Arthur Cayley and Mathematics Education

Arthur Cayley (1821-1895) was the epitome of the mathematical researcher and though he was not easily diverted from the pursuance of new mathematics he was also interested in wider issues and particularly mathematics education. He had definite views on what should be taught to both university students and to those at school level. He had little experience with the practice of teaching but was driven by the importance of topics within mathematics. His position as England’s leading pure mathematician of the late nineteenth century makes his views of significance in the history of mathematics education.

Cayley was from a comfortably off middle-class family. He showed an interest in mathematics from an early age, and at King’s College Senior School in London he was admitted at the age of fourteen entering the school two years earlier than the normal entrance age. This school gave him a superb mathematical background – the only rival school in England from this point of view was the University College School (UCL) across town where Augustus De Morgan led the way. The leading public schools in England (confusingly private schools such as Harrow, Eton, Winchester) of the time specialised in Classics and devoted very little time to mathematics, a few hours a week at most. The two London schools covered a whole range of modern subjects besides mathematics including avant-garde subjects as Chemistry and Physics as well as modern languages. Different in their guiding religious philosophies – King’s was staunchly Anglican, UCL not affiliated to any particular creed – both schools enlisted Cambridge graduates on their teaching staff and these came with a background informed by the famous mathematical tripus. Cayley had probably covered much of the Cambridge undergraduate course at school and in the first term at Cambridge in 1838 he was reading advanced French texts influenced by his tutor George Peacock. With a precocious talent he graduated as the top student in 1842. At Cambridge he gained a strong idea of what was valuable in mathematics and this informed
his attitudes to mathematics education when he was later called on to give advice.

In the 1850s Cayley was employed as a barrister in London but kept up his mathematics and wrote many research papers, including path breaking papers on invariant theory and matrix algebra. Until he was appointed as the first professor of pure mathematics at Cambridge in 1863 his own teaching experience was limited to teaching private pupils, taking tutorials and setting examinations at Cambridge, and conducting an evening course in London. When he was appointed to the professorship, he was required to apply himself to the advancement of his subject, but his duty also made it his responsibility to ‘explain and teach the principles of Pure Mathematics’. He actually had few teaching duties – a course of lectures in one term of the year on a subject of his own choosing. The matter of these lectures was rarely allied to the undergraduate teaching syllabus and he had few students. Few benefited from his erudition – an early exception was W. K. Clifford who proved an outstanding disciple.

Shortly after his appointment he was attacked by George Biddle Airy (the Astronomer Royal) for teaching abstruse mathematics which Airy categorised as ‘useless algebra’. Airy wanted students to be taught mathematics that was useful in physical subjects. Cayley objected that Airy’s plan only led students to mathematics which had yet to be developed and at that point they were helpless – they could only progress as far as the textbook solutions. Cayley had as an ideal, students who could develop their own mathematical ideas. He was implicitly thinking of students being independent in mathematics but could not see them taking a step that even professional mathematicians had not yet taken. The debate between these two leaders was the constant battle between practical ends and the means of reaching them. Airy might have been satisfied with his students being merely familiar with the mathematics which could be applied but Cayley was guided by the student’s potential for understanding the mathematics behind the applications.

Cayley’s principal connection with mathematics education was in the furore generated by the position of Euclid’s *Elements* in English school education. With the introduction of geometry into the school curriculum in the second half of the nineteenth century it was realised by teachers that the emphasis on Euclid had its limitations. Examinations required pupils to present Euclid’s arguments and to present them in the correct Euclidean logical order. The result was that geometry teaching was often reduced to a memory test and though the poor pupils ‘wrote it by rote they did not write it right’. Such was the concern that a committee of the British Association for the Advancement of Science was appointed in the late 1860s to look into the teaching of geometry. In true Victorian style it was top heavy with university academics but with few practising teachers. Cayley was there not for his experience in teaching but as a professor at Cambridge and the authority that this conferred.

Cayley is sometimes seen as the arch Conservative who held onto Euclid in the teeth of opposition, a crusty old man who dogmatically refused to break with tradition because he was against all change. He was actually guided in his opposition by his admiration for Euclid for its mathematical content, and in particular the famous Book V in which the theory of irrational numbers was set out.

The vexed question of Euclid’s *Elements* continued to simmer in the 1870s. The Association for the Improvement of Geometrical Teaching (AIGT) founded in 1870 by teachers (following an earlier ‘Anti-Euclid Association’, and the forerunner of the Mathematical Association) had produced an alternative school syllabus to the one based on Euclid and a textbook of its own. In continuing to campaign for the abolition of Euclid, the Association petitioned the University of Cambridge for a reduction of the emphasis on the text of Euclid’s sacred book in the school examinations. It was a very modest request. In 1887 an appeal to the
Cambridge Board of Mathematical Studies was signed by its members and prominent supporters:

We, the undersigned Members of the Association for the Improvement of Geometrical Teaching, and others interested in the teaching of Elementary Geometry, believing in the teaching of that subject than is consistent with a rigid adherence to the letter of Euclid’s *Elements* is highly desirable, would welcome such a change in the Examinations in Elementary Geometry conducted by the Universities and other Examining Bodies, as would admit of the subject being studied from Text-books, other than editions of Euclid, without the student being thereby placed at a disadvantage in those examinations.

To Cayley’s mind, Euclid’s treatment of geometry was superior to both the algebraic versions or the various simplified texts being made available. When one of the schoolteacher deputation pointed out that the authorised version of *Euclid* was Robert Simson’s edition, Cayley suggested striking out Simson’s additions and keeping strictly to the original treatise (such a proposal provoked a senior member of the University to whisper that to study *Euclid* in the original Greek would be better still).

The appeal of *Euclid* to Cayley, pure and unadulterated, reflected his own education at King’s College London and his undergraduate days at Cambridge. Euclid had been a firm fixture on the King’s curriculum when he was a pupil there, and, at Cambridge, a central plank of the undergraduate course, where Peacock taught that Euclid’s *Elements* approached perfection. In bringing together his thoughts on a Cambridge education, the architect of the modern mathematical tripods William Whewell had written in his Principles of English University Education (1837) that ‘Euclid has never been superseded, and never will be so without detriment to education’.

Cayley connected with the timelessness of *Euclid* as that other opponent, the erstwhile Oxford tutor Charles L. Dodgson, [the writer Lewis Carroll], who wrote: ‘neither thirty years, nor thirty centuries, affect the clearness, or the charm, of Geometrical truths’. The whole weight of tradition, both personally and institutionally, was therefore against change.

Cayley held an unwavering position, yet his excessive admiration for Euclid’s original text and his high minded approach underlined his lack of contact with the teaching of mathematics in schools. After the meeting, a member of the Board confessed to a schoolmaster delegate that ‘we cannot go against Cayley’:

He [Cayley] was absolutely irreconcilable. … [in his view the pupils] had the advantage of a thoroughly good book, and the further advantage of one single book instead of a number of books with slight variations. The sort of book the Association desired would really differ very little from Euclid; and while, if it were an open question, some parts of it might in themselves be improvements, they were unessential, and the gain would be far from compensating for the loss of the advantage of one book, and of studying geometry as it was studied by the Greeks. For the purpose of a liberal education it was best to study it in the form in which it had been studied from the first: as it stood, the form was part of the education.
In Cayley’s defence, he always admitted his little contact with the teaching of elementary mathematics. Cayley and schoolchildren with their ‘Euclid got by heart and not understood’ were separated by a gulf dividing two mutually exclusive viewpoints. When he was once asked about the parts of elementary mathematics in which advanced students were most deficient he answered with plain honesty: ‘My classes are very small and as I give lectures only I naturally assume in my hearers a sufficient acquaintance with the lower subjects’.

