

# Book 10

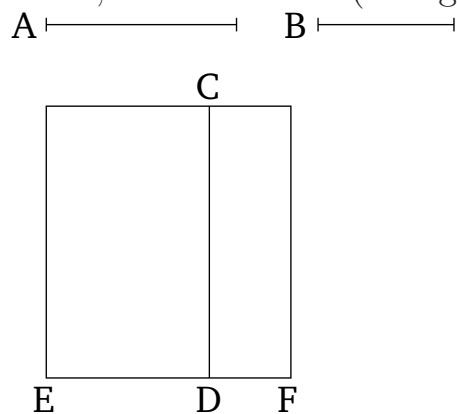
## Proposition 23

A (straight-line) commensurable with a medial (straight-line) is medial.

Let  $A$  be a medial (straight-line), and let  $B$  be commensurable with  $A$ . I say that  $B$  is also a medial (straight-line).

Let the rational (straight-line)  $CD$  be set out, and let the rectangular area  $CE$ , equal to the (square) on  $A$ , have been applied to  $CD$ , producing  $ED$  as width.  $ED$  is thus rational, and incommensurable in length with  $CD$  [Prop. 10.22]. And let the rectangular area  $CF$ , equal to the (square) on  $B$ , have been applied to  $CD$ , producing  $DF$  as width. Therefore, since  $A$  is commensurable with  $B$ , the (square) on  $A$  is also commensurable with the (square) on  $B$ . But,  $EC$  is equal to the (square) on  $A$ , and  $CF$  is equal to the (square) on  $B$ . Thus,  $EC$  is commensurable with  $CF$ . And as  $EC$  is to  $CF$ , so  $ED$  (is) to  $DF$  [Prop. 6.1]. Thus,  $ED$  is commensurable in length with  $DF$  [Prop. 10.11]. And  $ED$  is rational, and incommensurable in length with  $CD$ .  $DF$  is thus also rational [Def. 10.3], and incommensurable in length with  $DC$  [Prop. 10.13]. Thus,  $CD$  and  $DF$  are rational, and commensurable in square only. And the square-root of a (rectangle contained) by rational (straight-lines which are) commensurable in square only is medial [Prop. 10.21]. Thus, the square-root of the (rectangle contained) by  $CD$  and  $DF$  is medial. And the square on  $B$  is equal to the (rectangle contained) by

$CD$  and  $DF$ . Thus,  $B$  is a medial (straight-line).



## Corollary

And (it is) clear, from this, that an (area) commensurable with a medial area is medial.