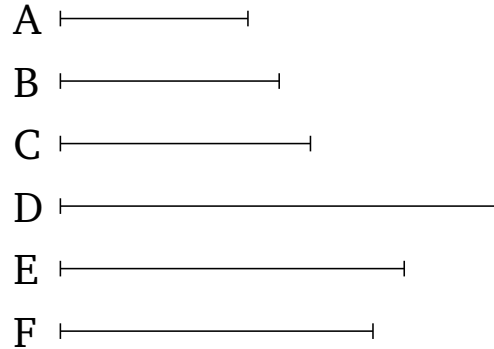


Book 7

Proposition 36

To find the least number which three given numbers (all) measure.

Let A , B , and C be the three given numbers. So it is required to find the least number which they (all) measure.



For let the least (number), D , measured by the two (numbers) A and B have been taken [Prop. 7.34]. So C either measures, or does not measure, D . Let it, first of all, measure (D). And A and B also measure D . Thus, A , B , and C (all) measure D . So I say that (D is) also the least (number measured by A , B , and C). For if not, A , B , and C will (all) measure [some] number which is less than D . Let them measure E (which is less than D). Since A , B , and C (all) measure E then A and B thus also measure E . Thus, the least (number) measured by A and B will also measure [E] [Prop. 7.35]. And D is the least (number) measured by A and B . Thus, D will measure E , the greater (measuring) the lesser. The very thing is impossible. Thus, A , B , and C cannot (all) measure some number which is less than D . Thus, A , B ,

and C (all) measure the least (number) D .

So, again, let C not measure D . And let the least number, E , measured by C and D have been taken [Prop. 7.34]. Since A and B measure D , and D measures E , A and B thus also measure E . And C also measures E . Thus, A , B , and C [also] measure E . So I say that (E is) also the least (number measured by A , B , and C). For if not, A , B , and C will (all) measure some (number) which is less than E . Let them measure F (which is less than E). Since A , B , and C (all) measure F , A and B thus also measure F . Thus, the least (number) measured by A and B will also measure F [Prop. 7.35]. And D is the least (number) measured by A and B . Thus, D measures F . And C also measures F . Thus, D and C (both) measure F . Hence, the least (number) measured by D and C will also measure F [Prop. 7.35]. And E is the least (number) measured by C and D . Thus, E measures F , the greater (measuring) the lesser. The very thing is impossible. Thus, A , B , and C cannot measure some number which is less than E . Thus, E (is) the least (number) which is measured by A , B , and C . (Which is) the very thing it was required to show.