

## **Mathematics and Mathematical Thought in the Quadrivium of Isidore of Seville**

As the intellectual descendent of Roman encyclopaedists,<sup>1</sup> Isidore of Seville explored a vast scope of learning in his Etymologies, including topics as varied as medicine and metallurgy, war and shipbuilding, grammar and augury. His approach presupposed the idea that understanding the origins of the word used to refer to a concept would lead to enhanced knowledge about the intrinsic nature of the concept itself. This belief can be traced back to ancient Greece, where writers Heraclides Ponticus (4<sup>th</sup> cent. B.C.E) and Apollodorus of Athens (2<sup>nd</sup> cent. B.C.E.) sought to develop the science of ἐτυμολογία in their works.<sup>2</sup> Roman author Terrentius Varro continued with this line of thought in his De Lingua Latina, and though Isidore likely lacked access to the majority of his texts, Varro represents Isidore's direct predecessor; the latter mimics the former's use of etymologizing as the basis of constructing an encyclopedia and freely incorporates the Varronian model of the seven liberal arts into the framework of his own writings.<sup>3</sup> This model, which was formally laid out in Martianus Capella's Marriage of Philology and Mercury, included both verbal and mathematical disciplines and separated them into two distinct parts: the "trivium," comprised of grammar, rhetoric and logic, and the "quadrivium," made up of arithmetic, geometry, astronomy and music.<sup>4</sup>

The content of Isidore's interpretation of this second section in his Etymologies provides a significant amount of insight into the state of mathematical thought during the Middle Ages. Decayed from want of use by the Romans and condensed into nearly

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<sup>1</sup> Grant, *The Foundations of Modern Science in the Middle Ages*. pp. 12-14.

<sup>2</sup> Barney et al. p. 11.

<sup>3</sup> Barney et al. pp. 11-12.

<sup>4</sup> Grant, *The Foundations of Modern Science in the Middle Ages*. p. 14.

useless fragments by medieval encyclopaedia writers,<sup>5</sup> mathematics existed in a form far removed from the advancements of Greek mathematicians, particularly those active during the Hellenistic period.<sup>6</sup> Isidore's attempts to illuminate the subjects of the quadrivium are certainly not to be belittled, but the ignorance of key concepts that he displays in his writings further indicate the depths to which these modes of thought had sunk.

Isidore was born around C.E. 560. At about this time, his family, originally from Cartagena, moved to Seville in order to escape invading Byzantine forces.<sup>7</sup> Isidore's family was orthodox Catholic and likely Hispano-Roman, which was the majority population of the country.<sup>8</sup> At this point in the early Middle Ages, Spain still bore the stamp of Roman occupation. During its time as part of the empire, it had contributed many figures notable to Roman culture and history, including the writers Martial and Lucan and emperors Trajan and Hadrian.<sup>9</sup> By the year of Isidore's birth, however, Rome was long fallen, and her Visigoth destroyers had ruled Spain for over a century.<sup>10</sup> Originally forced into Spain by Emperor Constantius in 412, two years after the sack of Rome, the Visigoths resettled in Gaul in 418. In 456, under Theodoric II, they moved once again into Spain, conquering most, but not all, of the peninsula and effectively

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<sup>5</sup> Grant, *Physical Science in the Middle Ages*. p. 11.

<sup>6</sup> Russo. p. 8.

<sup>7</sup> Barney et al. p. 7.

<sup>8</sup> Brehaut. p. 20.

<sup>9</sup> Brehaut. p. 18.

<sup>10</sup> Barney et al. p. 4.

instituting this area of Spain as the main region of the Visigothic kingdom.<sup>11</sup> Despite this dramatic governmental overhaul, Roman administrative structures persisted under the Visigoth kings, with Roman officials continuing to collect Roman taxes and to enforce Roman law.<sup>12</sup>

The first few kings of Isidore's lifetime included Athanagild, who died in 568, his successor Liuva, who died in 571 or 573, and Leovigild, the brother of Liuva.<sup>13</sup> Leovigild was an exceptionally strong military leader, restoring territory to the Visigoths from the rest of the peninsula. He also attempted to propagate a form of Christianity known as Arianism,<sup>14</sup> a belief system condemned by the council of Nicaea in 325 in which the Father and Son elements of the Trinity were held as unequal, with the Son lacking the eternal nature of the Father.<sup>15</sup> Leovigild used persuasion and rewards in his program of conversion, winning over such leading Catholics as the Bishop of Saragossa, but his campaign was ultimately unsuccessful. His own two sons, Hermenigild and Reccared, converted to Catholicism under the encouragement of none other than Isidore's older brother, the powerful bishop Leander.<sup>16</sup> Reccared, who assumed the throne after his father's death in 586, sought to dismantle the Arian Church. He introduced his plan to assimilate Arian congregations into the sees of Catholic bishops at the Council of Toledo in 589, in what seems to have been a decided success, as Arianism disappeared from the

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<sup>11</sup> Ibid.

