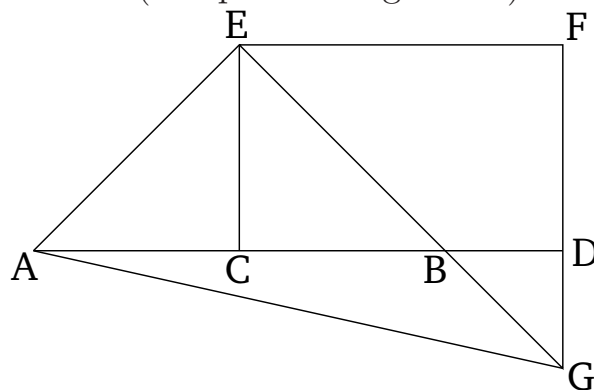


Book 2

Proposition 10

If a straight-line is cut in half, and any straight-line added to it straight-on, then the sum of the square on the whole (straight-line) with the (straight-line) having been added, and the (square) on the (straight-line) having been added, is double the (sum of the square) on half (the straight-line), and the square described on the sum of half (the straight-line) and (straight-line) having been added, as on one (complete straight-line).



For let any straight-line AB have been cut in half at (point) C , and let any straight-line BD have been added to it straight-on. I say that the (sum of the) squares on AD and DB is double the (sum of the) squares on AC and CD .

For let CE have been drawn from point C , at right-angles to AB [Prop. 1.11], and let it be made equal to each of AC and CB [Prop. 1.3], and let EA and EB have been joined. And let EF have been drawn through E , parallel to AD [Prop. 1.31], and let FD have been drawn through D , parallel to CE [Prop. 1.31].

And since some straight-line EF falls across the parallel straight-lines EC and FD , the (internal angles) CEF and EFD are thus equal to two right-angles [Prop. 1.29]. Thus, FEB and EFD are less than two right-angles. And (straight-lines) produced from (internal angles whose sum is) less than two right-angles meet together [Post. 5]. Thus, being produced in the direction of B and D , the (straight-lines) EB and FD will meet. Let them have been produced, and let them meet together at G , and let AG have been joined. And since AC is equal to CE , angle EAC is also equal to (angle) AEC [Prop. 1.5]. And the (angle) at C (is) a right-angle. Thus, EAC and AEC [are] each half a right-angle [Prop. 1.32]. So, for the same (reasons), CEB and EBC are also each half a right-angle. Thus, (angle) AEB is a right-angle. And since EBC is half a right-angle, DBG (is) thus also half a right-angle [Prop. 1.15]. And BDG is also a right-angle. For it is equal to DCE . For (they are) alternate (angles) [Prop. 1.29]. Thus, the remaining (angle) DGB is half a right-angle. Thus, DGB is equal to DBG . So side BD is also equal to side GD [Prop. 1.6]. Again, since EGF is half a right-angle, and the (angle) at F (is) a right-angle, for it is equal to the opposite (angle) at C [Prop. 1.34], the remaining (angle) FEG is thus half a right-angle. Thus, angle EGF (is) equal to FEG . So the side GF is also equal to the side EF [Prop. 1.6]. And since [EC is equal to CA] the square on EC is [also] equal to the square on CA . Thus, the (sum of the) squares on EC and CA is double the square on CA . And the (square) on EA is equal to the (sum of the squares) on EC and

CA [Prop. 1.47]. Thus, the square on EA is double the square on AC . Again, since FG is equal to EF , the (square) on FG is also equal to the (square) on FE . Thus, the (sum of the squares) on GF and FE is double the (square) on EF . And the (square) on EG is equal to the (sum of the squares) on GF and FE [Prop. 1.47]. Thus, the (square) on EG is double the (square) on EF . And EF (is) equal to CD [Prop. 1.34]. Thus, the square on EG is double the (square) on CD . But it was also shown that the (square) on EA (is) double the (square) on AC . Thus, the (sum of the) squares on AE and EG is double the (sum of the) squares on AC and CD . And the square on AG is equal to the (sum of the) squares on AE and EG [Prop. 1.47]. Thus, the (square) on AG is double the (sum of the squares) on AC and CD . And the (sum of the squares) on AD and DG is equal to the (square) on AG [Prop. 1.47]. Thus, the (sum of the) [squares] on AD and DG is double the (sum of the) [squares] on AC and CD . And DG (is) equal to DB . Thus, the (sum of the) [squares] on AD and DB is double the (sum of the) squares on AC and CD .

Thus, if a straight-line is cut in half, and any straight-line added to it straight-on, then the sum of the square on the whole (straight-line) with the (straight-line) having been added, and the (square) on the (straight-line) having been added, is double the (sum of the square) on half (the straight-line), and the square described on the sum of half (the straight-line) and (straight-line) having been added, as on one (complete straight-line). (Which is) the very thing it was required to show.