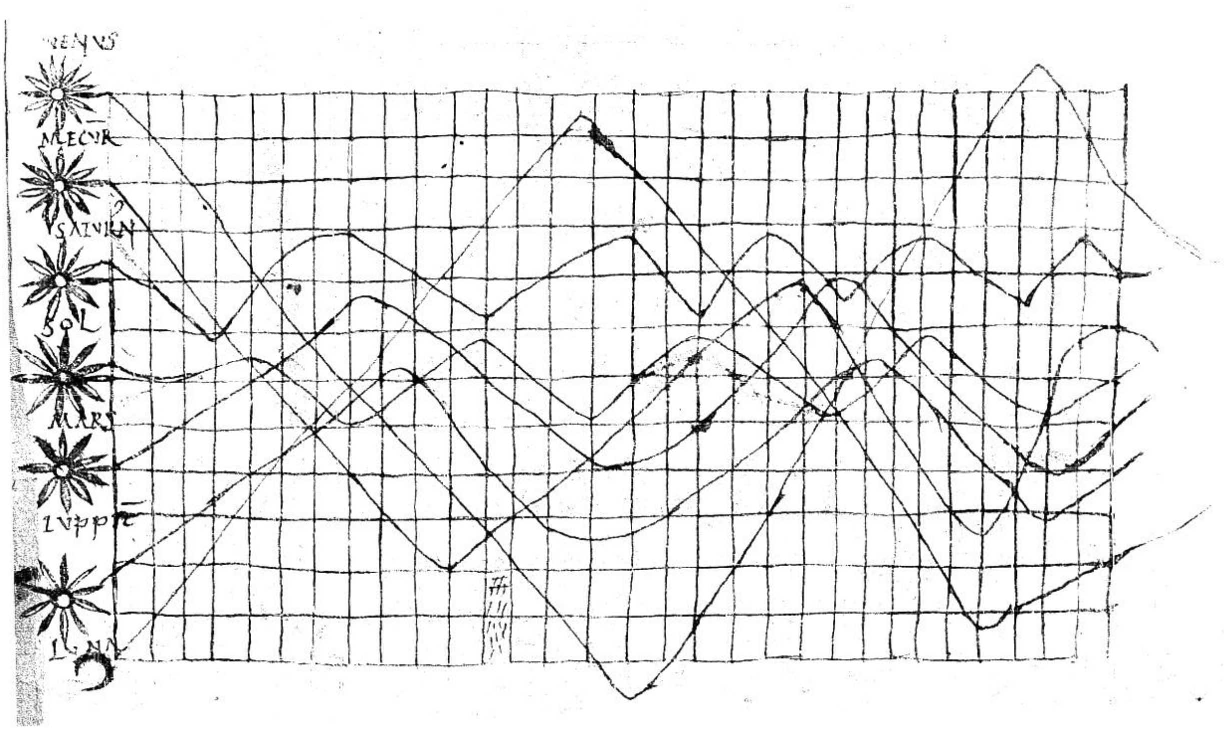


Figure 1. Time-series graph from the 10th or possibly 11th century [Unknown, 1010]

Task 1

(a) As you can see, the image has no labels on the x- or y-axis. What do you think the x-axis and y-axis represent?

(b) This graph is novel because a coordinate system was used to plot the various changes in orbits for the seven planets over time. Why do you think that a coordinate system was needed for this graph?

Now read the following discussion of this graph, written by a twentieth-century mathematician and historian.

The graph apparently was meant to represent a plot of the inclinations of the planetary orbits[2] as a function of the time. For this purpose the zone of the zodiac was represented on a plane with a horizontal line divided into thirty parts as the time or longitudinal axis. The vertical axis designates the width of the zodiac. The horizontal scale appears to have been chosen for each planet individually for the periods cannot be reconciled. The accompanying text refers only to the amplitudes. The curves are apparently not related in time. [Funkhouser, 1936]

Task 2 After reading Funkhouser's discussion of the graph in Figure 1, does your interpretation of the x- and y-axis change or stay the same? If you changed your interpretation, in what way?

3 Communicating Data Visually

In the mid-15th century, the invention of the moveable-type[3] printing press by Johannes Gutenberg had a large impact on the Renaissance, in that information was able to be spread throughout European civilization quickly and accurately compared to news previously carried by word of mouth. With the moveable-type printing system, components of a document that was composed of text were much more easily reproduced. However, the printing of graphical displays rather than summary tables or lists of numbers was much more difficult because the graphics did not have common components like the moveable type. Instead, a new graphical display had to be created for each printing which was very expensive, which in turn limited their use for several more centuries.

[2] The inclination of a planetary orbit is the angle between the horizon and the astronomical body being observed. The website `https://stellarium.org/` can be used as a virtual planetarium that you can use to find "star inclinations" from anywhere on Earth.

[3] This is a system of printing documents on paper using moveable components that are individual characters, numbers, and punctuation marks. Earlier versions of the moveable-type printing press made from wood, clay, porcelain, or bronze materials were known in China and Korea. Gutenberg's durable alloy and mold technique made his press commercially viable.

3.1 Distance Graph by van Langren

In 1644, Michael Florent van Langren (1598–1675) created a graph showing determinations of the distance (measured in degrees of longitude) from Toledo, Spain to Rome, Italy. Statisticians and historians credit this graph as one of the first visual representations of statistical data. Van Langren served as an astronomer to the royal house of Spain. Although astronomers at that time were able to determine latitude from star inclinations, longitude was much harder to determine precisely. Finding an accurate method for determining longitude was of political and economical value to Spain and other European nations interested in navigation at sea. Van Langren prepared his graph as part of a request to the Spanish court for financial support of his own efforts to solve the longitude problem.

