International Study Group on the Relations Between
the HISTORY and PEDAGOGY of MATHEMATICS
An Affiliate of the International Commission on
Mathematical Instruction

No. 68 July 2008

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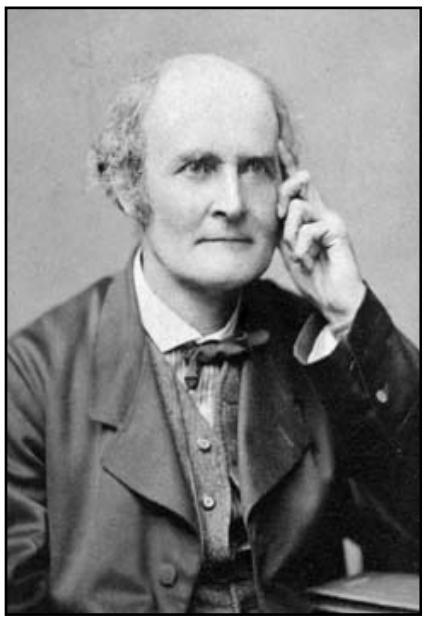
http://www.clab.edc.uoc.gr/hpm/

Arthur Cayley and Mathematics Education

Arthur Cayley (1821-1895) was the epitome of the mathematical researcher and though he was not easily diverted from the pursuance of new mathematics he was also interested in wider issues and particularly mathematics education. He had definite views on what should be taught to both university students and to those at school level. He had little experience with the practice of teaching but was driven by the importance of topics within mathematics. His position as England's leading pure mathematician of the late nineteenth century makes his views of significance in the history of mathematics education.



Cayley was from a comfortably off middle-class family. He showed an interest in mathematics from an early age, and at King's College Senior School in London he was admitted at the age of fourteen entering the school two years earlier than the normal entrance age. This school gave him a superb mathematical background – the only rival school in England from this point of view was the University College School (UCL) across town where Augustus De Morgan led the way. The leading public schools in England (confusingly *private* schools such as Harrow, Eton, Winchester) of the time specialised in Classics and devoted very little time to mathematics, a few hours a week at most. The two London schools covered a whole range of modern subjects besides mathematics including avant-garde subjects as Chemistry and Physics as well as modern languages. Different in their guiding religious philosophies – King's was staunchly Anglican, UCL not affiliated to any particular creed – both schools enlisted Cambridge graduates on their teaching staff and these came with a background informed by the famous mathematical tripos. Cayley had probably covered much of the Cambridge undergraduate course at school and in the first term at Cambridge in 1838 he was reading advanced French texts influenced by his tutor George Peacock. With a precocious talent he graduated as the top student in 1842. At Cambridge he gained a strong idea of what was valuable in mathematics and this informed

his attitudes to mathematics education when he was later called on to give advice.

In the 1850s Cayley was employed as a barrister in London but kept up his mathematics and wrote many research papers, including path breaking papers on invariant theory and matrix algebra. Until he was appointed as the first professor of pure mathematics at Cambridge in 1863 his own teaching experience was limited to teaching private pupils, taking tutorials and setting examinations at Cambridge, and conducting an evening course in London. When he was appointed to the professorship, he was required to apply himself to the advancement of his subject, but his duty also made it his responsibility to 'explain and teach the principles of Pure Mathematics'. He actually had few teaching duties – a course of lectures in one term of the year on a subject of his own choosing. The matter of these lectures was rarely allied to the undergraduate teaching syllabus and he had few students. Few benefited from his erudition – an early exception was W. K. Clifford who proved an outstanding disciple.

Shortly after his appointment he was attacked by George Biddle Airy (the Astronomer Royal) for teaching abstruse mathematics which Airy categorised as 'useless algebra'. Airy wanted students to be taught mathematics that was useful in physical subjects. Cayley objected that Airy's plan only led students to mathematics which had yet to be developed and at that point they were helpless – they could only progress as far as the textbook solutions. Cayley had as an ideal, students who could develop their own mathematical ideas. He was implicitly thinking of students being independent in mathematics but could not see them taking a step that even professional mathematicians had not yet taken. The debate between these two leaders was the constant battle between practical ends and the means of reaching them. Airy might have been satisfied with his students being merely familiar with the mathematics which could be applied but Cayley was guided by the student's potential for understanding the mathematics behind the applications.

