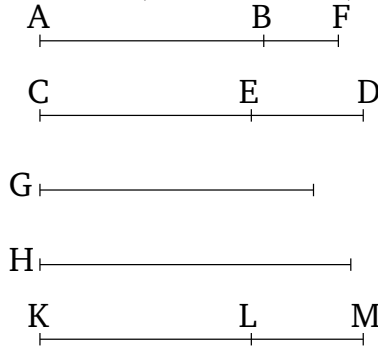


# Book 10

## Proposition 114

If an area is contained by an apotome, and a binomial whose terms are commensurable with, and in the same ratio as, the terms of the apotome then the square-root of the area is a rational (straight-line).



For let an area, the (rectangle contained) by  $AB$  and  $CD$ , have been contained by the apotome  $AB$ , and the binomial  $CD$ , of which let the greater term be  $CE$ . And let the terms of the binomial,  $CE$  and  $ED$ , be commensurable with the terms of the apotome,  $AF$  and  $FB$  (respectively), and in the same ratio. And let the square-root of the (rectangle contained) by  $AB$  and  $CD$  be  $G$ . I say that  $G$  is a rational (straight-line).

For let the rational (straight-line)  $H$  be laid down. And let (some rectangle), equal to the (square) on  $H$ , have been applied to  $CD$ , producing  $KL$  as breadth. Thus,  $KL$  is an apotome, of which let the terms,  $KM$  and  $ML$ , be commensurable with the terms of the binomial,  $CE$  and  $ED$  (respectively), and in the same ratio [Prop. 10.112]. But,  $CE$  and  $ED$  are also commensurable with  $AF$  and  $FB$  (respectively), and in the

same ratio. Thus, as  $AF$  is to  $FB$ , so  $KM$  (is) to  $ML$ . Thus, alternately, as  $AF$  is to  $KM$ , so  $BF$  (is) to  $LM$  [Prop. 5.16]. Thus, the remainder  $AB$  is also to the remainder  $KL$  as  $AF$  (is) to  $KM$  [Prop. 5.19]. And  $AF$  (is) commensurable with  $KM$  [Prop. 10.12].  $AB$  is thus also commensurable with  $KL$  [Prop. 10.11]. And as  $AB$  is to  $KL$ , so the (rectangle contained) by  $CD$  and  $AB$  (is) to the (rectangle contained) by  $CD$  and  $KL$  [Prop. 6.1]. Thus, the (rectangle contained) by  $CD$  and  $AB$  is also commensurable with the (rectangle contained) by  $CD$  and  $KL$  [Prop. 10.11]. And the (rectangle contained) by  $CD$  and  $KL$  (is) equal to the (square) on  $H$ . Thus, the (rectangle contained) by  $CD$  and  $AB$  is commensurable with the (square) on  $H$ . And the (square) on  $G$  is equal to the (rectangle contained) by  $CD$  and  $AB$ . The (square) on  $G$  is thus commensurable with the (square) on  $H$ . And the (square) on  $H$  (is) rational. Thus, the (square) on  $G$  is also rational.  $G$  is thus rational. And it is the square-root of the (rectangle contained) by  $CD$  and  $AB$ .

Thus, if an area is contained by an apotome, and a binomial whose terms are commensurable with, and in the same ratio as, the terms of the apotome, then the square-root of the area is a rational (straight-line).

## Corollary

And it has also been made clear to us, through this, that it is possible for a rational area to be contained by irrational straight-lines. (Which is) the very thing it was

required to show.