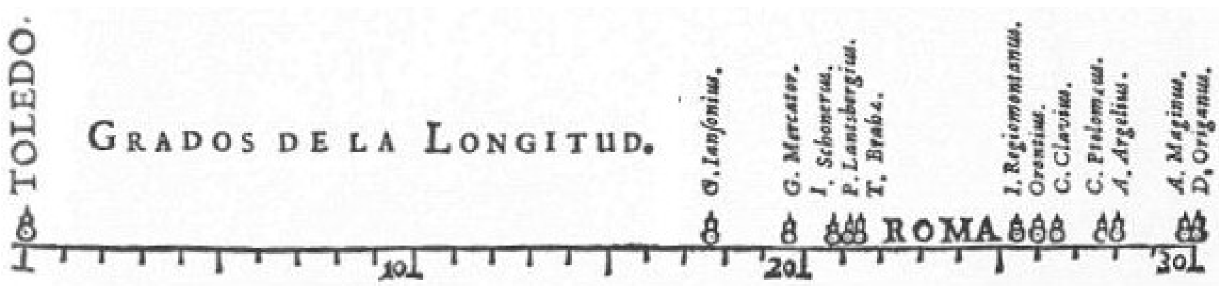


Figure 2. Graph of the distance from Toledo, Spain to Rome, Italy by Michael Florent van Langren [van Langren, 1643]

Task 3

(a) Take a few minutes to look at the graph in Figure 2. What do you think the different data points with names on the graph indicate?

(b) What do you gather about the determination of distance in longitude from Toledo to Rome based on the spread of data points? What do you notice about the data points and their relation to each other?

Task 4

Activity: As a class, choose two cities near your college or university that you would like to estimate the distance between. Then, have everyone in the class write their estimate in miles between the two cities on a piece of paper (rounding to the nearest tenth of a mile) along with their name. Have one or two students write all the estimates on the board as students call-out their guesses.

(a) In groups of three to four students, create a graph similar to van Langren's using the class estimates for the distance from the chosen city A to chosen city B.

(b) Then combine pairs of the groups (total of six to eight students) and discuss each group's graph. What are the similarities between the two graphs? Are there any differences? What does the spread of data points look like on each graph?

3.2 Common Statistical Graphs by Playfair

Historians of statistics consider William Playfair (1759–1823) to have been the first developer of many common statistical graphics used today, including the pie chart, the bar chart, and the statistical line graph. Through these inventions he was able to create a universal common language that was used from science to commerce as a way to understand and look at data. The bar graph below is from his 1786 book entitled *Commercial and Political Atlas*. Playfair's intention in this book was to represent data about the import/export of many countries that were prominent in foreign commerce at the time. The atlas did not have much success in England, but was very well received in France. Playfair reported in regard to King Louis XVI of France that "As his majesty made Geography a study, he at once understood the charts and was highly pleased. He said they spoke all languages and were very clear and easily understood" (as quoted in [Spence and Wainer, 2001, p. 110]). The graph below is for the country of Scotland for one year.

