

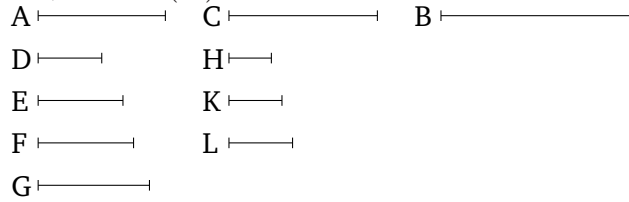
Book 10

Proposition 12

(Magnitudes) commensurable with the same magnitude are also commensurable with one another.

For let A and B each be commensurable with C . I say that A is also commensurable with B .

For since A is commensurable with C , A thus has to C the ratio which (some) number (has) to (some) number [Prop. 10.5]. Let it have (the ratio) which D (has) to E . Again, since C is commensurable with B , C thus has to B the ratio which (some) number (has) to (some) number [Prop. 10.5]. Let it have (the ratio) which F (has) to G . And for any multitude whatsoever of given ratios—(namely,) those which D has to E , and F to G —let the numbers H , K , L (which are) continuously (proportional) in the(se) given ratios have been taken [Prop. 8.4]. Hence, as D is to E , so H (is) to K , and as F (is) to G , so K (is) to L .



Therefore, since as A is to C , so D (is) to E , but as D (is) to E , so H (is) to K , thus also as A is to C , so H (is) to K [Prop. 5.11]. Again, since as C is to B , so F (is) to G , but as F (is) to G , [so] K (is) to L , thus also as C (is) to B , so K (is) to L [Prop. 5.11]. And also as A is to C , so H (is) to K . Thus, via equality, as A is to B , so H (is) to L [Prop. 5.22]. Thus, A has to B the

ratio which the number H (has) to the number L . Thus, A is commensurable with B [Prop. 10.6].

Thus, (magnitudes) commensurable with the same magnitude are also commensurable with one another. (Which is) the very thing it was required to show.