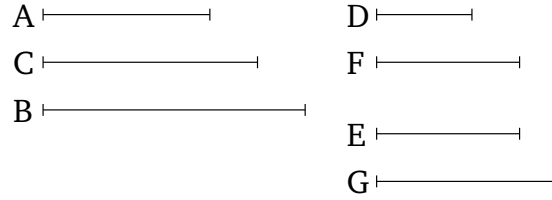


Book 8

Proposition 20

If one number falls between two numbers in mean proportion then the numbers will be similar plane (numbers).

For let one number C fall between the two numbers A and B in mean proportion. I say that A and B are similar plane numbers.



[For] let the least numbers, D and E , having the same ratio as A and C have been taken [Prop. 7.33]. Thus, D measures A as many times as E (measures) C [Prop. 7.20]. So as many times as D measures A , so many units let there be in F . Thus, F has made A (by) multiplying D [Def. 7.15]. Hence, A is plane, and D , F (are) its sides. Again, since D and E are the least of those (numbers) having the same ratio as C and B , D thus measures C as many times as E (measures) B [Prop. 7.20]. So as many times as E measures B , so many units let there be in G . Thus, E measures B according to the units in G . Thus, G has made B (by) multiplying E [Def. 7.15]. Thus, B is plane, and E , G are its sides. Thus, A and B are (both) plane numbers. So I say that (they are) also similar. For since F has made A (by) multiplying D , and has made C (by) multiplying E , thus as D is to E , so A (is) to C —that is to say, C to B [Prop. 7.17].[†] Again, since E has made C , B (by) multiplying F , G , respectively, thus as F is to G , so C (is) to B [Prop. 7.17].

And as C (is) to B , so D (is) to E . And thus as D (is) to E , so F (is) to G . And, alternately, as D (is) to F , so E (is) to G [Prop. 7.13]. Thus, A and B are similar plane numbers. For their sides are proportional [Def. 7.21]. (Which is) the very thing it was required to show.