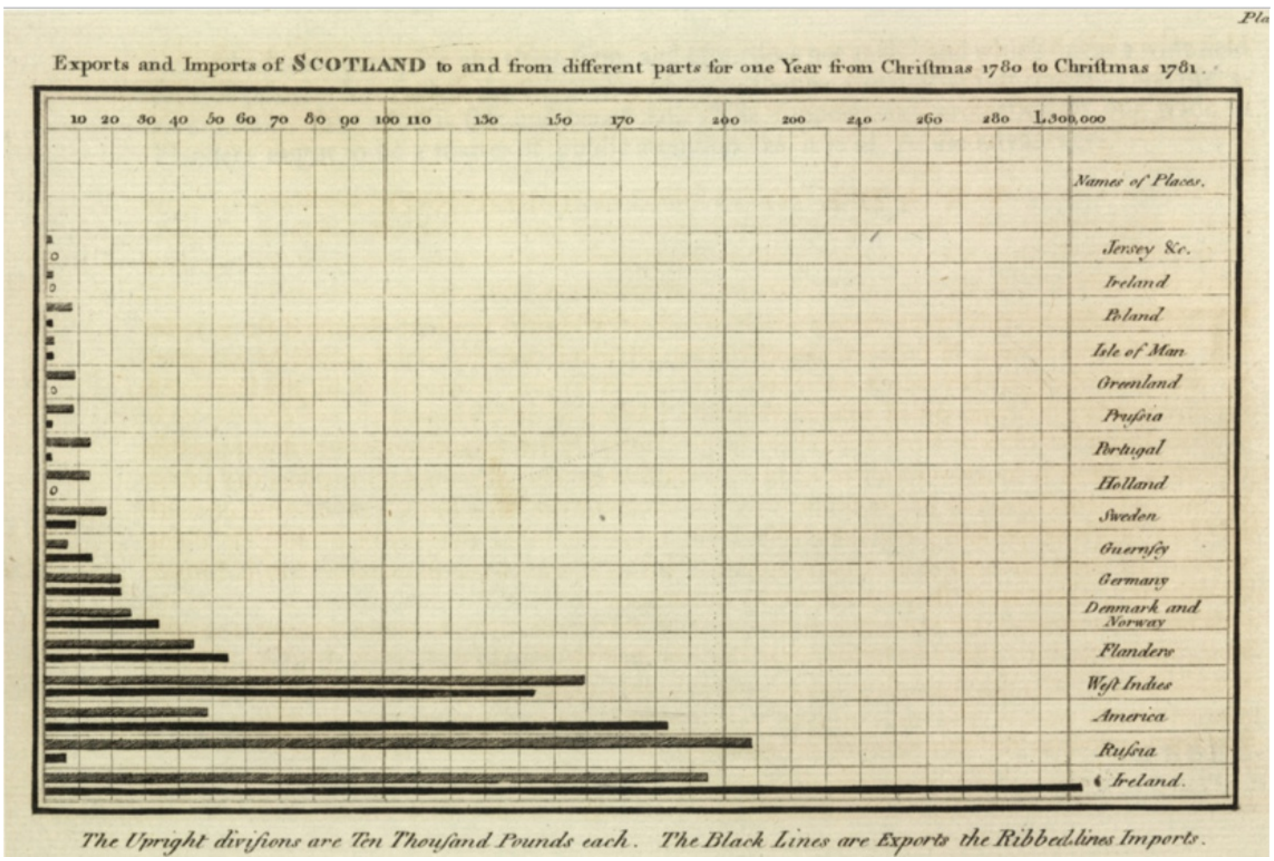


Figure 3. Playfair Bar Graph [Playfair, 1786]

Task 5 Compare and contrast this bar graph with the bar graphs we use today in the newspaper and other media. What do you see that is different between the bar graph in Figure 3 compared to a bar graph today? Does how we interpret the bar graph in Figure 3 differ from how we interpret bar graphs made today?

The following graph is a pie chart that was also created by William Playfair. Playfair included this graph in his 1805 translation of D. F. Donnant's 1802 French text, titled *Statistical Account of the United States of America*.

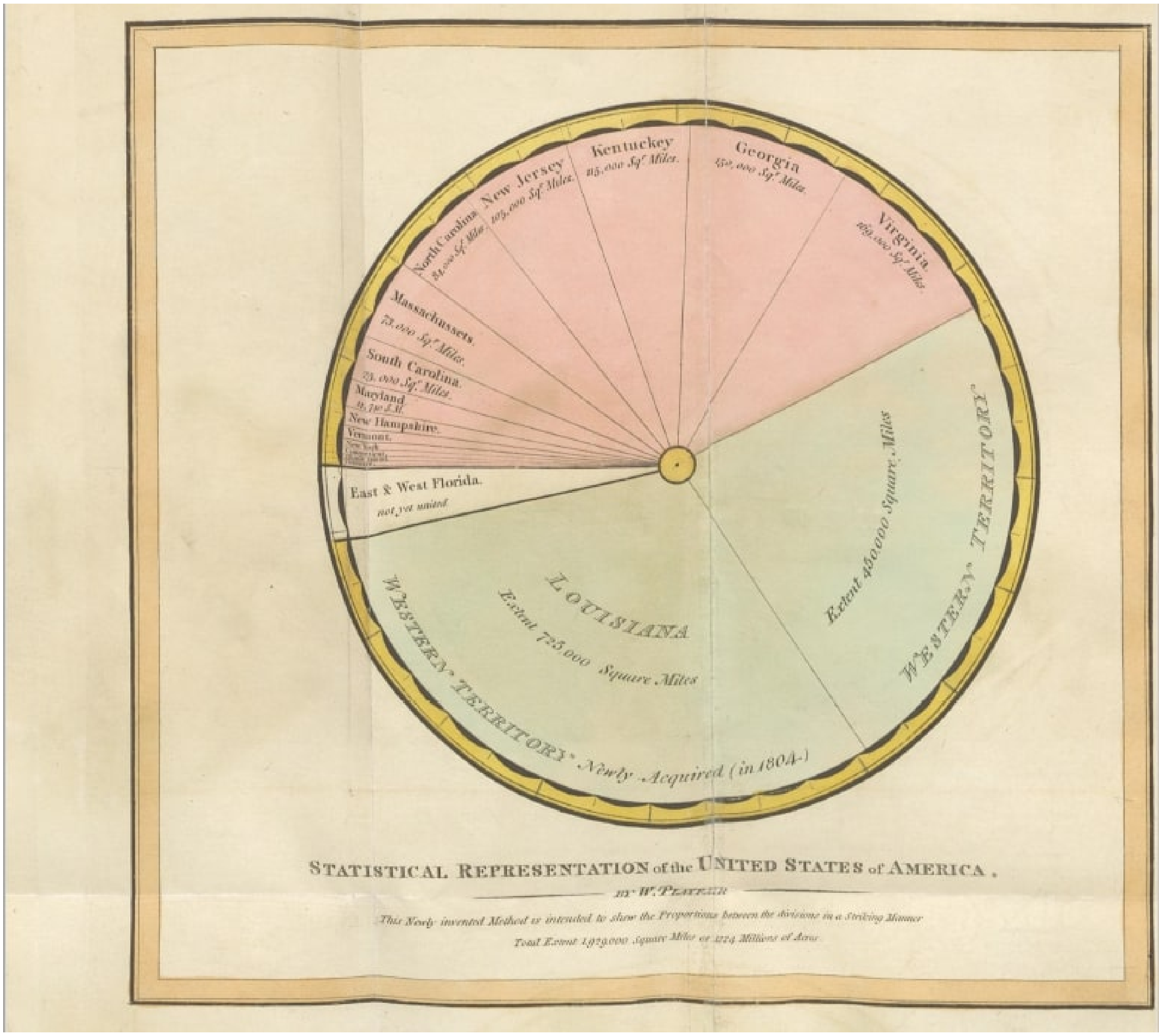


Figure 4. Playfair Pie Chart [Playfair, 1805]

Task 6 Given that this pie chart was created in 1805, what do you think each slice of the pie chart represents?

Task 7 A map of the United States in 1804 is shown below. Why do the proportions represented by the slices of the pie chart in Figure 4 look different than how we would expect those of a pie chart of the present day United States to look?

