

# Book 10

## Proposition 80

Only one medial straight-line, which is commensurable in square only with the whole, and contains a rational (area) with the whole, can be attached to a first apotome of a medial (straight-line).<sup>†</sup>



For let  $AB$  be a first apotome of a medial (straight-line), and let  $BC$  be (so) attached to  $AB$ . Thus,  $AC$  and  $CB$  are medial (straight-lines which are) commensurable in square only, containing a rational (area)—(namely, that contained) by  $AC$  and  $CB$  [Prop. 10.74]. I say that a(nother) medial (straight-line), which is commensurable in square only with the whole, and contains a rational (area) with the whole, cannot be attached to  $AB$ .

For, if possible, let  $DB$  also be (so) attached to  $AB$ . Thus,  $AD$  and  $DB$  are medial (straight-lines which are) commensurable in square only, containing a rational (area)—(namely, that) contained by  $AD$  and  $DB$  [Prop. 10.74]. And since by whatever (area) the (sum of the squares) on  $AD$  and  $DB$  exceeds twice the (rectangle contained) by  $AD$  and  $DB$ , the (sum of the squares) on  $AC$  and  $CB$  also exceeds twice the (rectangle contained) by  $AC$  and  $CB$  by this (same area). For [again] both exceed by the same (area)—(namely), the (square) on  $AB$  [Prop. 2.7]. Thus, alternately, by whatever (area) the (sum of the squares) on  $AD$  and  $DB$  exceeds the (sum of the squares) on  $AC$  and  $CB$ , twice the (rectangle contained) by  $AD$  and  $DB$  also exceeds twice the (rectangle contained) by

$AC$  and  $CB$  by this (same area). And twice the (rectangle contained) by  $AD$  and  $DB$  exceeds twice the (rectangle contained) by  $AC$  and  $CB$  by a rational (area). For both (are) rational (areas). Thus, the (sum of the squares) on  $AD$  and  $DB$  also exceeds the (sum of the [squares] on  $AC$  and  $CB$  by a rational (area). The very thing is impossible. For both are medial (areas) [Props. 10.15, 10.23 corr.], and a medial (area) cannot exceed a(nother) medial (area) by a rational (area) [Prop. 10.26].

Thus, only one medial (straight-line), which is commensurable in square only with the whole, and contains a rational (area) with the whole, can be attached to a first apotome of a medial (straight-line). (Which is) the very thing it was required to show.