It was clear that Cayley’s lofty position was untenable in the long run. In hindsight one can appreciate the wisdom of his counterpart at Oxford, Henry’s Smith, who thought that Euclid was written for men while in England it was administered to children. After Cayley’s death in 1895, the academic argument for its retention unraveled. His successor to the Cambridge professorship A. R. Forsyth sided with the revolutionaries and with the new century a new era began.

**Tony Crilly, UK**

[mailto:t.crilly@btinternet.com](mailto:t.crilly@btinternet.com)

---

**Conference reports**

**History and Pedagogy of Mathematics**

The Americas Section of the HPM held its annual meeting at the MAA Carriage House in Washington, DC, April 19, 2008.

The program for the meeting was:

**Amy Ackerberg Hastings** (University of Maryland University College):

“The Acknowledged National Standard”:

**Charles Davies, A. S. Barnes and Textbooks as Teaching Tools.**

Book historians have added a number of dimensions to our understanding of texts in the history of science and mathematics, including how readers and publishers participate alongside authors in the transmission of knowledge, how patterns of use indicate intellectual reception, and how textbooks communicate scientific ideas to popular audiences. However, promotion has been at least as important a factor as pedagogical and intellectual superiority in determining which objects have become widely established instruments for teaching mathematics and science. This talk explores the evolution of the textbook into a commercialized teaching tool by concentrating on how the partnership of Charles Davies (1798-1876) and Alfred Smith Barnes (1817-1888) shaped mathematics instruction in the United States. Davies parlayed his reputation as a professor at the United States Military Academy at West Point into a successful career of defining himself primarily as a producer of textbooks. Barnes, his publisher, organized the books into graded series and utilized aggressive marketing techniques. Together, the men sought to enlarge their audience of American students and laid claim to national status as the standard for the nascent mathematics textbook industry. This talk is based upon the first chapter of Tools of American Mathematics Teaching, 1800-2000, a forthcoming book prepared jointly with Peggy Aldrich Kidwell and David Lindsay Roberts, and will include a few highlights from the entire volume.

**Alain Touwaide** (President, Washington Academy of Sciences Historian, Smithsonian Institution):

*Leafing through History: An Imaginary Walk through an Ancient Library.*

This will be an overview of ancient scientific books, their writing, their illustration, their production, and their history, with a presentation of some important mathematical collections. The talk will be illustrated with images of ancient books, from papyrus to 16th and early 17th-century printed works.
Florence Fasanelli (American Association for the Advancement of Science): Some Frontispieces in the Michalowicz Collection at American University.

These frontispieces dating from 1647 to 1792 played an important role in the design of the mathematics texts that accompanied them by portraying images of the works that followed. Who made them and why as well as where and how will be discussed as we look carefully at the pages.

Peggy Aldrich Kidwell (National Museum of American History, Smithsonian Institution): The Chinese Tangram, Mathematical Recreations, and Mathematics Education in the United States

The tangram is a flat, seven-piece puzzle consisting of a square, a rhombus, and 5 isosceles right triangles of differing sizes. Invented in China in the late eighteenth century, the tangram may well have been the first mathematical recreation that could be described as a global fad. The puzzles reached the United States from Europe in 1817. In this same period, there was considerable emphasis on teaching arithmetic and even the elements of geometry to young children. At mid-century, Unitarian minister and textbook author Thomas Hill proposed using simplified tangrams in introductory geometry, and designed a set of puzzle pieces and pattern cards for school use. Hill's cards were only modestly successful, but fit nicely with the general growth in apparatus for elementary mathematics teaching at the time, as well as kindergarten apparatus being developed in Germany by Friedrich Froebel. Tangrams have been a mathematical recreation, sometimes used in mathematics teaching, ever since.

Betty Mayfield and Kimber Tysdal (Hood College):

Last summer we supervised a group of four undergraduate students in research that focused on women and mathematics in 18th century Europe. We structured our program like a traditional REU, with speakers and field trips and collaboration as well as individual research projects. We will report on our experiences and on what our students learned about the history of mathematics.

Maryam Voulis (University of New Haven):
History of Arab Cryptanalysis.

This presentation will discuss the contributions of medieval Muslim scientists to cryptography and cryptanalysis.

Ilhan Izmirli (American University):
Necessary Are Proofs in Mathematics? A Brief History.

This talk will trace the idea of proof from Babylonian to Greek, medieval, and modern mathematics to Lakatosian ideas. It will also address the role of proof in mathematical pedagogy.

Bob Stein (California State University, San Bernardino):
The Math Wars and Culture.

In recent decades, attempts to improve mathematics education in the US have been marked by fundamental differences that flare up as “math wars”. We will consider those differences in terms of Hofstede’s dimensions of
culture and their implications for mathematics education. The talk will be illustrated with personal experiences, notably with the California Mathematics Framework revision of 1997.

Next year HPM Americas Section will again meet at the MAA Carriage House in Washington, DC, either on the weekend of March 13-14 or March 20-21.

Bob Stein  
San Bernardino, US

IVth HTEM  
The IVth HTEM – Workshop on history and technology in the teaching of mathematics was held in Rio the Janeiro, Brazil, on the campus of the Federal University of Rio de Janeiro (UFRJ), May 5th to 9th, 2008.  
The focus of the workshop was continued education of mathematics teachers, celebrating the 200th anniversary of the arrival of the Portuguese Cortes in Brazil, the first step for independence of Brazil from Portugal. The transfer of the ruling house of the Portuguese empire to its colony signaled widespread changes in culture, education and the sciences in Brazil.  
The main speakers of the workshop were  
David Tall (UK, University of Warwick): Historical and Individual Development of Mathematical Thinking: Ideas that are Set-Before and Met-Before  
Evelyne Barbin (France, Université de Nantes): The notion of Magnitude in History and in Teaching  
Fidel Oteiza (Chile, Universidade de Santiago de Chile: La Aplicación de Un Modelo para el Aprendizaje Matemático y sus Implicaciones para la Formación Inicial y Continua de Docentes  
Gert Schubring (Germany, Universität Bielefeld): Gauss e a Tábua dos Logaritmos  
Masami Isoda (Japan, University of Tsukuba): Challenges for Developing Pedagogical Content Knowledge Through Lesson Study – using historical sources for teacher education.  
Michelle Artigue (France, Université Paris VII): Digital technologies and mathematics education: reflecting on twenty years of research in that area  

René Grothmann (Germany, Katolischen Universität Eichstatt-Ingolstadt): Past, Present and Future of Computers in Schools  
See http://limc.ufrj.br/htem_en/index.php/Program for the full program.  
Joao Bosco Pitombeira  
PUC do Rio de Janeiro, Brazil

The editors welcome reports from conferences.

Work in progress  
We encourage young researchers in fields related to HPM to send us a brief description of their work in progress or a brief description of their dissertation.

New Books  

Table des matières:  
Introduction.  
Friederich GAUSS. La présentation axiomatique de William Rowan HAMILTON.

Bibliographie

**Synopsis:**
This book examines the evolution of machine design methodology from the Renaissance to the Age of Machines in the 19th century. This premise is based in part on the scholarly work of da Vinci scholar Ladislo Reti who translated the last discovered work of Leonardo da Vinci in 1967. In the Codex Madrid, Reti found evidence that Leonardo planned to write a book on basic machine elements and compared the great artist-engineer's drawings to the work of 19th C. machine theorist Franz Reuleaux of Berlin. Reuleaux is credited with classifying the basic elements of machine design and also enumerating six basic classes of mechanisms to change motion from one form to another.