Figure 5. Map of the United States in 1804 [Golbez, 2006]

The following circle graph was created by William Playfair and was published in his book titled *Statistical Breviary: Shewing, on a Principle Entirely New, the Resources of Every State and Kingdom in Europe* that was published in 1801.

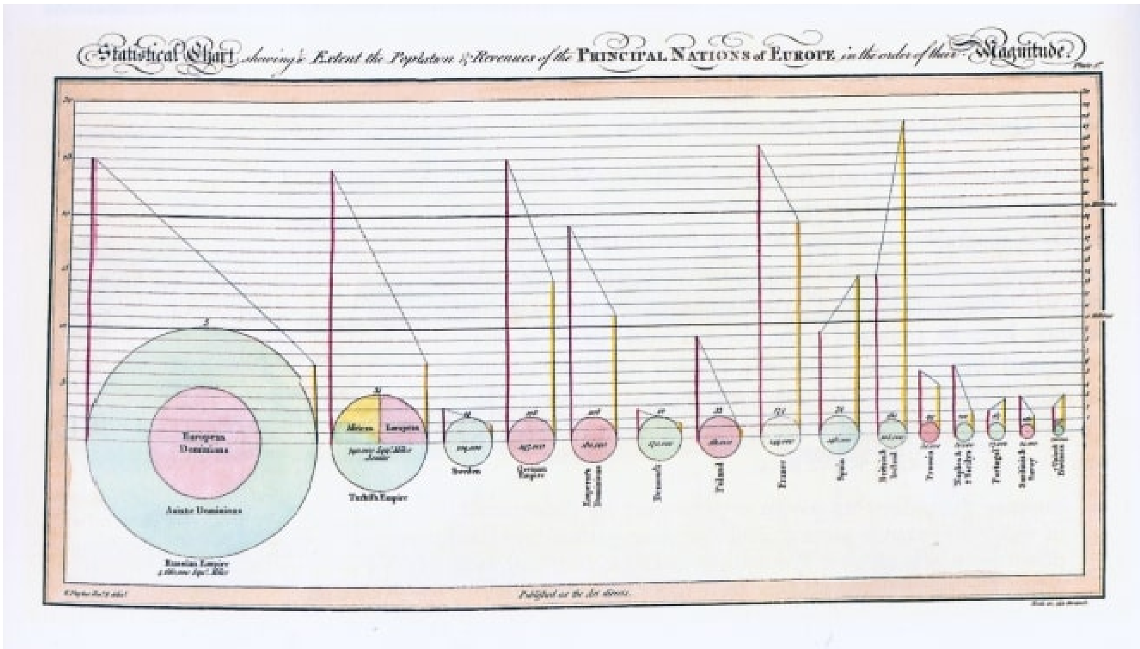


Figure 6. Playfair Circle Graph [Playfair, 1801]

Task 8

(a) Now this circle graph is a complicated graph. Take about five minutes to study the graph taking note that the axis on the left side is on a different scale compared to the axis on the right side of the graph.

  (i) What does each circle represent?

  (ii) Does the size of each circle mean something?

  (iii) What does the axis on the left-side represent?

  (iv) What does the axis on the right-side represent?

  (v) What do each of the lines on the left and right of each circle represent?

  (vi) Why is there a line connecting the left and right line for each circle?

(b) Have you seen any recent data displayed in this fashion? If so, do you recall the subject matter? If not, why do you think we don't use this type of graph anymore?

4 Making an Argument by Telling a Data Story

Playfair's work brought data to a much wider audience than just mathematicians used to working with large collections of numbers or summary tables. It was all about describing social and economic conditions without overwhelming the reader with lists of numbers. His ideas were overlooked by his contemporaries, but they would be revisited in the 19th century with greater success.

4.1 Graphs by Florence Nightingale

The well-to-do parents of Florence Nightingale (1820–1910) knew there was something "wrong" with their youngest daughter, who preferred studying mathematics to dancing and had no interest in accepting the several offers of marriage made to her. Victorian high society would condone no other occupation for a young lady than keeping a husband's house (through the labor of servants) and raising his children. Nightingale, however, longed to care for the sick and related her unusual desire for public service to a religious calling. Nineteenth-century British society considered nursing a degrading occupation, often associated with drunkenness, squalor, and promiscuity. After years of denials from her mother, Nightingale took an important step toward fulfilling her dream by going to the Deaconess School at Kaiserwerth in Germany to train as a nurse in 1851. She spent a year as the superintendent at the Institute for the Care of Sick Gentlewomen, probably the only place she could ever have worked with the consent of her ever-proper mother.

The outbreak of war in the Crimea presented an opportunity for Nightingale to combine her unconventional desire for occupation, medical training, and social connections in a patriotic cause. When the first reports from the Battle of Alma included descriptions of the disastrous state of the field hospitals, Nightingale wrote a letter to the Minister of War (a social acquaintance) volunteering her services as a professional nurse, which actually crossed in the mail with his request to her to lead a party of nurses to the Crimea. What she and her nurses found upon their arrival at Scutari exceeded even the most shocking newspaper reports. In addition to directing the nurses in her charge regarding the nutrition and hygiene for the thousands of wounded soldiers, she organized the chaotic administrative records and collected data that would change the British and, subsequently, the United States' Army hospitals.[4]

The Minister of War once again wrote to Nightingale after her return to Britain, soliciting her opinions on the state of the Army hospitals. Her response was more than 500 pages long and included a striking illustration of the data she had gathered at the city of Scutari (Figure 7).

[4] This data collection became the basis for her report to the Minister of War which earned her an honorary fellowship in the American Statistical Association in 1874. She was also the first woman elected as a fellow in the Royal Statistical Society in 1859.