<sup>12</sup> Ibid.

<sup>13</sup> Barney et al. p. 5.

<sup>14</sup> Ibid.

<sup>15</sup> Oxford Classical Dictionary, 3<sup>rd</sup> ed. rev. p. 156.

<sup>16</sup> Barney et al. p. 5.

region within a generation.<sup>17</sup> Reccared's commitment to Catholicism resulted in a solid relationship between the monarchy and the Church. Leander exercised a fair amount of influence in affairs concerning these institutions, and his clout extended to Isidore, who was appointed Bishop of Seville after Leander's death.<sup>18</sup> In his new role, Isidore presided over the Council of Seville in 619 and the Council of Toledo in 633.<sup>19</sup> He also developed a deep friendship with one of the key monarchs of his time, King Sisebut, a man of intellectual interests who in fact authored a poem on lunar eclipses in response to some of Isidore's writings, as well as a biography of St. Desiderius.<sup>20</sup> During his lifetime of service to the Church, Isidore wrote voluminously. In the *Renotatio Isidori*, his colleague Braulio listed seventeen major works and a smattering of minor works in Isidore's corpus of writings and praised his "incomparable elegance" and "great knowledge."<sup>21</sup> He spent the last part of his literary career working on The Etymologies, and left it unfinished at his death in 636. The Catholic Church canonized him as a saint in 1598.<sup>22</sup>

The mathematical component of The Etymologies begins in Book III, with the quadrivium subjects of arithmetic, geometry, music, and astronomy appropriately following the trivium subjects of grammar, rhetoric, and logic, which Isidore covers in Books I and II. Although the subjects of the quadrivium are combined into one Book,

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<sup>17</sup> Barney et al. pp. 5-6.

<sup>18</sup> Barney et al. p. 7.

<sup>19</sup> Ibid.

<sup>20</sup> Barney et al. pp. 6-7.

<sup>21</sup> Barney et al. p. 9.

<sup>22</sup> Barney et al. p. 10.

entitled *De Mathematica*, Isidore explains to the reader that each subject makes up its own specific part of this body of knowledge.

Mathematica Latine dicitur doctrinalis scientia, quae abstractam considerat quantitatem...Cuius species sunt quattuor: id est Arithmetica, Musica, Geometria et Astronomia. Arithmetica est disciplina quantitatis numerabilis secundum se. Musica est disciplina quae de numeris loquitur, qui inveniuntur in sonis. Geometria est disciplina magnitudinis et formarum. Astronomia est disciplina quae cursus caelestium siderum atque figuras contemplatur omnes atque habitudines stellarum.<sup>23</sup>

*The doctrinal knowledge which examines abstract quantity is called Mathematics in Latin...Of this there are four types: that is, Arithmetic, Music, Geometry, and Astronomy. Arithmetic is the discipline of numeric quantity in accordance with itself. Music is the discipline which discusses the numbers which are found in sounds. Geometry is the discipline of magnitude and shapes. Astronomy is the discipline which observes the direction of the heavenly constellations and all the figures and appearances of the stars.*<sup>24</sup>

This format, as has been discussed, comes directly from the Varronian model. Isidore proceeds to give each subject the etymological treatment, often using the Greek roots as tools. He seems to have known little of Greek grammar and specific verbal structure,<sup>25</sup> but just as a modern educated individual might use a smattering of Latin knowledge to discern the meanings of English words, so too did Isidore likely use the same process to discuss the histories of Latin words.

Arithmetica est disciplina numerorum. Graeci enim numerum ἀριθμόν dicunt. Quam scriptores saecularium litterarum inter disciplinas mathematicas ideo primam esse voluerunt, quoniam ipsa ut sit nullam aliam indiget disciplinam. Musica autem et Geometria et Astronomia, quae sequuntur, ut sint atque subsistant istius egent auxilium.<sup>26</sup>

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<sup>23</sup> Isidore, *Etymologiarum*. III.

<sup>24</sup> Trans. Costrell, Sarah.

<sup>25</sup> Brehaut. pp. 35-36.