**The Machines of Leonardo Da Vinci and Franz Reuleaux**
Kinematics of Machines from the Renaissance to the 20th Century,
by Francis C. Moon,
Springer, 2007
The book hopes to convince the reader that the development of a rational design methodology for machines that grew from the time of Leonardo to the early 20th century was as great a feat as the invention of the machines themselves.

**Have you read these?**


---

HPM Newsletter No. 68 July 2008
HPM webpage: [http://www.elab.edc.uoc.gr/hpm/](http://www.elab.edc.uoc.gr/hpm/)


---

**Have you been here?**

**New links in this issue**

**Digitization of the oldest extant manuscript of Euclid's Elements**

A complete digital edition of the manuscript of Euclid's Elements copied by Stephen the Clerk in Constantinople in the year 888 AD is available in [http://librarieswithoutwalls.org/bookviewer/](http://librarieswithoutwalls.org/bookviewer/)

---

*The digitization of this manuscript was a joint project of the Bodleian Library, the Clay Mathematics Institute, and Octavo.com.*

**Oxford Museum of the History of Science**

[www.mhs.ox.ac.uk/exhibits/](http://www.mhs.ox.ac.uk/exhibits/)

[http://www.mhs.ox.ac.uk/measurer/text/title.htm](http://www.mhs.ox.ac.uk/measurer/text/title.htm)

[http://www.mhs.ox.ac.uk/geometry/title.htm](http://www.mhs.ox.ac.uk/geometry/title.htm)

[http://www.mhs.ox.ac.uk/scienceislam/](http://www.mhs.ox.ac.uk/scienceislam/)

**Societies and organisations**

**Commission on the History of Mathematics in Africa** (including newsletter) [http://www.math.buffalo.edu/mad/AMU/amumha_online.html](http://www.math.buffalo.edu/mad/AMU/amumha_online.html)

**Association des Professeurs de Mathematiques de l'Enseignement Public [APMEP]** History site: [http://www.apmep.asso.fr/BMhist.html](http://www.apmep.asso.fr/BMhist.html)

HOMSIGMAA - History of Mathematics
Special Interest Group of the MAA
http://www.maa.org/sigmaca/hom

HPM Americas
http://www.hpm-americas.org/

Italian Society of History of Mathematics
http://www.dm.unito.it/sism/indexeng.html

Association pour la Recherche en Didactique des Mathématiques:
http://www.arbm.asso.fr/

Commission Française pour l'Enseignement des Mathématiques: http://www.cfem.asso.fr/

Instituts de Recherche sur l'Enseignement des Mathématiques (IREM):
http://www.univ-irem.fr/

Canadian Society for History and Philosophy of Mathematics
http://www.cshpm.org

Brazilian Society for History of Mathematics
http://www.sbhmat.com.br

Nuncius Newsletter
http://brunelleschi.imss.fi.it/nuncius/inln.asp?c =5302

International History, Philosophy and Science Teaching Group
www.ihpst.org

Centre for the History of the Mathematical Sciences.
The Open University, UK
http://puremaths.open.ac.uk/pmd_research/CHMS/index.html

Topics and Resources

MATHS for EUROPE: The history of some aspects of mathematics like: history of mathematical persons, symbols, algorithms...
http://mathsforeurope.digibel.be/index.html
http://mathsforeurope.digibel.be/list.htm
http://mathsforeurope.digibel.be/olvp.htm
http://mathsforeurope.digibel.be/olvp2.htm
http://mathsforeurope.digibel.be/olvp3.htm

Ethnomathematics on the Web
http://www.rpi.edu/%7Eeglash/isgem.dir/links.htm

About Medieval Arabic Numbers
http://www.geocities.com/rmlyra/Numbers.html
http://www.geocities.com/rmlyra/arabic.html

Annotated Bibliography on Proof in Mathematics Education
http://fcis.oise.utoronto.ca/~ghanna/educationabstracts.html

BibM@th
http://www.bibmath.net/dico/index.php3?action=rub&quoi=0

Centro Virtual de Divulgación de las Matemáticas, esta siendo desarrollada por la Comisión de Divulgación de la Real Sociedad Matemática Española (R.S.M.E.)
http://www.divulgamat.net/index.asp

History of Statistics
http://www.stat.ucla.edu/history/

Images of Lobachevsky’s context
http://www.ksu.ru/eng/museum/page0.htm

Images of Mathematicians on Postage Stamps
http://members.tripod.com/jeff560/index.html

Photos of Mathematicians
http://www.math.uni-hamburg.de/home/grothkopf/fotos/math-ges/
Numdam-Digitization of ancient mathematics documents  

The Montana Mathematics Enthusiast (journal)  
http://www.montanamath.org/TMME/

Convergence: an online magazine of the MAA providing resources to teach mathematics through its history  
http://convergence.mathdl.org/

International Journal for Mathematics Teaching and Learning,  
http://www.cimt.plymouth.ac.uk/journal/default.htm

Homepage of International Journal for the History of Mathematics Education  
http://www.tc.edu/centers/ijhmt/index.asp?Id=Journal+Home

Documents for the History of the teaching of mathematics in Italy  
http://www.dm.unito.it/mathesis/documents.html

Ethnomathematics Digital Library  
http://www.ethnomath.org/

Some Japanese Mathematical Landscapes:  
The results of wandering in a beautiful country, with a mathematical eye, aided by a digital camera, by A. Arcavi  

Wann-Sheng Horng's webpage  
with HPM related materials in Chinese.  
http://math.ntnu.edu.tw/~horng/

Fred Rickey's History of Mathematics Page  
http://www.dean.usma.edu/math/people/rickey/hm/default.htm

CultureMATH. Ressources pour les enseignants de Mathématiques  
www.dma.ens.fr/culturemath/actu/livres.htm

The French INRP (National Institute for Pedagogical Research) is developing a website on questions related to mathematics teaching:  
EducMath  
http://educmath.inrp.fr

Homepage of Albrecht Heeffer  
http://logica.ugent.be/albrecht/

Homepage of Jens Hoyrup  
http://www.akira.ruc.dk/~jensh/

L’Enseignement Mathématique, Archive  
http://retro.seals.ch/digbib/vollist?UID=ensmat-001

Homepage of Prof. Leo Corry  
http://www.tau.ac.il/~corry/

Opera Mathematica of Christoph Clavius  
http://mathematics.library.nd.edu/clavius/

Geometrical books and instruments from 15th to 18th century  
http://www.geometricum.com/

David Henderson’s Home Page  
[Educational and Historical Topics on Geometry]  
http://www.math.cornell.edu/~dwh/

Archimedes Project [Some famous mathematical books of the Renaissance period are available on line, i.e. Pacioli’s Summa]  
http://archimedes2.mpiwg-berlin.mpg.de/archimedes_templates

Simon Stevin’s De Meetdaet [The Practice of Measuring]  
http://www.math.leidenuniv.nl/~wiskonst/meetdaet/index.html

and The Principal Works of Simon Stevin  
http://www.historyofscience.nl/works_detail.cfm?RecordId=2702

Mathematicians Gallery  
History of Mathematics
http://www.otterbein.edu/resources/library/libpages/subject/maththis.htm

The Garden of Archimedes. A museum for Mathematics
http://web.math.unifi.it/archimede/archimede_NEW_inglese/

Mathematical instruments
http://brunelleschi.imss.fi.it/museum/esim.asp?c=500164
and
http://web.mat.bham.ac.uk/C.J.Sangwin/Sliderules/sliderules.html
and
http://www.mhs.ox.ac.uk/epact/catalogue.php?ENumber=52265

Homepage of Eleanor Robson
http://www.hps.cam.ac.uk/dept/robson.html

We would like to provide a more comprehensive list of websites containing resources useful to researchers and students (not necessarily in English). If there are any you use, or you know are useful for students or researchers, please send your recommendations to the editors.