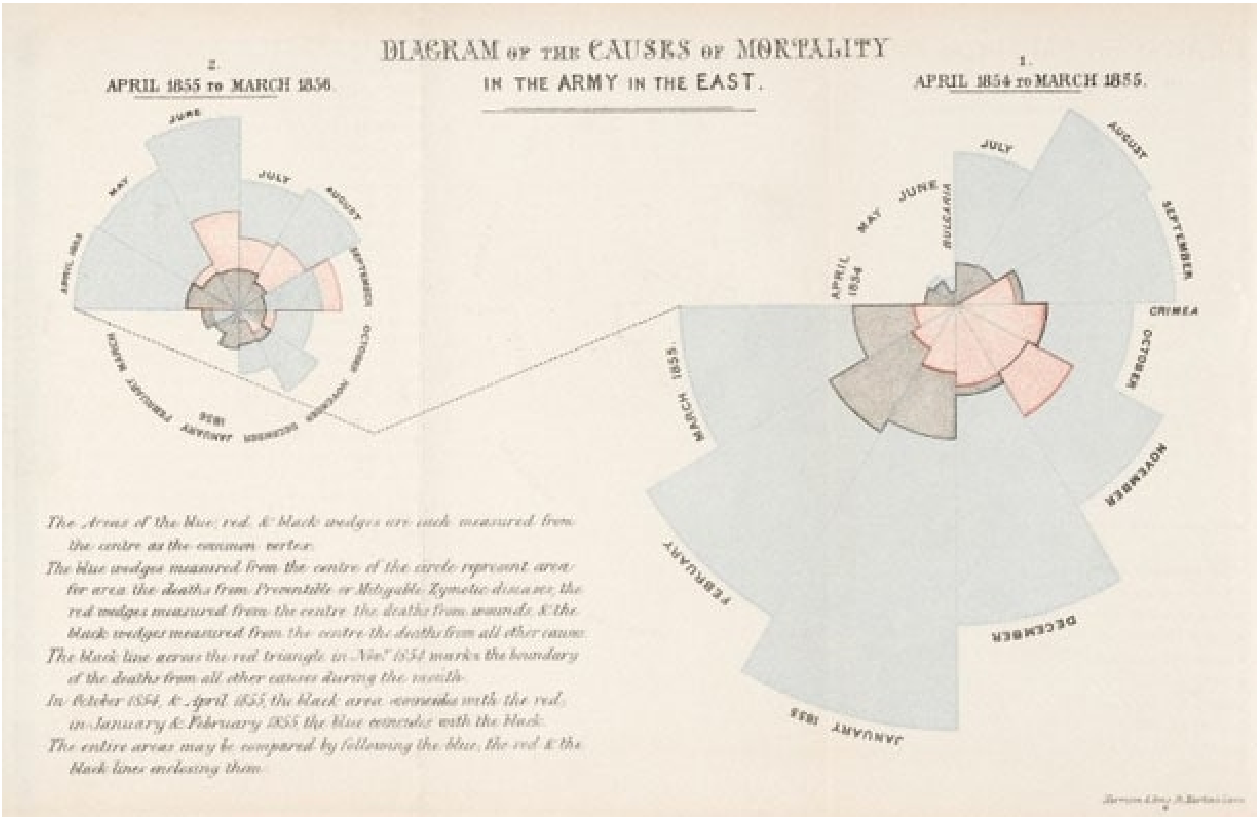


Figure 7. Florence Nightingale's "coxcombs" [Nightingale, 1858]

The text in the bottom left of the illustration serves as a legend. Within each wedge of this odd pie chart (also called a polar area diagram or rose diagram), the area is computed from the common center. The blue represents deaths by preventable diseases, the red represents deaths resulting from wounds, and the black represents deaths by all other causes. The left circle displays the deaths of soldiers in the first year of the war and the right one shows the deaths in the second year.

Task 9

(a) Compare the wedges for the same months in the two years, paying particular attention to the proportion of each color within the wedge. Consult a timeline of the major battles and discuss how they align with the size of wedges in either year.

(b) What data did Nightingale need in order to calculate the areas to draw and to shade? She did not have a computer to calculate or draw this illustration for her. Speculate on the tools she needed in order to draw these illustrations.

(c) Have you seen any recent data displayed in this fashion? If so, do you recall the subject matter? If not, can you speculate on a modern issue that *could* be displayed like this?

4.2 Graph by Charles Minard

Arguably one of the most elegant displays of data ever produced is Charles Joseph Minard's (1781–1870) *Figurative map of the successive losses in men of the French Army in the Russian campaign 1812–13* shown below.

