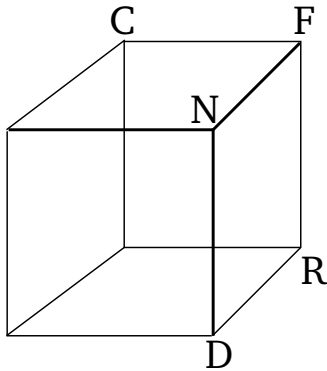


Book 11  
Proposition 33

Similar parallelepiped solids are to one another as the cubed ratio of their corresponding sides.

Let  $AB$  and  $CD$  be similar parallelepiped solids, and let  $AE$  correspond to  $CF$ . I say that solid  $AB$  has to solid  $CD$  the cubed ratio that  $AE$  (has) to  $CF$ .

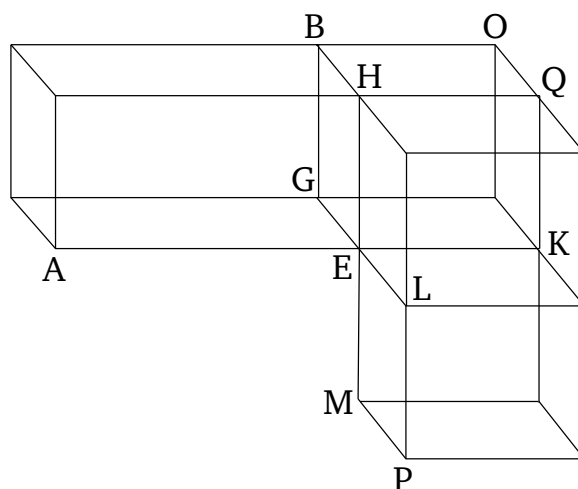
For let  $EK$ ,  $EL$ , and  $EM$  have been produced in a straight-line with  $AE$ ,  $GE$ , and  $HE$  (respectively). And let  $EK$  be made equal to  $CF$ , and  $EL$  equal to  $FN$ , and, further,  $EM$  equal to  $FR$ . And let the parallelogram  $KL$  have been completed, and the solid  $KP$ .



And since the two (straight-lines)  $KE$  and  $EL$  are equal to the two (straight-lines)  $CF$  and  $FN$ , but angle  $KEL$  is also equal to angle  $CFN$ , inasmuch as  $AEG$  is also equal to  $CFN$ , on account of the similarity of the solids  $AB$  and  $CD$ , parallelogram  $KL$  is thus equal [and similar] to parallelogram  $CN$ . So, for the same (reasons), parallelogram  $KM$  is also equal and similar to [parallelogram]  $CR$ , and, further,  $EP$  to  $DF$ . Thus, three parallelograms of solid  $KP$  are equal and similar to three

parallelograms of solid  $CD$ . But the three (former parallelograms) are equal and similar to the three opposite (parallelograms), and the three (latter parallelograms) are equal and similar to the three opposite (parallelograms) [Prop. 11.24]. Thus, the whole of solid  $KP$  is equal and similar to the whole of solid  $CD$  [Def. 11.10]. Let parallelogram  $GK$  have been completed. And let the solids  $EO$  and  $LQ$ , with bases the parallelograms  $GK$  and  $KL$  (respectively), and with the same height as  $AB$ , have been completed. And since, on account of the similarity of solids  $AB$  and  $CD$ , as  $AE$  is to  $CF$ , so  $EG$  (is) to  $FN$ , and  $EH$  to  $FR$  [Defs. 6.1, 11.9], and  $CF$  (is) equal to  $EK$ , and  $FN$  to  $EL$ , and  $FR$  to  $EM$ , thus as  $AE$  is to  $EK$ , so  $GE$  (is) to  $EL$ , and  $HE$  to  $EM$ . But, as  $AE$  (is) to  $EK$ , so [parallelogram]  $AG$  (is) to parallelogram  $GK$ , and as  $GE$  (is) to  $EL$ , so  $GK$  (is) to  $KL$ , and as  $HE$  (is) to  $EM$ , so  $QE$  (is) to  $KM$  [Prop. 6.1]. And thus as parallelogram  $AG$  (is) to  $GK$ , so  $GK$  (is) to  $KL$ , and  $QE$  (is) to  $KM$ . But, as  $AG$  (is) to  $GK$ , so solid  $AB$  (is) to solid  $EO$ , and as  $GK$  (is) to  $KL$ , so solid  $OE$  (is) to solid  $QL$ , and as  $QE$  (is) to  $KM$ , so solid  $QL$  (is) to solid  $KP$  [Prop. 11.32]. And, thus, as solid  $AB$  is to  $EO$ , so  $EO$  (is) to  $QL$ , and  $QL$  to  $KP$ . And if four magnitudes are continuously proportional then the first has to the fourth the cubed ratio that (it has) to the second [Def. 5.10]. Thus, solid  $AB$  has to  $KP$  the cubed ratio which  $AB$  (has) to  $EO$ . But, as  $AB$  (is) to  $EO$ , so parallelogram  $AG$  (is) to  $GK$ , and the straight-line  $AE$  to  $EK$  [Prop. 6.1]. Hence, solid  $AB$  also has to  $KP$  the cubed ratio that  $AE$  (has) to  $EK$ . And solid  $KP$  (is)

equal to solid  $CD$ , and straight-line  $EK$  to  $CF$ . Thus, solid  $AB$  also has to solid  $CD$  the cubed ratio which its corresponding side  $AE$  (has) to the corresponding side  $CF$ .



Thus, similar parallelepiped solids are to one another as the cubed ratio of their corresponding sides. (Which is) the very thing it was required to show.

## Corollary

So, (it is) clear, from this, that if four straight-lines are (continuously) proportional then as the first is to the fourth, so the parallelepiped solid on the first will be to the similar, and similarly described, parallelepiped solid on the second, since the first also has to the fourth the cubed ratio that (it has) to the second.