Notices

The Mathematical Education of Greek Women during the 19th century

The education of women in Greek territory, namely Greek communities of the Ottoman Empire and Greek state, first appears at the beginning of the 19th century with the operation of a small number of non-systematically organized schools. Women’s systematic education begins in 1830s and 1840s for the state and communities respectively, with the operation of a) communal, self-sustained and “national” educational institutions in the communities’ territory, b) state institutions – under the responsibility of municipalities – in the Greek state, and c) private schools in both of the aforementioned spaces of Hellenism.

Women’s education constitutes a distinct educational system, separate from that of men, not only as regards its structure, the courses provided and the duration of studies, but also as regards the school curricula (taught subjects and their content). This differentiation results from the fact that female education was organized on the basis of the “different nature” and the “different social destination” of women – gendered perceptions that were further reinforced during 19th century by the socio-biological theories (Social Darwinism) that had a great impact on Greek scholars and pedagogues.

With respect to the mathematic education of women, judging by the Math’s teaching goals and textbooks, the school curricula, the timetables and the educators’ published opinions with regard to the role of Mathematics and Sciences in female’s education, it emerges that their Mathematics education was limited at both range and type of content. For example, Algebra was rarely taught, Geometry when appeared in the syllabuses embodied only very elementary units, whereas girls were taught from Arithmetic only the four basic skills, i.e. addition, subtraction, multiplication, and division, and in exceptional cases women’s instruction included the teaching of decimal numbers. Also, their mathematical education was practically oriented towards their future social destination and the needs that women would be called to meet in the private sphere as wives, mothers and house-keepers. The shrunken provision of mathematical knowledge was founded on ‘scientific’ stances of the era concerning the limited intellectual capabilities of women, along with the damage that intellectual strain causes to their reproductive (!) ability.

This overall attitude towards the mathematical education of women was differentiated to some extent on the basis of:

a) The time period of the schools’ operation. More particularly, up to 1830 (for the areas that comprised the first dominion of the Greek state) or to 1840 (for the Greek
communities remained within the dominion of the Ottoman Empire) Mathematics wasn’t always among the courses taught in the schools for girls, as the primary purpose of those few, short-lived first girls’ schools was the systematic instruction of the “female duties and works”. During the first period of girls’ systematic education, namely 1840 up to decade of 1860, Mathematics was included in all schools’ syllabuses and their relevant education was shaped wider and more enhanced in perspective to the following period (1870 -1900), due to the influence of innovatory principles regarding women’s education of Greek Enlightenment Practical [Elementary] and Theoretical Arithmetic, Geometry, Cosmography were included in the schools’ syllabuses and the hours devoted to girls’ mathematical education were 8,3% to 10,5% of the weekly total teaching hours. Thus from 1870 onwards despite the completion of girls’ educational system and the extension of their studies duration, mathematical education was reduced in regard with the taught units and teaching hours devoted to it. Algebra was excluded from the syllabuses, Theoretical Arithmetic was taught in few only schools (usually in the self-sustained schools) and Geometry was limited to Descriptive and Plane Geometry, meanwhile it continued to be taught only in one school year as it used to during the previous educational period. Also the hours in the school timetable devoted to Mathematics were decreased to 6% as the subject stopped being included in the syllabus of all school years.

b) The geographical region where schools operated. In the area of Smyrna (Izmir, in Turkish) for instance, the competitive functioning of Greek schools towards the missionary and western schools, that were first founded at the area, the strong impact of the New Greek Enlightenment’s tenets and especially of some celebrated Greek Enlighteners who took action at the area (foundation of innovative schools or restructuring of the curricula of existing establishments), as well as the endorsement and financial support to “the new ideas and new knowledge” by the emerging middle class lead to more radical educational attitudes towards girls education in general and their mathematical education particularly. As soon as the first Greek schools for girls were founded their curricula provided an ample knowledge in Mathematics, (including Algebra, major number of units in Geometry and in one case Trigonometry) as well as in Sciences. It is worth to be mentioned that the necessity of the Sciences’ and the Mathematics’ instruction in the schools for girls is frequently underlined by women mostly pedagogues in their public speeches and their published opinions.

c) The administrative type of schools, which was related to their social class orientation. Regardless of the geographical region and – to some extent – the educational period, in the self-sustained schools, where girls of the superior social class studied, not only the selection of the provided knowledge but also the teaching methodology and the media of instruction were more innovative and wider. The syllabuses of these schools exceeded the mathematical and Sciences knowledge provided – apart from the stereotypical mathematical units taught in all schools for girls – to the instruction of a greater number of units in Geometry, to Algebra, Astronomy, Trigonometry and – very rarely – to the instruction of Nautical Theory.

Katerina Dalakoura, Greece
dalakoura@phl.uoc.gr

Dividing and Composing the Squares

The Islamic craftsman, faced with a belief system that restricted forms of visual display, demonstrated his celebration of Islam by creating attractive abstract geometrical patterns. These very patterns show the role of geometry in Islamic arts. Numbers also played an important part in association with geometry, giving a personality to a shape; the number four, associated with the square, represented stability.
Abu l-Wafa (4th century hejira, 10th century AD) describes innovative cut and paste constructions in his famous book on applied geometry: *On those parts of geometry needed by craftsman*. In part 9 of the book he explains methods for dividing and re-assembling squares that do not rely on Pythagoras’ Theorem (also known as the Bride’s Chair theorem by Arabic writers). He also worked with artisans and provided solutions to their questions. There is evidence to suggest that this approach to geometry was not restricted to 4th century (10th century AD) Baghdad but was common throughout the Islamic world. The examples explained by Abu’l Wafa provide descriptive proofs that make a link between theoretical and applied geometry, particularly useful for craftsmen. In addition, he seems well informed of links between number theory and geometry.

The main purpose of my study was to investigate the applied methods by Abu l-Wafa for dividing and composing squares. In addition, I have attempted to explain such patterns related to squares in attractive tiling and wooden arts in Isfahan as a workshop for my students. Then students could see applied geometry in Islamic arts clearly.

Educationally, since geometry is so appealing, we can use the subject to motivate students to take more interest in mathematics and particularly geometry learning. This is very important for increasing and fostering the creative power of students. In addition, searching historical fundamentals through manuscripts and Islamic constructions increased their self-dependence.

**Methods for Dividing and Composing Squares**
Abu l-Wafa classified numbers into two classes, *squares* and *non-squares*. In addition, numbers were classified into two classes: first those that are equal to the sum of two other square numbers, such as, $13 = 4 + 9$, or $41 = 16 + 25$; second those that are not equal to the sum of two squares such as seven. Where numbers are squares or equal to two squares, changing it to smaller squares is very simple.

Here, I explain Abu l-Wafa’s methods through activities suitable for the classroom:

**Activity 1)** Divide squares with two and three lengths side to some congruent squares. (Figure 1)

**Activity 2)** Compose 16 congruent squares to a square. (Figure 2)

**Activity 3)** For squares that are equal to the sum of two squares, make a square.

a) **If two squares are equal:**

**Activity 3-1)** we want make a square from two $1 \times 1$ squares (figure 3).

(Since $2 = 1^2 + 1^2$ we can divide any $1 \times 1$ square in half along a diagonal; these diagonals become the sides of the new square.)