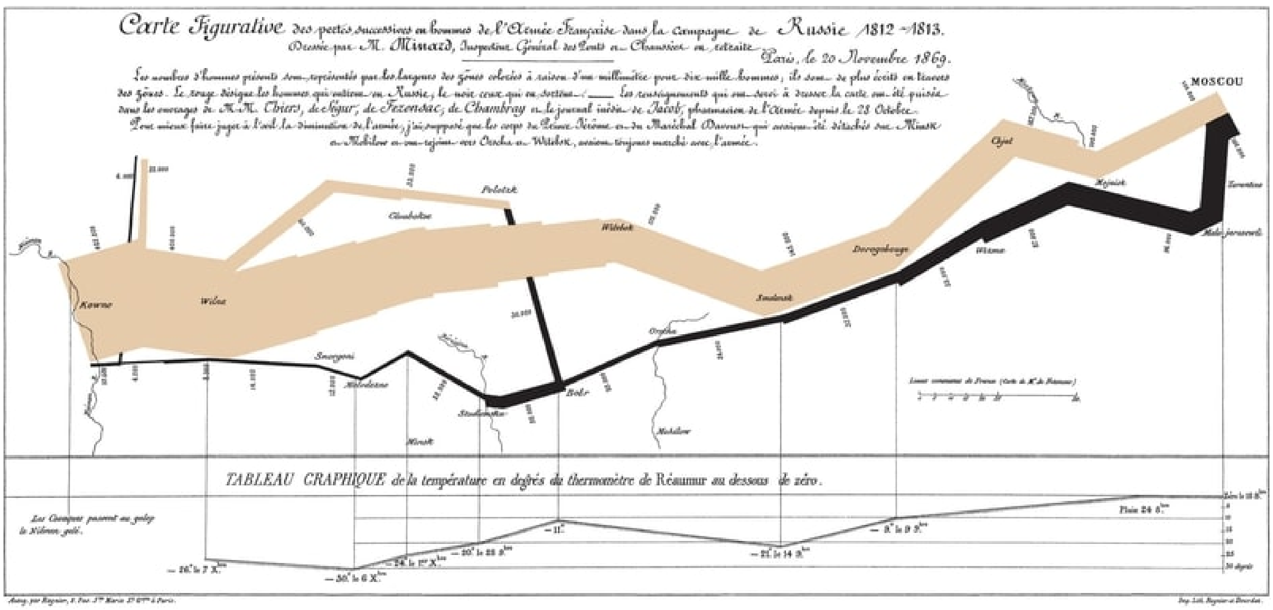


Figure 8. *Carte figurative des pertes successives en hommes de l'Armée Française dans la campagne de Russie 1812–1813* by Charles Joseph Minard [Minard, 1869]

Minard was a civil engineer by training and trade during the Napoleonic era in France (1799–1815), and retired from government service in 1851 as the Inspector General of Bridges and Roads. In his retirement, he created a collection of visualizations that culminated in this astounding piece drawn in 1869 and still admired today. Here is a link to a larger version of the graph: `https://upload.wikimedia.org/wikipedia/commons/e/e2/Minard_Update.png`

Task 10

(a) Count and describe the variables that Minard included in this visualization.

(b) The combination of so many variables on a two-dimensional representation is part of this display's long-lasting appeal. Compare your list of variables with those of classmates and discuss why each variable was important to Minard in the telling of this data story of the French Army.

You probably cannot read the tiny print of the legend at the top of the graph but it makes no mention whatsoever of Napoleon! Minard's interest was in the soldiers who suffered in this misguided march. In the graph's legend, he also gave credit for the sources of his data and stated that the scale on the bands is one millimeter for every 10,000 men.

5 Beyond Paper and Ink

Nightingale and Minard provided examples of beautiful data displays with their laboriously constructed visualizations that told a story almost without words. As the utility of statistics expanded through various sciences over the course of the 19th and early 20th centuries, refinement of Playfair's simpler constructions led to the almost ubiquitous use of data displays that have become standard in both public and scholarly publications: bar chart, pie chart, histogram, line chart, and scatter plot.

5.1 Tufte's Principles of Graphical Excellence

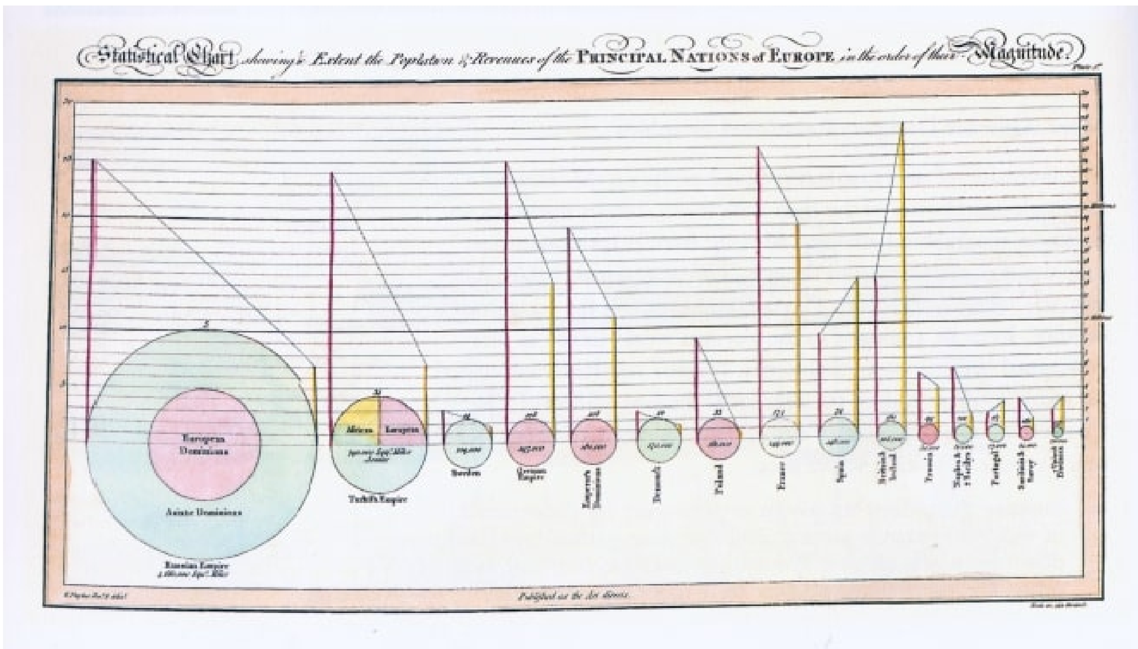
In the preface to the second edition of *The Visual Display of Quantitative Information*, Edward Tufte (1942–) describes the genesis of the first edition as part of a seminar series with John Tukey (1915–2000) at Princeton University. Tukey's interest in "exploratory data analysis" focused on easy-to-construct (by hand after minimal arithmetic) displays of data to complement statistical analysis. Two such displays are still commonplace: the boxplot and the stem-and-leaf plot.