Cayley's principal connection with mathematics education was in the furore generated by the position of Euclid's *Elements* in English school education. With the introduction of geometry into the school curriculum in the second half of the nineteenth century it was realised by teachers that the emphasis on Euclid had its limitations. Examinations required pupils to present Euclid's arguments and to present them in the correct Euclidean logical order. The result was that geometry teaching was often reduced to a memory test and though the poor pupils 'wrote it by rote they did not write it right'. Such was the concern that a committee of the British Association for the Advancement of Science was appointed in the late 1860s to look into the teaching of geometry. In true Victorian style it was top heavy with university academics but with few practising teachers. Cayley was there not for his experience in teaching but as a professor at Cambridge and the authority that this conferred.

Cayley is sometimes seen as the arch Conservative who held onto Euclid in the teeth of opposition, a crusty old man who dogmatically refused to break with tradition because he was against all change. He was actually guided in his opposition by his admiration for Euclid for its mathematical content, and in particular the famous Book V in which the theory of irrational numbers was set out.

The vexed question of Euclid's *Elements* continued to simmer in the 1870s. The Association for the Improvement of Geometrical Teaching (AIGT) founded in 1870 by teachers (following an earlier 'Anti-Euclid Association', and the forerunner of the Mathematical Association) had produced an alternative school syllabus to the one based on Euclid and a textbook of its own. In continuing to campaign for the abolition of Euclid, the Association petitioned the University of Cambridge for a reduction of the emphasis on the text of Euclid's sacred book in the school examinations. It was a very modest request. In 1887 an appeal to the

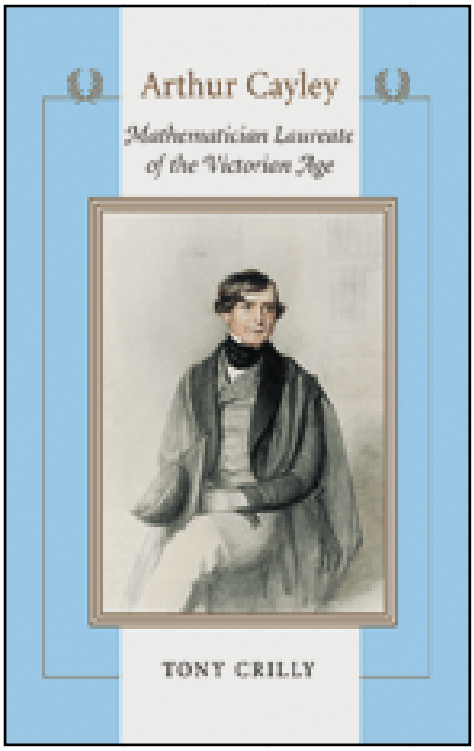
Cambridge Board of Mathematical Studies was signed by its members and prominent supporters:

> We, the undersigned Members of the Association for the Improvement of Geometrical Teaching, and others interested in the teaching of Elementary Geometry, believing in the teaching of that subject than is consistent with a rigid adherence to the letter of Euclid's *Elements* is highly desirable, would welcome such a change in the Examinations in Elementary Geometry conducted by the Universities and other Examining Bodies, as would admit of the subject being studied from Text-books, other than editions of Euclid, without the student being thereby placed at a disadvantage in those examinations.

To Cayley's mind, Euclid's treatment of geometry was superior to both the algebraic versions or the various simplified texts being made available. When one of the schoolteacher deputation pointed out that the authorised version of *Euclid* was Robert Simson's edition, Cayley suggested striking out Simson's additions and keeping strictly to the original treatise (such a proposal provoked a senior member of the University to whisper that to study *Euclid* in the original Greek would be better still).