**Activity 3-2)** we want make a square from two $2 \times 2$ squares (figure 4).

(Since $8 = 2^2 + 2^2$ we can divide any $2 \times 2$ square in half along a diagonal; these diagonals become the sides of the new square.)
Figure 4: composing a square from two $2 \times 2$ squares.

b) If two squares are not equal:

Activity 3-3) $13 = 4 + 9 = 2^2 + 3^2$

(Since $13 = 6 + 6 + 1$ first, divide into two $2 \times 3$ rectangles and a unit square, then divide the rectangles along their diagonal; arrange the resulting triangles around the unit square.) (Figure 5)

Figure 5: composing $2 \times 2$ and $3 \times 3$ squares.

Activity 3-4) $20 = 16 + 4$

(Since $16 = 8 + 8 + 4$ first, divide into two $2 \times 4$ rectangles and a $2 \times 2$ square, then divide the rectangles along their diagonals, then arrange resulting triangles around the square.) (Figure 6)

Figure 6: composing squares.

The following patterns can result from the above figures:

Activity 4) Using a non-square number of equal squares to make a square, for example making a square from three equal squares. In this section Abu l-Wafa explained that artisans made errors when applying this problem. He investigated their methods then rejects some solutions, and provided better solutions for this problem.

First, we cut two squares in half along their diagonals. Then we will lay any part on the sides of the third square as in figure 8:

Figure 8: composing three square to make a ‘square’.

Activity 5) Using some unequal squares to make a square. This method applies generally when we use two squares and to generate others. First, we will lay the smaller square on the other square. Then cut increased parts from larger square. We will cut a new small square and two rectangles and divide along their diagonal, and then cut those and arrange the resulted part as figure 9:
Activity 6) We want to cut a square with certain side length from another square. First, draw semi-circles with square side length and then by drawing arcs with radius equal to the desired square side mark points on the arcs. Then we will join these points. The resulting square is smaller than the desired square. Now, we will cut these parts and make a rectangle from any two triangles and then cut small square from one rectangle and arrange those as figure 10:

Figure 10: cutting a square from other square.

The following pattern is from an applied geometry book. It is from an appendix written several centuries after Buzjani (Jazbi believes the writer is Koobnaani, but Alpay believes the author is unknown). Consider squares and rectangles.

Activity 7) In this section, students have provided some artful samples by following pattern. By survey tiling, wooden arts and other Islamic arts in Isfahan we found very attractive samples.

Samples of geometrical patterns in Isfahan constructions

Finally, here are some illustrations of applied square patterns that the students are able to see around them in Isfahan.

Isfahan Friday mosque: the date of building construction referred to 4th to 11th hejira (AD 10th-17th) century.
Here are some more four squares tilings.

**Figure 12:** North Iwan, four symmetric divided square.

These tilings have applied pattern figure 8.

**Figure 13:** An attractive square tiling, South Iwan.
**Figure 14:** South Iwan perspective.

**Figure 15:** West Iwan perspective.
**Figure 16:** West Iwan, and square tiling.
Figure 17: West Iwan, and square tiling.
Figure 18: West Iwan, and square tiling that covered by Ali name.

In the next picture, you can see a geometrical shape that covered by poem that known as *katibeh khate bannaee* and consist of a poem about Imam Ali:

Figure 19: *katibeh* in West Iwan.

Sheikh lotf ol lah mosque: this construction built in 11th hejira (AD 17th century) by order of *Shah Abbas Safavi*. One of archaeologists said: 'hardly, I can say that this building has been constructed with human hands'.

Figure 20: repeated pattern.

Figure 21: wooden door covered by repeated pattern.

**Khan mosque:** the date of building khan mosque referred to 1090 hejira (AD 17th century) by order of *Shah Soleiman Safavi*. The brick tiling constructions in khan mosque is very attractive and unique.
**Jame’e Abbasi mosque:** building mosque started in *Safavi age* (1020 hejira/17 AD century).

**Chahar bagh madrasa:** the beautiful building in *Shah Soltan Hosein Safavi* (1116-1126 hejira/AD 17th century).

**Conclusion**

The results of this research indicate, by implication, that with a historical background
in mathematics education, particularly geometry, the learner can make links between the arts and mathematics. Our students generally ask where is mathematics used, then we can state and analyse such valuable samples to improve their mathematical thinking, and so foster their creativity power. By using these patterns, we will teach abstract subjects as applied subjects. In addition, by using the geometrical patterns, students could have a better understanding about geometrical concepts such as rotation, symmetry, reflection, transformation, mapping, translation and dilation.

References

Narges Assarzadegan, Iran
narges.assarzadegan@gmail.com

Narges Assarzadegan, a high school teacher from Isfahan, Iran has written before (Newsletter 62) about the Persian manuscript Labb Al-Hisab (Book of Reckoning). Here she describes some cut and paste activities she designed for her class (Grade 11, age 14), based on a book by Abul- Wafa Buzdani (4th Hejira/AD 10th century).

An introduction to wasan, native Japanese mathematics

Japanese culture is much indebted to the Chinese one, including its mathematics. However, at the beginning of the seventeenth century, under the Tokugawa shogunate, Japan entered a period of sakuko, seclusion from the outside world. Japanese were not allowed to go abroad and foreigners were either forbidden in Japan or very restricted in their movement. The Dutch were limited in number and confined to a small island before Nagasaki. Remaining Chinese and Koreans were placed under strict supervision. Despite this seclusion, but thanks to this period of peace and security, Japan enjoyed a period of development of arts and crafts. It is in this era that wasan developed as a native kind of mathematics distinct from its Chinese heritage. Some philosophers have raised the question: could mathematics develop different from the way it did in history? In a way, because of its isolated development, the study of wasan provides an almost experimental situation to explore this question. There were many schools in the major cities of Japan teaching wasan in a way somewhat similar to European abaco schools or rechenmeister in Germany and Flanders. The best known schools are those of Seki and later Aida. Contrary to the European context however, wasan served little practical purpose. It was a sort of mathematics mainly pursued for its intellectual and esthetic aspects. Good examples of this are the so-called sangaku problems. These are problems of geometrical nature which were displayed on wooden tablets at the entrance of Shinto shrines and Buddhist temples. An impressive collection of them including a mathematical analysis was published by Fukugawa and Pedoe (1989). See figure 1 for an example.


