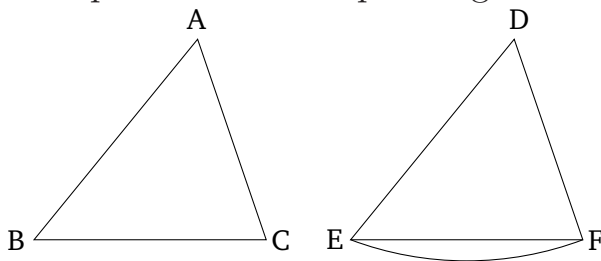


Book 1

Proposition 4

If two triangles have two sides equal to two sides, respectively, and have the angle(s) enclosed by the equal straight-lines equal, then they will also have the base equal to the base, and the triangle will be equal to the triangle, and the remaining angles subtended by the equal sides will be equal to the corresponding remaining angles.



Let ABC and DEF be two triangles having the two sides AB and AC equal to the two sides DE and DF , respectively. (That is) AB to DE , and AC to DF . And (let) the angle BAC (be) equal to the angle EDF . I say that the base BC is also equal to the base EF , and triangle ABC will be equal to triangle DEF , and the remaining angles subtended by the equal sides will be equal to the corresponding remaining angles. (That is) ABC to DEF , and ACB to DFE .

For if triangle ABC is applied to triangle DEF ,[†] the point A being placed on the point D , and the straight-line AB on DE , then the point B will also coincide with E , on account of AB being equal to DE . So (because of) AB coinciding with DE , the straight-line AC will also coincide with DF , on account of the angle BAC being equal to EDF . So the point C will also coincide

with the point F , again on account of AC being equal to DF . But, point B certainly also coincided with point E , so that the base BC will coincide with the base EF . For if B coincides with E , and C with F , and the base BC does not coincide with EF , then two straight-lines will encompass an area. The very thing is impossible [Post. 1].[‡] Thus, the base BC will coincide with EF , and will be equal to it [C.N. 4]. So the whole triangle ABC will coincide with the whole triangle DEF , and will be equal to it [C.N. 4]. And the remaining angles will coincide with the remaining angles, and will be equal to them [C.N. 4]. (That is) ABC to DEF , and ACB to DFE [C.N. 4].

Thus, if two triangles have two sides equal to two sides, respectively, and have the angle(s) enclosed by the equal straight-line equal, then they will also have the base equal to the base, and the triangle will be equal to the triangle, and the remaining angles subtended by the equal sides will be equal to the corresponding remaining angles. (Which is) the very thing it was required to show.