<sup>26</sup> Isidore, *Etymologiarum*. III.i.1 – III.i.2.

*Arithmetic is the discipline of numbers. For the Greeks call number(s) ἀριθμόν. It is this that the writers of secular literature have wished to be first among mathematical disciplines, because this very discipline requires no other discipline in order to exist. Music, Geometry, and Astronomy, however, which follow (Arithmetic), are in need of the aid of this discipline in order that they might exist and stand firm.<sup>27</sup>*

Isidore's knowledge of arithmetic came largely from the writings of Flavius Magnus Aurelius Cassiodorus, a Roman statesman who served under Theodoric the Great during the 6<sup>th</sup> century C.E.<sup>28</sup> Cassiodorus' writings in turn borrowed liberally from Ancius Manlius Severinus Boethius, who wrote a detailed translation of the Hellenistic-era Greek philosopher and mathematician Nicomachus' Introduction to Arithmetic.<sup>29</sup> Although Isidore celebrates arithmetic as the sole mathematical discipline that can stand on its own, his main interest in it stems from his belief in numerology, specifically as related to the Holy Scriptures. His Liber Numerorum is concerned entirely with the mystic significance and interpretation of numbers in the Bible, and arithmetic, in the modern sense of the word, makes no appearance.<sup>30</sup> Thus, Isidore's entry on arithmetic in The Etymologies is a rather peculiar amalgam of his diverse influences and his own personal beliefs in the pseudo-science of numerology.

Quid praestent numeri: Ratio numerorum contemnenda non est. In multis enim sanctorum scripturarum locis quantum mysterium habent elucet. Non enim frustra in laudibus Dei dictum est: 'Omnia in mensura et numero et pondere fecisti.' Senarius namque [numerus] qui partibus suis perfectus est, perfectionem mundi quadam numeri [sui] significatione declarat. Similiter et quadraginta dies, quibus Moyses et Helias et ipse Dominus ieiunaverunt, sine numerorum cognitione non intelleguntur...Per numerum siquidem ne confundamur instruimur. Tolle numerum in rebus omnibus, et omnia pereunt. Adime saeculo conputum, et cuncta ignorantia caeca

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<sup>27</sup> Trans. Costrell, Sarah.

<sup>28</sup> Grant, *The Foundations of Modern Science in the Middle Ages*. p. 14-15.

<sup>29</sup> Brehaut. p. 124.

<sup>30</sup> Brehaut. p. 29.

conplectitur, nec differri potest a ceteris animalibus, qui calculi nesciunt rationem.<sup>31</sup>

*What numbers show: The reckoning of numbers ought not to be condemned. For in many passages of the Holy Scriptures it illuminates how much mystery the Scriptures hold. For not in vain has it been said in praises of God: 'You have made all things in measure, number, and weight.' For the number six which is perfect in its own parts, declares the perfection of the world in a certain signification of its own number. Similarly, in the forty days, in which Moses and Elijah and God himself fasted, without the knowing of numbers, this would not be understandable...In fact, through number we are instructed lest we be confused. Take number out from all things, and all things perish. Take away computation from time-span, and blind ignorance encircles all things, and he cannot be different from the other animals, who do not know the reckoning of calculations.<sup>32</sup>*

Isidore's praise of the merits of computational knowledge perhaps indicates a vestigial awareness of the heights to which "reckoning of numbers" was raised during the Classical and Hellenistic ages of Greek thought, but the example he uses, of being able to ascertain the full meaning of the forty days that Moses, Elijah and God spent fasting, reassigns this importance to numerology, a body of knowledge that was not truly part of the learning of the quadrivium. Thus, although he acknowledges the subject area, Isidore here seems to be unable to appropriately explicate the concepts that were of such great import to Greek mathematical advancement.

Isidore places music among the mathematical disciplines, a choice made by some of his contemporaries, but by no means an obvious one; Plato, for example, includes music as a type of literary learning, beneficial for youths in an ideally-governed society.<sup>33</sup> Once music had been placed in the quadrivium, however, it remained there, among

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<sup>31</sup> Isidore, *Etymologiarum*. III.iv.1 – III.iv.4.

<sup>32</sup> Trans. Costrell, Sarah.

