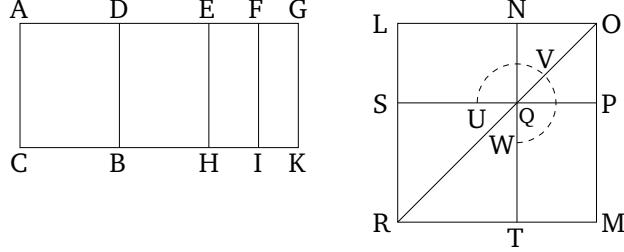


# Book 10

## Proposition 93

If an area is contained by a rational (straight-line) and a third apotome then the square-root of the area is a second apotome of a medial (straight-line).



For let the area  $AB$  have been contained by the rational (straight-line)  $AC$  and the third apotome  $AD$ . I say that the square-root of area  $AB$  is the second apotome of a medial (straight-line).

For let  $DG$  be an attachment to  $AD$ . Thus,  $AG$  and  $GD$  are rational (straight-lines which are) commensurable in square only [Prop. 10.73], and neither of  $AG$  and  $GD$  is commensurable in length with the (previously) laid down rational (straight-line)  $AC$ , and the square on the whole,  $AG$ , is greater than (the square on) the attachment,  $DG$ , by the (square) on (some straight-line) commensurable (in length) with ( $AG$ ) [Def. 10.13]. Therefore, since the square on  $AG$  is greater than (the square on)  $GD$  by the (square) on (some straight-line) commensurable (in length) with ( $AG$ ), thus if (an area) equal to the fourth part of the square on  $DG$  is applied to  $AG$ , falling short by a square figure, then it divides ( $AG$ ) into (parts which are) commensurable (in length) [Prop. 10.17]. Therefore, let  $DG$  have been cut in half

at  $E$ . And let (an area) equal to the (square) on  $EG$  have been applied to  $AG$ , falling short by a square figure. And let it be the (rectangle contained) by  $AF$  and  $FG$ . And let  $EH$ ,  $FI$ , and  $GK$  have been drawn through points  $E$ ,  $F$ , and  $G$  (respectively), parallel to  $AC$ . Thus,  $AF$  and  $FG$  are commensurable (in length).  $AI$  (is) thus also commensurable with  $FK$  [Props. 6.1, 10.11]. And since  $AF$  and  $FG$  are commensurable in length,  $AG$  is thus also commensurable in length with each of  $AF$  and  $FG$  [Prop. 10.15]. And  $AG$  (is) rational, and incommensurable in length with  $AC$ . Hence,  $AF$  and  $FG$  (are) also (rational, and incommensurable in length with  $AC$ ) [Prop. 10.13]. Thus,  $AI$  and  $FK$  are each medial (areas) [Prop. 10.21]. Again, since  $DE$  is commensurable in length with  $EG$ ,  $DG$  is also commensurable in length with each of  $DE$  and  $EG$  [Prop. 10.15]. And  $GD$  (is) rational, and incommensurable in length with  $AC$ . Thus,  $DE$  and  $EG$  (are) each also rational, and incommensurable in length with  $AC$  [Prop. 10.13].  $DH$  and  $EK$  are thus each medial (areas) [Prop. 10.21]. And since  $AG$  and  $GD$  are commensurable in square only,  $AG$  is thus incommensurable in length with  $GD$ . But,  $AG$  is commensurable in length with  $AF$ , and  $DG$  with  $EG$ . Thus,  $AF$  is incommensurable in length with  $EG$  [Prop. 10.13]. And as  $AF$  (is) to  $EG$ , so  $AI$  is to  $EK$  [Prop. 6.1]. Thus,  $AI$  is incommensurable with  $EK$  [Prop. 10.11].

Therefore, let the square  $LM$ , equal to  $AI$ , have been constructed. And let  $NO$ , equal to  $FK$ , which is about the same angle as  $LM$ , have been subtracted (from  $LM$ ). Thus,  $LM$  and  $NO$  are about the same diagonal [Prop. 6.26].

Let  $PR$  be their (common) diagonal, and let the (rest of the) figure have been drawn. Therefore, since the (rectangle contained) by  $AF$  and  $FG$  is equal to the (square) on  $EG$ , thus as  $AF$  is to  $EG$ , so  $EG$  (is) to  $FG$  [Prop. 6.17]. But, as  $AF$  (is) to  $EG$ , so  $AI$  is to  $EK$  [Prop. 6.1]. And as  $EG$  (is) to  $FG$ , so  $EK$  is to  $FK$  [Prop. 6.1]. And thus as  $AI$  (is) to  $EK$ , so  $EK$  (is) to  $FK$  [Prop. 5.11]. Thus,  $EK$  is the mean proportional to  $AI$  and  $FK$ . And  $MN$  is also the mean proportional to the squares  $LM$  and  $NO$  [Prop. 10.53 lem.]. And  $AI$  is equal to  $LM$ , and  $FK$  to  $NO$ . Thus,  $EK$  is also equal to  $MN$ . But,  $MN$  is equal to  $LO$ , and  $EK$  [is] equal to  $DH$  [Prop. 1.43]. And thus the whole of  $DK$  is equal to the gnomon  $UVW$  and  $NO$ . And  $AK$  (is) also equal to  $LM$  and  $NO$ . Thus, the remainder  $AB$  is equal to  $ST$ —that is to say, to the square on  $LN$ . Thus,  $LN$  is the square-root of area  $AB$ . I say that  $LN$  is the second apotome of a medial (straight-line).

For since  $AI$  and  $FK$  were shown (to be) medial (areas), and are equal to the (squares) on  $LP$  and  $PN$  (respectively), the (squares) on each of  $LP$  and  $PN$  (are) thus also medial. Thus,  $LP$  and  $PN$  (are) each medial (straight-lines). And since  $AI$  is commensurable with  $FK$  [Props. 6.1, 10.11], the (square) on  $LP$  (is) thus also commensurable with the (square) on  $PN$ . Again, since  $AI$  was shown (to be) incommensurable with  $EK$ ,  $LM$  is thus also incommensurable with  $MN$ —that is to say, the (square) on  $LP$  with the (rectangle contained) by  $LP$  and  $PN$ . Hence,  $LP$  is also incommensurable in length with  $PN$  [Props. 6.1, 10.11]. Thus,  $LP$  and  $PN$

are medial (straight-lines which are) commensurable in square only. So, I say that they also contain a medial (area).

For since  $EK$  was shown (to be) a medial (area), and is equal to the (rectangle contained) by  $LP$  and  $PN$ , the (rectangle contained) by  $LP$  and  $PN$  is thus also medial. Hence,  $LP$  and  $PN$  are medial (straight-lines which are) commensurable in square only, and which contain a medial (area). Thus,  $LN$  is the second apotome of a medial (straight-line) [Prop. 10.75]. And it is the square-root of area  $AB$ .

Thus, the square-root of area  $AB$  is the second apotome of a medial (straight-line). (Which is) the very thing it was required to show.