Wasan thrived until the second half of the nineteenth century. At the beginning of the eighteenth century under Yoshimune...
Tokugawa,\textsuperscript{1} who reigned from 1716 to 1744, the prohibition of foreign books became somewhat relaxed. The shogun himself showed an interest in Western astronomy and calendar composition and started building a library, of which later a catalogue as been found (Heeffer 2008). In the middle of the nineteenth century Kyō Uchida taught mathematics to several hundreds of students in Tokyo, adopting the Dutch word \textit{Mathematische} for the name of his school. Any Dutch book that could be found on one of the ships in Nagasaki became a precious item and circulated within a small group of Japanese scholars called \textit{rangakusha}. These consisted of medical doctors, astronomers, gunnery instructors and mathematicians of the samurai class in service of feudal lords. Early nineteenth century there even was a book dealer in Yedo (now Tokyo) who specialized in Dutch books. In 1857 the \textit{Yōsan Yōhō} \textit{(洋算用法, The method of Western arithmetic)} was published by Yanagawa Shunzō. It introduces Japan to the Western (Dutch) method of calculating. Shortly after the Meiji restoration of 1868 it was decided that only \textit{yōzan} \textit{(Western mathematics)} was to be taught at Japanese schools. In a matter of a few years \textit{wasan} became completely neglected and \textit{wasan} books were replaced by schoolbooks translated from French, English and German. Many \textit{wasan} books were discarded and only used for their paper. However, some scholars collected these precious books and manuscripts, One of them being Tsuruichi Hayashi (1873 – 1935) who became the first historian to publish on \textit{wasan} in a Western language.\textsuperscript{2} He left an enormous collection of 14,470 \textit{wasan} books and manuscripts, now the bulk part of the \textit{wasan} collection at Tohoku University. Together with two other minor collections, Tohoku university library now holds the largest collection of \textit{wasan} books in the world with 18,335 items. It is estimated that this accounts for two-thirds of the books produced on this subject. It gives us an idea of the spread of this tradition in Japanese society and the influence it must have had.

Hayashi published a well-received series of articles in the Dutch mathematical journal \textit{Nieuw Archief voor Wiskunde}. His “Brief History of the Japanese Mathematics” \textit{(sic)} is the first published account of the \textit{wasan} tradition in a Western language (Hayashi 1905a, b). Mikami (1909) published a reaction to Hayashi correcting him on several points, later followed by a more extensive history on \textit{wasan} (Mikami 1913). This publication was further augmented by Smith (1914) and turned into a popular book, at this time still the only English book on the subject. There are not so many Western publications on \textit{wasan} since then.\textsuperscript{3} The most extensive study to date is in French by Annick Horiuchi (1994) who primarily studied the works of Seki and his student Katahiro Takebe. While Martzloff (1997) writes on Chinese mathematics, he also provides valuable links to \textit{wasan}, especially on its Chinese heritage. Despite its limited appearance in Western publications on the history of mathematics the study of \textit{wasan} in Japan is blossoming more than ever since the Meiji restoration. There are several academic centers in Japan where scholars study the original sources and there is The Wasan Institute in Tokyo which publishes a dedicated journal and regular books with facsimiles and commentaries. In addition, there is a lively scene of amateurs, collecting and studying \textit{wasan} literature.\textsuperscript{4}

\textsuperscript{1} Contrary to Japanese custom, I will list Japanese names with their first name first.
\textsuperscript{2} The parallel with Western book collectors and historians of mathematics is obvious. Guillaume Libri, who published the classic \textit{History of mathematics in Italy} (Libri 1838) owned thousands of books and manuscripts, most of which were sold during his lifetime. At some point he was accused of having stolen them from libraries. The many appendices in this three-volume work contain transcriptions from books and manuscripts of his own library, including the \textit{Trattato d’abaco} by Pierro della Francesca. The other example is David Eugene Smith who wrote the two volume work \textit{History of mathematics}. According to an inventory made in 1940 he owned over 20,000 items on mathematics, including 11,000 rare books, numerous manuscripts, autographs and portraits, now part of the Columbia university library collection (Simons 1945).
\textsuperscript{3} For a list of \textit{wasan} publications in Western languages see http://logica.ugent.be/albrecht/wasanbib.htm.
\textsuperscript{4} This combination of academic and amateur efforts may seem exceptional but there is also a similar
So what is now so special about *wasan*? What makes it different from Western mathematics? Let us first look at the subjects treated. They can be broadly divided into three categories: arithmetic and algebra, geometry and recreational problems. Typical arithmetical problems include merchant problems as we also know from Renaissance arithmetic. However, within *wasan* they are mostly related to the use of the *soroban*, the Japanese equivalent of the abacus. Calculation with Hindu-Arabic numerals was not generally known before the 1880’s. I have seen the *soroban* still in use in today’s high-tech Japan. Other recurring themes are the approximation of fractions, infinite series and Bernoulli numbers. Geometrical problems include the calculation of the area of triangles, polygons and rings. In the process of devising techniques for the calculation of the area under a curve, Seki became close to what developed as calculus in the West. Recreational problems concern mostly magic squares and circles of an intricate nature and the *mamakodate* problem, known in the West as the Josephus or Christians and Turks problem. Possibly this last one has been imported into Japan by Portuguese Jesuits but it appears that the problem was known in Japan already from the tenth century! And finally there is algebra which deserves our closer attention.

We can rephrase our question about an alternative mathematics with respect to algebra. Did algebraic solutions to arithmetical and geometrical problems within *wasan* develop into a symbolic algebra as it did in Europe? Opinions on this differ widely, also among Japanese scholars. Ken’ichi Sato devoted an article to the comparison of *wasan* algebra with its Chinese predecessor and concludes that the Japanese version cannot be considered algebra at all and that it “should be recognized as a category independent on that of Western mathematics” (Sato 1995, 65). Chikara Sasaki, on the other hand, in an article comparing Japanese with Western algebra, concludes that precisely because of its symbolic characteristics “Seki’s school of algebra can be compared with that of Viète” (Sasaki 1999, 19). We tend closer to the second position but the situation is more complex than that.

Japanese algebra is derived from the so-called *celestial element method* in China. It first appeared in a Chinese work of 1248 by Li Ye (or Li Shi), *Sea-mirror of the circle measurements*. More important for Japan however was a book written in 1299 and titled *The Introduction to mathematical studies* (*Suan xue qi meng* (SQ), 算学啓蒙) (Lam Lay-Yong, 1979). The Chinese edition of the SQ was lost for a long time until 1839 when a Korean reprint of 1660 was discovered. Though lost in China, it was translated into Japanese in 1658, printed in several editions and widely read. The original Chinese method solves higher degree equations by laying out and manipulating the coefficients of the unknown terms using colored rods, called *sangi*, on a grid. Black rods represent positive values, red ones negatives. In Japan the method was called *tengen jutsu*. Early Japanese texts follow the Chinese method but print the rods as Chinese characters for numbers. For example, the equation $11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$ is represented (using Hindu-Arabic numerals) as in table 1.

| TABEL 1: THE REPRESENTATION OF AN EQUATION BY COEFFICIENTS ON A GRID |
|---|---|---|---|---|---|---|---|
| 0000 | 1000 | 100 | 10 | 1 |   |
| 1 | 1 | 5 | 2 | 0 | 実 |
| - | 4 | 3 | 2 | x | 方 |
| - | 2 | 3 | 6 | $x^2$ | 廉 |
| 4 | $x^3$ | 偶 |
| 1 | $x^4$ |   |
So, moving from a tangible form of performing algebra to a printed representation involves already a step towards symbolization. The rods, as material means, were replaced by Chinese characters. The red colored rods become replaced by a negative sign (a diagonal dash).

A further development introduced by Seki was the method of side writing. The idea probably evolved from the custom of annotating Chinese characters with hiragana (the Japanese phonetic script) to show how to pronounce them. At some point Seki started adding literals to the numerical coefficients in the grids to tackle more general problems. For example: given a rectangle with an area of $A$ and the sum of its length and width being $B$, how to determine the length and width. Take $x$ (the celestial element) to be the length, then the width will be $(B - x)$ and the area $x(B - x) = A$. The coefficients to be placed on the grid would then be $-1$, $1B$ and $-1A$ respectively, representing the equation $-x^2 + Bx - A = 0$.

While the method allowed expressing complex general problems in equations, it was kept secret within Seki’s school. In de Hatsubi Sanpō of 1674 Seki solved 15 problems posed in an earlier book without revealing his method of side writing. It was his student Katahiro Takebe who would first explain the method in a commentary published in 1687.