<sup>33</sup> Plato. *The Republic*.

arithmetic, geometry and astronomy, until the end of the Middle Ages.<sup>34</sup> This is not an unfounded decision, as thinkers going back as far as the Pythagoreans in the 6<sup>th</sup> cent. B.C.E. were able to discern a connection between harmony and ratios of whole numbers.<sup>35</sup> Isidore, however, seems to have no conception of the actual mathematical characteristics of music. He makes oblique references to its importance as regards some sort of “universal harmony,”<sup>36</sup> but he lacks the evidence or methodology to back up this type of sweeping claim. Furthermore, he clearly does not understand basic music theory, as he makes no mention of modes, already studied by his Greek predecessors, and is unable to define keys without conflating them with other musical subjects.<sup>37</sup> He alludes to the Pythagorean explorations in harmony and ratio by trying to explain a sort of “musical mean.”

De numeris musicis: Numeros autem secundum musicam ita quaeris. Positis extremis, utputa VI et decas dipondius, vides quot monadibus superetur VI a XII, et est VI monadibus: ducis per quadratum, sexies seni faciunt XXXVI. Coniungis extrema illa prima, VI ad XII, simul efficiunt XVIII. Partiris tricies sexies per decas octo, efficit dipondius. Hos iungis cum summa minore, id est sexies, erunt VIII et erit medium inter VI et XII. Quapropter VIII superant VI duabus monadibus, id est tertia de VI, et superantur VIII a XII quattuor monadibus, tertia portione. Qua parte ergo superat, eadem superatur. Sed haec ratio quemadmodum in mundo est ex volubilitate circulorum, ita et in microcosmo in tantum praeter vocem valet, ut sine ipsius perfectione etiam homo symphoniis carens non constet. Eiusdem musicae perfectione etiam metra consistunt in arsi et thesi, id est elevatione et positione.<sup>38</sup>

*On musical numbers: You seek numbers supporting music thus. With extremes (high and low numbers) having been set out, as for an example,*

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<sup>34</sup> Brehaut. p. 134.

<sup>35</sup> Oxford Classical Dictionary, 3<sup>rd</sup> ed. rev. p. 1283 – 1284.

<sup>36</sup> Isidore, Etymologiarum. III.xvii.1.

<sup>37</sup> Woodridge. p. 33.

<sup>38</sup> Isidore, Etymologiarum. III.xxiii.1 – III.xxiii.2.

*six and twelve, you see by how many monads six is surmounted by twelve, and it that is by six monads: You guide this into a square, and six in six groups makes thirty-six. You add together the first extremes, six and twelve, and together they make eighteen. You divide apart thirty-six through eighteen, and this makes two. You add these with main smaller one, that is, six, and they are eight, and that is the middle between six and twelve. Wherefore eight surmounts six by two monads, that is, one-third of six and these are surmounted: eight from twelve by four units, by a third portion. Therefore this surmounts by this part, the same part it is surmounted by. But just as this proportion in the world is from the whirling motion of the circles, thus in a microcosm it is powerful beyond such a voice, that without the perfection of this very thing, man, lacking harmony, does not exist. Also by the perfection of this same music, meters occur in arsis and thesis, that is, by going up and settling back down.<sup>39</sup>*

It takes much effort to glean anything of value from this passage. The Pythagorean method of mathematical harmony concerned the ratios 2:1, 3:2, and 4:3, which represented the lengths of strings corresponding to the octave, the fifth, and the fourth.<sup>40</sup> Yet for some reason, Isidore's meandering takes him in and around the realm of even numbers between two and thirty-six. He sometimes seems to be honing in a rational, i.e., ratio-based, relationship, but he never quite gets there. In addition, he attempts to direct his non-explanation about harmony towards the consideration of meter along the same terms, but the only connection he can make is that meter oscillates up and down, much in the same way that his strange interpretation of ratios leads him through the high and low numbers. As noted, this is indicative of the character of the mathematical portion of this text. Isidore makes cognitive contact with the fruits of Greek thought, but fails to reach a meaningful interpretation of that which he has found.

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<sup>39</sup> Trans. Costrell, Sarah.

<sup>40</sup> Oxford Classical Dictionary, 3<sup>rd</sup> ed. rev. p. 1283 – 1284.

The science of astronomy flourished particularly in Hellenistic Alexandria, led by the advanced thinking of Geminus and the elusive yet influential Cleomedes.<sup>41</sup> Isidore, while preserving some of the notions promoted by pre – Classical and Classical thinkers, omits the body of work produced in later periods.

