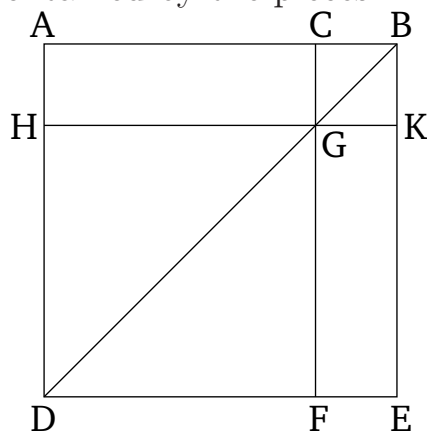


Book 2

Proposition 4

If a straight-line is cut at random then the square on the whole (straight-line) is equal to the (sum of the) squares on the pieces (of the straight-line), and twice the rectangle contained by the pieces.



For let the straight-line AB have been cut, at random, at (point) C . I say that the square on AB is equal to the (sum of the) squares on AC and CB , and twice the rectangle contained by AC and CB .

For let the square $ADEB$ have been described on AB [Prop. 1.46], and let BD have been joined, and let CF have been drawn through C , parallel to either of AD or EB [Prop. 1.31], and let HK have been drawn through G , parallel to either of AB or DE [Prop. 1.31]. And since CF is parallel to AD , and BD has fallen across them, the external angle CGB is equal to the internal and opposite (angle) ADB [Prop. 1.29]. But, ADB is equal to ABD , since the side BA is also equal to AD [Prop. 1.5]. Thus, angle CGB is also equal to GBC . So

the side BC is equal to the side CG [Prop. 1.6]. But, CB is equal to GK , and CG to KB [Prop. 1.34]. Thus, GK is also equal to KB . Thus, $CGKB$ is equilateral. So I say that (it is) also right-angled. For since CG is parallel to BK [and the straight-line CB has fallen across them], the angles KBC and GCB are thus equal to two right-angles [Prop. 1.29]. But KBC (is) a right-angle. Thus, BCG (is) also a right-angle. So the opposite (angles) CGK and GKB are also right-angles [Prop. 1.34]. Thus, $CGKB$ is right-angled. And it was also shown (to be) equilateral. Thus, it is a square. And it is on CB . So, for the same (reasons), HF is also a square. And it is on HG , that is to say [on] AC [Prop. 1.34]. Thus, the squares HF and KC are on AC and CB (respectively). And the (rectangle) AG is equal to the (rectangle) GE [Prop. 1.43]. And AG is the (rectangle contained) by AC and CB . For GC (is) equal to CB . Thus, GE is also equal to the (rectangle contained) by AC and CB . Thus, the (rectangles) AG and GE are equal to twice the (rectangle contained) by AC and CB . And HF and CK are the squares on AC and CB (respectively). Thus, the four (figures) HF , CK , AG , and GE are equal to the (sum of the) squares on AC and BC , and twice the rectangle contained by AC and CB . But, the (figures) HF , CK , AG , and GE are (equivalent to) the whole of $ADEB$, which is the square on AB . Thus, the square on AB is equal to the (sum of the) squares on AC and CB , and twice the rectangle contained by AC and CB .

Thus, if a straight-line is cut at random then the square on the whole (straight-line) is equal to the (sum of the)

squares on the pieces (of the straight-line), and twice the rectangle contained by the pieces. (Which is) the very thing it was required to show.