In the 2001 edition of *The Visual Display of Quantitative Information*, Tufte also repeats the nine qualities of graphical excellence from the groundbreaking 1983 edition [Tufte, 2001, p. 13]:

> Excellence in statistical graphics consists of complex ideas communicated with clarity, precision, and efficiency. Graphical displays should
>
> - show the data
> - induce the viewer to think about the substance rather than about methodology, graphic design, the technology of graphic production, or something else
> - avoid distorting what the data have to say
> - present many numbers in a small space
> - make large data sets coherent
> - encourage the eye to compare different pieces of data
> - reveal the data at several levels of detail, from a broad overview to the fine structure
> - serve a reasonably clear purpose: description, exploration, tabulation, or decoration
> - be closely integrated with the statistical and verbal descriptions of a data set.
>
> Graphics *reveal* data. Indeed graphics can be more precise and revealing than conventional statistical computations.

Task 11 Review each of the seven figures shown in the previous sections, choosing two about which to write a paragraph comparing the data display with Tufte's list of graphical excellence qualities.

Another idea initiated by Tufte is “data-ink ratio.” Data-ink is the part of the graph that cannot be erased without loss of data. For example, the Playfair Circle Graph in Figure 6 (repeated below) includes more gridlines than necessary to communicate the data story, thereby using too much ink on non-data.



Playfair Circle Graph [Playfair, 1801]

Task 12 Keeping in mind all nine of the hallmarks of graphical excellence, review the other data displays in this project for ink that has been spent on non-data. Write a short description of each to describe the data-ink vs. non-data ink expended in the graphic. Support an opinion of the “best” graphic with reference to both graphical excellence and data-ink.

5.2 Data visualizations of the 21st century

Informative and compelling displays of data in the 21st century are far less likely to require actual ink; instead, we might be concerned about data-pixels. The challenges of printing graphics on a press have been largely overcome as print has lost its place as the primary medium for information exchange. The demand for digital rather than physical pages has paralleled the technological advancements that have made the visual experience on a monitor nearly equivalent to the printed page.

Numerous software packages that include the production of standard statistical displays offer nearly universal opportunity for users to create visual displays of data. Anyone can make a graph; the question of quality, however, remains.

In the early 2000s, a new level of sophistication arrived to include more than bivariate data and even larger data sets. A pioneer (and engaging showman) in this area was Hans Rosling (1948–2017), who brought innovation to the display of public health data intended for a general audience.

Task 13

(a) Watch Rosling's *200 Countries, 200 Years, 4 Minutes* at https://youtu.be/jbkSRLYSojo.

(b) Visit the Gapminder tool https://www.gapminder.org/tools/#$chart-type=bubbles online to re-play Rosling's visualization at your own speed. Explore the data for a single country by selecting it in the rightmost column. Write a paragraph about the story of the world when the dynamic data includes all countries and another paragraph about how the one country you watched fits or deviates from that story.

6 Conclusion

From the days when mathematicians preferred long lists of numbers to standard graphs at the click of a mouse (or tap of a screen), visualizations have transformed the way we communicate data. King Louis XVI's response to Playfair's work still rings true in the 21st century: "they [speak] all languages and [are] very clear and easily understood" (as quoted in [Spence and Wainer, 2001, p. 110]). One need not be a mathematician, statistician, or king to see and understand the story that data has to tell.

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Michael Florent van Langren. Graph of statistical data, showing the wide range of estimates of the distance in longitude between Toledo and Rome, 1643. URL `https://en.wikipedia.org/wiki/Michael_van_Langren/media/File:Grados_de_la_Longitud.jpg`. [Online; accessed November 2, 2018].

Notes to Instructors

PSP Content: Topics and Goals

Many textbooks for Introductory Statistics (majors and non-majors alike) continue to include chapters that “introduce” visual displays of data focused on the charts and graphs common in publications. For decades, elementary school students have been constructing pie charts and histograms and middle school students have added box plots and scatter plots to their repertoire. This Primary Source Project offers an alternative to more of the same (though doesn’t necessarily preclude traditional practice) through the investigation of early attempts to communicate data in pictures. The displays that have been chosen are pre-cursors to the common displays that can now be created with a few clicks in a computer program; thus, they can serve as a novel approach to data organization techniques with which students may already be familiar.

The suggested courses for this PSP are introductory statistics courses at all levels and introductory data science courses. It is also suitable for use with mathematics education students and general student audiences enrolled in courses that include some treatment of data displays.

Student Prerequisites

The student prerequisites for this PSP are none. This PSP is designed to be available to all students and can be used as a way to refresh students’ memory or reinforce concepts regarding graphs that students would have learned in middle and high school.

PSP Design, and Task Commentary

The PSP is broken down into four sections. The notes below include suggestions or tips for implementing each of the sections.

- **Section 2:** “Ancient” Visualization

  The 10th century graph was novel in its time because the Cartesian coordinate system was not invented until seven centuries later (named for Rene Descartes but used by Pierre de Fermat decades earlier; neither man used axes in the way that we do today). The coordinate system also allowed plotting of more than one line on the graph as a means to compare the seven planets.

- **Section 3.1:** Distance Graph by van Langren

  The data points each represent a different estimate of the distance in longitude between Toledo and Rome. The names above each data point indicate the name of each astronomer (Mercator, Tycho Brahe, Ptolemy, etc.).

  The purpose of the tasks in this section is to have the students understand that there is variability in the estimates of the distance in longitude between Toledo and Rome made by each of the astronomers. Also, there is wide variation in that the estimates span about half the length of the scale [Friendly, 2006]. Another reference for learning more about this statistical graph is an article that discusses the issues of longitude at the time [Friendly et al., 2010].