It may be all too easy to draw similarities between Seki and Viète on basis of their use of generalized coefficients. Important differences withhold us to attribute the qualification of symbolic algebra to wasan. With Viète, European algebra evolved away from problem solving to the study of the structure of equations. Important discoveries such as the relations between de roots of an equation and its coefficients, the number of roots and the degree of an equation, and the relation between algebra and geometry are missing in wasan. By the end of the sixteenth century the foundations of European algebra became completely transformed as part of the humanist program of providing more solid foundations based on Greek roots. Algebra became a normative model for a *mathesis universalis*, a universal symbolic language to approach the laws of nature, and as such had a decisive influence on the development of Western science. In contrast, wasan contributed nothing to the understanding of nature. Despite its ability to solve complex problems in an efficient way, the wasan tradition failed to produce a single law on mechanics, hydraulics or kinematics. Even when Dutch books on Newtonian physics became available during the eighteenth century, Japanese scholars did not resort to wasan methods of algebra for its mathematical foundations and instead struggled with ways to represent Western symbols (Ravina 1993, 213). The failure of embedding wasan within other intellectual traditions made it possible to replace it by Western mathematics in a very short period during the Meiji restoration.

The current growth of interest in wasan is not only from a historical perspective. The rich history of wasan and its enormous legacy of woodblock printed books provide ample opportunities of using historical material in the classroom. Being an integral part of Japanese culture and its close relation with religion (*sangaku*) makes wasan an excellent example of the use of history for mathematics education. Several projects have been started along this line (Leung e.a. 2006). Specific lessons have been developed by specialized institutes as the CRICED from Tsukuba University.\(^5\) Placing traditional wasan methods of problem solving against their Western counterparts provides students with a better understanding of the development of mathematics and the way mathematics is part of our social and cultural legacy.

**References**


---

\(^5\) For a lesson example of teaching algebra by means of *sangi*, see the website of the CRICED at Tsukuba. http://math-info.criced.tsukuba.ac.jp/Forall/project/history/2003/sangi/sangi_index.html


Albrecht Heeffer, Belgium
albrecht.heeffer@ugent.be

---

**Announcements of events**

**ICME-11**

**July 6-13, 2008**

Monterrey, Mexico


of special interest is the

**TSG 23: The role of history of mathematics in mathematics education**

See HPM Newsletter 67 or [http://tsg.icme11.org/tsg/show/24](http://tsg.icme11.org/tsg/show/24) for more information on this.

**HPM 2008**

**History and Pedagogy of Mathematics**

The HPM Satellite Meeting of ICME 11

**July 14-18, 2008**

Mexico City, Mexico


**Plenary Activities:**

The following plenary lectures will be given:

David Pengelley (New Mexico State University, USA): *Teaching mathematics with primary historical sources: Should it go mainstream? Can it?*

---

6 Fellow of the Research Foundation Flanders (FWO). This paper was written during a research visit at Center of Cross-cultural Studies of Kobe University, Japan. The author wishes to thank his host professor Nobuo Miura for the support and hospitality.


Glen Van Brummelen (Quest University, Canada): Crossing Cultures, Oceans, Religions, and the Cosmos: In Search of the Origins of Trigonometry.

Alejandro García Diego (FC, UNAM), Ricardo Quintero (DME – Cinvestav IPN), Alberto Camacho (Instituto Tecnológico de Chihuahua II), Francisco Cordero (DME – CINVESTAV, IPN), Gisela Montiel (Prome – Cicata, IPN). Title to be announced.

Evelyne Barbin (Université de Nantes, France): Dialogism in mathematical writing: historical, philosophical and pedagogical issues.

Ubiratan d’Ambrosio (Pontificia Universidade, Catolica de Sao Paulo, Brazil): The transmission and acquisition of Mathematics in Colonial and Early Independent Countries in the Americas.


Participants wishing to register after June 15, 2008, should pay on the spot 250$ / 150$ (for students).

HPM2008@red-cimates.org.mx

Models in Developing Mathematics Education (10th International Conference of The Mathematics Education into the 21st Century Project)

September 12-18, 2008
Dresden, Germany
For further information contact arogerson@inetia.pl

A note from the Editors

The Newsletter of HPM is primarily a tool for passing on information about forthcoming events, recent activities and publications, and current work and research in the broad field of history and pedagogy of mathematics. The Newsletter also publishes brief articles which they think may be of interest. Contributions from readers are welcome on the understanding that they may be shortened and edited to suit the compass of this publication.
Distributors:
If you wish to be a distributor in a new or unstaffed area please contact the editor.