De caelo et eius nomine: Caelum philosophi rotundum, volubile atque ardens esse dixerunt; vocatumque hoc nomine, eo quod tamquam vas caelatum inpressa signa habeat stellarum. Distinxit enim eum Deus claris luminibus, et inplevit sole scilet et lunae orbe fulgenti, et astrorum micantium splendentibus signis adornavit...<sup>42</sup>

*On heaven and its name: The philosophers said that heaven is round, spinning and burning; and is called by this name, because, just as if a heavenly dish, it has the signs of the stars pressed into it. For god distinguished heaven with white lights, and evidently covered it with the sun and with the gleaming orb of the moon, and adorned it with the splendors of twinkling stars...*<sup>43</sup>

In this passage, Isidore seems to be referring to the “fixed – sphere theory,” in which the earth is posited to be surrounded by a fixed sphere of stars and planets. This hypothesis was put forth by Parmenides, Plato, and Aristotle, among others, and further transmitted by Ptolemy.<sup>44</sup> Several notable astronomers of the later period, however, such as Aristarchus of Samos and Seleucus of Seleucia, endorsed a heliocentric view of the universe, thereby expanding the solar system past a closed sphere of stars and out into the possibility of an infinite cosmos.<sup>45</sup> Thus, Isidore takes his information from the earlier tradition, but ignores innovations made closer to his own time.

De magnitudine solis: Magnitudo solis fortior terrae est, unde et eodem momento, quum oritur, et orienti simul et occidenti aequaliter apparet...<sup>46</sup>

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<sup>41</sup> Russo. pp. 88-91.

<sup>42</sup> Isidore, Etymologiarum. III.xxxi.1 – III.xxxi.2.

<sup>43</sup> Trans. Costrell, Sarah.

<sup>44</sup> Russo. p. 86.

<sup>45</sup> Russo. p. 88.

<sup>46</sup> Isidore, Etymologiarum. III.xlvii.1.

*On the vastness of the sun: The vastness of the sun is greater than the earth, whence at the same moment, when it rises, it appears equally to the east and to the west at the same time...*<sup>47</sup>

De itinere solis: Sol oriens per meridiem iter habet. Qui postquam ad occasum venerit et Oceano se tinxerit, per incognitas sub terra vias vadit et rursus ad orientem recurrit.<sup>48</sup>

*On the journey of the sun: The sun, rising, has a journey through the south, after which it comes to the west and plunges itself in the ocean, and wanders under the earth in unknown paths, and again hurries back to the east.*<sup>49</sup>

Isidore's statements here can likely be construed as products of his geocentric worldview. Some issues are rather apparent, however; in the first selection he states that the sun, upon rising, "appears equally to the east and to the west at the same time." This seems to be factually inaccurate and his goal in including this observation is contextually unclear; perhaps he wishes to remark on the sun's shape or the way it stretches across the sky, showing vastness even at a distance. Another problem arises in Isidore's dual assumption that the sun is vaster than the earth and that the sun is part of a fixed celestial sphere. He acknowledges the spherical nature of the earth and in his remarks on the "vastness" of the sun he may be acknowledging its sphericity, as well. To fix this vast sun in a rotating sphere, while still keeping it close enough to hug the earth in its nightly travels, seems to suggest an incompatibility of radius lengths. It is questionable as to whether Isidore's logic went this far; it is certain that he is continuing to endorse an outmoded geocentric universe without taking the comparatively recently developed heliocentric model into account.

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<sup>47</sup> Trans. Costrell, Sarah.

<sup>48</sup> Isidore, Etymologiarum. III.lii.1.

<sup>49</sup> Trans. Costrell, Sarah.

In Isidore's writings, we see evidence of a vast degeneration of the body of scholarship developed in the Classical era. In the centuries following the end of ancient science, a point often dated to the 415 A.D. lynching of the female philosopher-mathematician Hypatia of Alexandria by a mob of fanatical Christians, it is clear that even those who sought specifically to pursue new channels of learning were unable to do so, owing to the massive destruction of books and systematic killing of those who espoused so-called "pagan" areas of thought.<sup>50</sup> Isidore's position as part of the educated clergy would seem to make him well-equipped for the job of encyclopaedist, but beshadowed as he was by the cloak of dominant Christian thought and impaired by the lack of key Classical texts, his works could not possibly stand as the collection of knowledge he sought to represent. Indeed, as part of the learned and literate class of his age, his logical fallacies and factual misrepresentations show that even the intellectuals of this period could not resurrect the magnificent feats of learning lost scant centuries before. Even at this relatively early point in the Middle Ages, it is clear that key scientific and mathematical ideas, formerly disseminated across Europe and North Africa, were no longer fully understood by those who wished to make use of them.

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<sup>50</sup> Russo, p. 15.

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