  In-class activity: Make sure that the students do not use an app for their estimate. The students should have a “productive struggle” in trying to estimate the distance between the two cities without GPS.

Optional activities:

1. This activity can also be done having students look up the distance between the two cities using Google Maps. This would allow for more accurate estimates of the distance but there would still be variation based on where students place the pins on the map.
2. This activity can also be done as a homework activity by having students drive the distance between the two cities and record their distance using their mileage reading. You would have variation between the estimates based on the different start and stop locations the students would choose.

- **Section 3.2:** Common Statistical Graphs by Playfair

  We want the students to see that the scale of the heights of the bars is at the top of Figure 3 rather than on the left side of the graph where we are normally used to seeing the y-axis of a bar chart. Also, the labels for what each bar represents are on the right side of the bar graph and we are normally used to seeing these labels on the x-axis. Even with the labels for the bar graph in Figure 3 in a position we're not used to, we still interpret the bar graph in the same way we would a bar graph created in present day.

  We want students to understand that each slice of the pie in Figure 4 represents the total land mass for that state or territory proportional to the total size of the whole country. Also, we want students to understand that in 1805, the United States did not yet have 50 states, and that the pie chart has a large portion that represents the Louisiana Territory.

  Optional Activity for the Pie Chart: Have the students collect the total land mass for each state in the present-day United States. Then, have the students take the data that they have collected and create their own pie chart. The class can then compare the pie chart the students create and the pie chart by Playfair. An online version of the Pie Chart by Playfair can be found at https://www.researchgate.net/publication/228401311_No_Humble_Pie_The_Origins_and_Usage_of_a_Statistical_Chart, on page 364 of the article.

  In Figure 6, Playfair drew the circles proportional to the geographic area of the nations. The vertical line tangent to the left of the circle is scaled along the left of the graph for the population of that nation. The other vertical line is scaled along the right edge of the graph for the nation's tax revenue. The sloped line connecting them illustrates the varying tax burden to the citizenry of these nations. An online version of the Circle Graph by Playfair can be found at this link: https://commons.wikimedia.org/wiki/William_Playfair#/media/File:Playfair_piecharts.jpg.

- **Section 4.1:** Graphs by Florence Nightingale

  Consider providing some geo-political information about the Crimean war, particularly the dates of the major battles. Optional activity 1 (below) suggests that students collect such facts ahead of this lesson.

  A compass and straightedge may spring to the mind of any student (or instructor) who has studied Euclid. Nightingale almost certainly would have had access to a protractor as well. The National Museum of American History (url: http://americanhistory.si.edu/collections/object-groups/protractors) collection holds some protractors as old as the early 1700s and describes much earlier documentation of the use of protractors in the making of 16th-century maps. Examples of other early modern drawing instruments that Nightingale would have used to draw her graphs can be seen at this link: https://www.maa.org/press/periodicals/convergence/mathematical-treasure-early-modern-drawing-instruments.

An online version of the "coxcomb" graph by Nightingale is here: https://en.wikipedia.org/wiki/Pie_chart#/media/File:Nightingale-mortality.jpg.

Optional activities for the "coxcomb":

1. As homework, have the students individually research the Crimean War and how Florence Nightingale got the epithet "Lady with the Lamp". Compile the facts within student groups or as a class.
2. To add a humanistic perspective, extend the research to the poem "Charge of the Light Brigade" by Lord Alfred Tennyson and the painting "The Lady with the Lamp" by Henrietta Rae.
3. The improvement of sanitation and hygiene in Army hospitals was not the end of Florence Nightingale's efforts to use data to effect change on public health issues. Research other areas in which she fulfilled her calling: nursing as a profession, sanitation in the British colonies, design of hospitals, midwifery, and trained nurses in the workhouse system.

- **Section 4.2:** Graph by Charles Minard

  Consider providing a larger version of the "map" from Figure 8 in class or projecting one in the classroom. A modern redrawing (in English) is available at https://upload.wikimedia.org/wikipedia/commons/e/e2/Minard_Update.png. The Wikipedia entry for Charles Minard has a clean graphic (in French) that could be projected. Beware that enlarging the image comes with a caption that gives away the answer to the first question of Task 10. Quickly switch to the full-screen display for both additional size and to hide the caption.

  Google Earth or even just a map of Eastern Europe may help to contextualize the distance component of the Army's travel. You might compare the longitude and latitude information to where you are located to make the date and temperature connection.

  An optional activity in this section would be to add John Snow's map of cholera outbreak that influenced the growing ideas about germ theory; see https://en.wikipedia.org/wiki/John_Snow.

- **Section 5** Beyond Paper and Ink

  Depending on the availability of internet resources in your classroom, these activities may have to be homework. For Task 13(b), you may wish to ensure some variety in the selection of a country not in the first dozen visible in the list. There is at least one country for every letter of the alphabet (except W) so matching first or last name could diversify the selections. Alternatively, the students/groups could be randomly assigned a country. Over time, you may find a subset of interest (politically, historically, statistically, or meaningful to your location and students) for use in future implementations.

Suggestions for Classroom Implementation

This PSP can be implemented in two or three class periods, depending on the length of class session and how much is assigned for out-of-class work. Day 1 might include the introduction and first two sections, possibly assigning Subsection 2.2 as homework. Sections 3 and 4 could be finished in a day. Note that Subsection 4.2 requires computer access and may be best for individual homework, though it could be done in small groups if some in-class access is available.

LaTeX code of this entire PSP is available from the authors by request to facilitate adaptations to meet individual instructor goals for the course. Examples of adaptations include slide presentations, reading guides, homework assignments, and in-class worksheets based on the tasks presented in this document.

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