<table>
<thead>
<tr>
<th>Area</th>
<th>Name and address</th>
<th>Email address</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>Juan E. Nápoles Valdés, Lamadrid 549, (3400) Corrientes, ARGENTINA</td>
<td><a href="mailto:idic@ucp.edu.ar">idic@ucp.edu.ar</a></td>
</tr>
<tr>
<td>Australia</td>
<td>G. FitzSimons, Faculty of Education, P.O.Box 6, Monash University, 3800 Victoria, AUSTRALIA</td>
<td><a href="mailto:gail.fitzsimons@education.monnash.edu.au">gail.fitzsimons@education.monnash.edu.au</a></td>
</tr>
<tr>
<td>Austria</td>
<td>Manfred Kronfellner, Institute of Discrete Mathematics and Geometry, Vienna University of Technology, Wiedner Haupstr. 8-10, A-1040 Wien, AUSTRIA</td>
<td><a href="mailto:m.kronfellner@tuwien.ac.at">m.kronfellner@tuwien.ac.at</a></td>
</tr>
<tr>
<td>Belgium and The Netherlands</td>
<td>Sylvia Eerhart, Freudenthal Instituut, Aidadreef 12, 3561 GE Utrecht, THE NETHERLANDS</td>
<td><a href="mailto:S.Eerhart@fi.uu.nl">S.Eerhart@fi.uu.nl</a></td>
</tr>
<tr>
<td>Canada</td>
<td>Thomas Archibald, Mathematics Department, Acadia University, Wolfville, NS B0P1X0, CANADA</td>
<td><a href="mailto:Tom.Archibald@acadiau.ca">Tom.Archibald@acadiau.ca</a></td>
</tr>
<tr>
<td>China</td>
<td>Ma Li, Linkoping University, ITN, SE - 601 74 Norrkoping, SWEDEN</td>
<td><a href="mailto:mali@itn.liu.se">mali@itn.liu.se</a></td>
</tr>
<tr>
<td>Eastern Europe</td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>Evelyne Barbin, Centre François Viète, Faculté des sciences et des techniques, 2 Chemin de la Houssinière, BP 92208, 44322 Nantes cedex, FRANCE</td>
<td><a href="mailto:evelyne.barbin@wanadoo.fr">evelyne.barbin@wanadoo.fr</a></td>
</tr>
<tr>
<td>Germany</td>
<td>Gert Schubring, Inst. f. Didaktik der Math., Universitaet Bielefeld, Postfach 100 131, D-33501, Bielefeld, GERMANY</td>
<td><a href="mailto:gert.schubring@uni-bielefeld.de">gert.schubring@uni-bielefeld.de</a></td>
</tr>
<tr>
<td>Iran</td>
<td>Mohammad Bagheri, P.O.Box 13145-1785, Tehran, IRAN</td>
<td><a href="mailto:saii5@sina.sharif.edu">saii5@sina.sharif.edu</a></td>
</tr>
<tr>
<td>Israel</td>
<td>Ted Eisenberg, Mathematics Department, Ben Gurion University of the Negev, Beer-Sheva 84105, ISRAEL</td>
<td><a href="mailto:eisen@math.bgu.ac.il">eisen@math.bgu.ac.il</a> <a href="mailto:eisenbt@barak-online.net">eisenbt@barak-online.net</a></td>
</tr>
<tr>
<td>Italy</td>
<td>Giorgio T. Bagni, Department of Mathematics and Computer Science, University of Udine, Polo Rizzi, via delle Scienze 206, I-33100 Udine, ITALY and Marta Menghini, Dipartimento di Matematica (Universita’ La Sapienza), Piazzale A. Moro 5, 00185 Roma ITALY</td>
<td><a href="mailto:bagni@didi.uniud.it">bagni@didi.uniud.it</a> <a href="mailto:marta.menghini@uniroma1.it">marta.menghini@uniroma1.it</a></td>
</tr>
<tr>
<td>Japan</td>
<td>Osamu Kota, 3-8-3 Kajiwara, Kamakura Kanagawa-ken, 247-0063 JAPAN</td>
<td><a href="mailto:kota@asa.email.ne.jp">kota@asa.email.ne.jp</a></td>
</tr>
<tr>
<td>Malaysia</td>
<td>Mohamed Mohini, Department of Science and Mathematical Education, Universiti Teknologi Malaysia, 81310 Johor, MALAYSIA</td>
<td><a href="mailto:mohini@fp.utm.my">mohini@fp.utm.my</a></td>
</tr>
<tr>
<td>Mexico</td>
<td>Alejandro R. Garciadiego, Caravaggio 24, Col. Nonoalco Mixcoac Del. Benito Juárez 03700 México, D. F. México</td>
<td><a href="mailto:gardan@servidor.unam.mx">gardan@servidor.unam.mx</a></td>
</tr>
<tr>
<td>Morocco</td>
<td>Abdellah El Idrissi, E.N.S. B.P: 2400 Marrakech, C.P: 40 000, MOROCCO</td>
<td><a href="mailto:a_elidrissi@hotmail.com">a_elidrissi@hotmail.com</a></td>
</tr>
<tr>
<td>New Zealand</td>
<td>Bill Barton, Mathematics Education Unit, Dept of Mathematics and Statistics University of Auckland, Private Bag 92-019, Auckland, NEW ZEALAND</td>
<td><a href="mailto:b.barton@auckland.ac.nz">b.barton@auckland.ac.nz</a></td>
</tr>
<tr>
<td>Other East Asia</td>
<td>Gloria Benigno, Department of Education, Culture and Sports, Region X, Division of Misamis Occidental, Oroquieta City, PHILIPPINES</td>
<td><a href="mailto:glorya4444@yahoo.com">glorya4444@yahoo.com</a></td>
</tr>
<tr>
<td>Russia</td>
<td>Vasili Mikhailovich Busev</td>
<td><a href="mailto:vbusev@yandex.ru">vbusev@yandex.ru</a></td>
</tr>
<tr>
<td>Scandinavia</td>
<td>Sten Kajser, Department of Mathematics, P.O. Box 480, SE-751 06 Uppsala, SWEDEN</td>
<td><a href="mailto:sten@math.uu.se">sten@math.uu.se</a></td>
</tr>
<tr>
<td>South America</td>
<td>Marcos Vieira Teixeira , Departamento de Matemática , IGCE - UNESP, Postal 178 13 500 – 230 Rio Claro, SP BRAZIL</td>
<td><a href="mailto:marti@rc.unesp.br">marti@rc.unesp.br</a></td>
</tr>
<tr>
<td>South Asia</td>
<td>Prof. R. C. Gupta, Ganita Bharati Academy, R-20, Ras Bahar Colony, Jhansi-284003, U.P. INDIA</td>
<td></td>
</tr>
<tr>
<td>Region</td>
<td>Contact Details</td>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td><strong>South East Europe</strong></td>
<td>Nikos Kastanis, Department of Mathematics, Aristotle University of Thessaloniki, Thessaloniki 54006, GREECE</td>
<td></td>
</tr>
<tr>
<td><strong>Southern Africa</strong></td>
<td>Paulus Gerdes, Mozambican Ethnomaths Research Centre, C.P. 915, Maputo, MOZAMBIQUE</td>
<td></td>
</tr>
<tr>
<td><strong>Spain and Portugal</strong></td>
<td>Carlos Correia de Sá, Departamento de Matemática Pura; Faculdade de Ciências da Universidade do Porto; Rua do Campo Alegre, 687 P - 4169 - 007 Porto; Portugal</td>
<td></td>
</tr>
<tr>
<td><strong>Taiwan</strong></td>
<td>Wann-sheng Horng, Math dept NTNU, 88 Sec.4, Tingchou Rd., Taipei, TAIWAN</td>
<td></td>
</tr>
<tr>
<td><strong>Turkey</strong></td>
<td>Funda Gonulates, Bagazici Universitesi, Egitim Fakultesi, Bebek- Istanbul, TURKEY</td>
<td></td>
</tr>
<tr>
<td><strong>United Kingdom</strong></td>
<td>Chris Weeks, Downeycroft, Virginstow, Beaworthy, GB - EX21 5EA, GREAT BRITAIN</td>
<td></td>
</tr>
<tr>
<td><strong>United States of America</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

HPM Newsletter editorial team

Nikos Kastanis  Chris Weeks  Bjørn Smestad
Table of contents

Arthur Cayley and Mathematics Ed p. 1
Conference Reports p. 4
History and Pedagogy of Math. p. 4
IVth HTEM p. 6
New Books p. 6
Have you read these? p. 8
Have you been here? p. 9
Notices p. 12
The Mathematical Education... p. 12
Dividing and Composing... p. 13
An introduction to wasan... p. 20
Announcements of events p. 24
Distributors p. 26

The views expressed in this Newsletter may not necessarily be those of the HPM Advisory Board.
Please pass on news of the existence of this newsletter to any interested parties.
This and previous newsletters can be downloaded from our website:
http://www.clab.edc.uoc.gr/hpm/

Items for the Newsletter should be sent to the editors, preferably by email (see addresses below).
The Newsletter appears three times a year with the following deadlines for next year.

<table>
<thead>
<tr>
<th>No.</th>
<th>Deadline for material</th>
<th>Sent to distributors</th>
</tr>
</thead>
<tbody>
<tr>
<td>69</td>
<td>12 October 2008</td>
<td>1 November 2008</td>
</tr>
<tr>
<td>70</td>
<td>12 February 2009</td>
<td>1 March 2009</td>
</tr>
<tr>
<td>71</td>
<td>12 June 2009</td>
<td>1 July 2009</td>
</tr>
</tbody>
</table>

The Newsletter is the communication of the International Study Group on the Relations between the History and Pedagogy of Mathematics, an affiliate of the International Commission on Mathematical Instruction.
The Newsletter is free of charge, available upon request from the distributor for your area, and may be reproduced with acknowledgement.

Editors:

Nikos Kastanis, nikos@math.auth.gr (Department of Mathematics, Faculty of Sciences, Aristotle University of Thessaloniki, Thessaloniki 541 24, Greece)
Chris Weeks, chris.weeks@virgin.net (Downeycroft, Virginstow, Beaworthy, GB - EX21 5EA, Great Britain)
Bjørn Smestad, bjorn.smestad@lu.hio.no (Faculty of Education, Oslo University College, Postbox 4 St. Olavs plass, N-0130 Oslo, Norway)