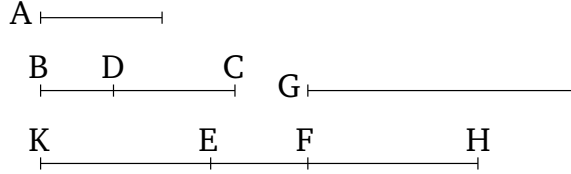


# Book 10

## Proposition 112

The (square) on a rational (straight-line), applied to a binomial (straight-line), produces as breadth an apotome whose terms are commensurable (in length) with the terms of the binomial, and, furthermore, in the same ratio. Moreover, the created apotome will have the same order as the binomial.



Let  $A$  be a rational (straight-line), and  $BC$  a binomial (straight-line), of which let  $DC$  be the greater term. And let the (rectangle contained) by  $BC$  and  $EF$  be equal to the (square) on  $A$ . I say that  $EF$  is an apotome whose terms are commensurable (in length) with  $CD$  and  $DB$ , and in the same ratio, and, moreover, that  $EF$  will have the same order as  $BC$ .

For, again, let the (rectangle contained) by  $BD$  and  $G$  be equal to the (square) on  $A$ . Therefore, since the (rectangle contained) by  $BC$  and  $EF$  is equal to the (rectangle contained) by  $BD$  and  $G$ , thus as  $CB$  is to  $BD$ , so  $G$  (is) to  $EF$  [Prop. 6.16]. And  $CB$  (is) greater than  $BD$ . Thus,  $G$  is also greater than  $EF$  [Props. 5.16, 5.14]. Let  $EH$  be equal to  $G$ . Thus, as  $CB$  is to  $BD$ , so  $HE$  (is) to  $EF$ . Thus, via separation, as  $CD$  is to  $BD$ , so  $HF$  (is) to  $FE$  [Prop. 5.17]. Let it have been contrived that as  $HF$  (is) to  $FE$ , so  $FK$  (is) to  $KE$ . And, thus, the whole  $HK$  is to the whole  $KF$ , as  $FK$  (is) to  $KE$ . For as one of the

leading (proportional magnitudes is) to one of the following, so all of the leading (magnitudes) are to all of the following [Prop. 5.12]. And as  $FK$  (is) to  $KE$ , so  $CD$  is to  $DB$  [Prop. 5.11]. And, thus, as  $HK$  (is) to  $KF$ , so  $CD$  is to  $DB$  [Prop. 5.11]. And the (square) on  $CD$  (is) commensurable with the (square) on  $DB$  [Prop. 10.36]. The (square) on  $HK$  is thus also commensurable with the (square) on  $KF$  [Props. 6.22, 10.11]. And as the (square) on  $HK$  is to the (square) on  $KF$ , so  $HK$  (is) to  $KE$ , since the three (straight-lines)  $HK$ ,  $KF$ , and  $KE$  are proportional [Def. 5.9].  $HK$  is thus commensurable in length with  $KE$  [Prop. 10.11]. Hence,  $HE$  is also commensurable in length with  $EK$  [Prop. 10.15]. And since the (square) on  $A$  is equal to the (rectangle contained) by  $EH$  and  $BD$ , and the (square) on  $A$  is rational, the (rectangle contained) by  $EH$  and  $BD$  is thus also rational. And it is applied to the rational (straight-line)  $BD$ . Thus,  $EH$  is rational, and commensurable in length with  $BD$  [Prop. 10.20]. And, hence, the (straight-line) commensurable (in length) with it,  $EK$ , is also rational [Def. 10.3], and commensurable in length with  $BD$  [Prop. 10.12]. Therefore, since as  $CD$  is to  $DB$ , so  $FK$  (is) to  $KE$ , and  $CD$  and  $DB$  are (straight-lines which are) commensurable in square only,  $FK$  and  $KE$  are also commensurable in square only [Prop. 10.11]. And  $KE$  is rational. Thus,  $FK$  is also rational.  $FK$  and  $KE$  are thus rational (straight-lines which are) commensurable in square only. Thus,  $EF$  is an apotome [Prop. 10.73].

And the square on  $CD$  is greater than (the square

on)  $DB$  either by the (square) on (some straight-line) commensurable, or by the (square) on (some straight-line) incommensurable, (in length) with  $(CD)$ .

Therefore, if the square on  $CD$  is greater than (the square on)  $DB$  by the (square) on (some straight-line) commensurable (in length) with  $[CD]$  then the square on  $FK$  will also be greater than (the square on)  $KE$  by the (square) on (some straight-line) commensurable (in length) with  $(FK)$  [Prop. 10.14]. And if  $CD$  is commensurable in length with  $DB$  then the square on  $FK$  will also be greater than (the square on)  $KE$  by the (square) on (some straight-line) commensurable (in length) with  $(FK)$  [Prop. 10.14]. And if neither of  $CD$  or  $DB$  (is commensurable), neither also (are) either of  $FK$  or  $KE$ .

And if the square on  $CD$  is greater than (the square on)  $DB$  by the (square) on (some straight-line) incommensurable (in length) with  $(CD)$  then the square on  $FK$  will also be greater than (the square on)  $KE$  by the (square) on (some straight-line) incommensurable (in length) with  $(FK)$  [Prop. 10.14]. And if  $CD$  is commensurable in length with  $DB$  then the square on  $FK$  will also be greater than (the square on)  $KE$  by the (square) on (some straight-line) commensurable (in length) with  $(FK)$  [Prop. 10.14]. And if neither of  $CD$  or  $DB$  (is commensurable), neither also (are) either of  $FK$  or  $KE$ . Hence,  $FE$  is an apotome whose terms,  $FK$  and  $KE$ , are commensurable (in length) with the terms,  $CD$  and  $DB$ , of the binomial, and in the same ratio. And  $(FE)$  has the same order as  $BC$  [Defs. 10.5—10.10]. (Which is) the very thing it was required to show.