The appeal of *Euclid* to Cayley, pure and unadulterated, reflected his own education at King's College London and his undergraduate days at Cambridge. Euclid had been a firm fixture on the King's curriculum when he was a pupil there, and, at Cambridge, a central plank of the undergraduate course, where Peacock taught that Euclid's *Elements* approached perfection. In bringing together his thoughts on a Cambridge education, the architect of the modern mathematical tripos William Whewell had written in his Principles of English University Education (1837) that 'Euclid has never been superseded, and never will be so without detriment to education'.

Cayley connected with the timelessness of *Euclid* as that other opponent, the erstwhile Oxford tutor Charles L. Dodgson, [the writer Lewis Carroll], who wrote: 'neither thirty years, nor thirty centuries, affect the clearness, or the charm, of Geometrical truths'. *The whole weight of tradition, both personally and institutionally, was therefore against change.*



Cayley held an unwavering position, yet his excessive admiration for Euclid's original text and his high minded approach underlined his lack of contact with the teaching of mathematics in schools. After the meeting, a member of the Board confessed to a schoolmaster delegate that 'we cannot go against Cayley':

> He [Cayley] was absolutely irreconcilable. … [in his view the pupils] had the advantage of a thoroughly good book, and the further advantage of one single book instead of a number of books with slight variations. The sort of book the Association desired would really differ very little from Euclid; and while, if it were an open question, some parts of it might in themselves be improvements, they were unessential, and the gain would be far from compensating for the loss of the advantage of one book, and of studying geometry as it was studied by the Greeks. For the purpose of a liberal education it was best to study it in the form in which it had been studied from the first: as it stood, the form was part of the education.

In Cayley's defence, he always admitted his little contact with the teaching of elementary mathematics. Cayley and schoolchildren with their 'Euclid got by heart and not understood' were separated by a gulf dividing two mutually exclusive viewpoints. When he was once asked about the parts of elementary mathematics in which advanced students were most deficient he answered with plain honesty: 'My classes are very small and as I give lectures only I naturally assume in my hearers a sufficient acquaintance with the lower subjects'.

It was clear that Cayley's lofty position was untenable in the long run. In hindsight one can appreciate the wisdom of his counterpart at Oxford, Henry's Smith, who thought that *Euclid* was written for men while in England it was administered to children. After Cayley's death in 1895, the academic argument for its retention unraveled. His successor to the Cambridge professorship A. R. Forsyth sided with the revolutionaries and with the new century a new era began.

Tony Crilly, UK
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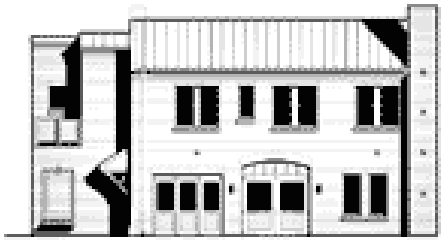
Conference reports

History and Pedagogy of Mathematics

The Americas Section of the HPM held its annual meeting at the MAA Carriage House in Washington, DC, April 19, 2008.



The program for the meeting was:

Amy Ackerberg Hastings (University of Maryland University College):
"The Acknowledged National Standard": Charles Davies, A. S. Barnes and Textbooks as Teaching Tools.

Book historians have added a number of dimensions to our understanding of texts in the history of science and mathematics, including how readers and publishers participate alongside authors in the transmission of knowledge, how patterns of use indicate intellectual reception, and how textbooks communicate scientific ideas to popular audiences. However, promotion has been at least as important a factor as pedagogical and intellectual superiority in determining which objects have become widely established instruments for teaching mathematics and science. This talk explores the evolution of the textbook into a commercialized teaching tool by concentrating on how the partnership of Charles Davies (1798-1876) and Alfred Smith Barnes (1817-1888) shaped mathematics instruction in the United States. Davies parlayed his reputation as a professor at the United States Military Academy at West Point into a successful career of defining himself primarily as a producer of textbooks. Barnes, his publisher, organized the books into graded series and utilized aggressive marketing techniques. Together, the men sought to enlarge their audience of American students and laid claim to national status as the standard for the nascent mathematics textbook industry. This talk is based upon the first chapter of Tools of American Mathematics Teaching, 1800-2000, a forthcoming book prepared jointly with Peggy Aldrich Kidwell and David Lindsay Roberts, and will include a few highlights from the entire volume.

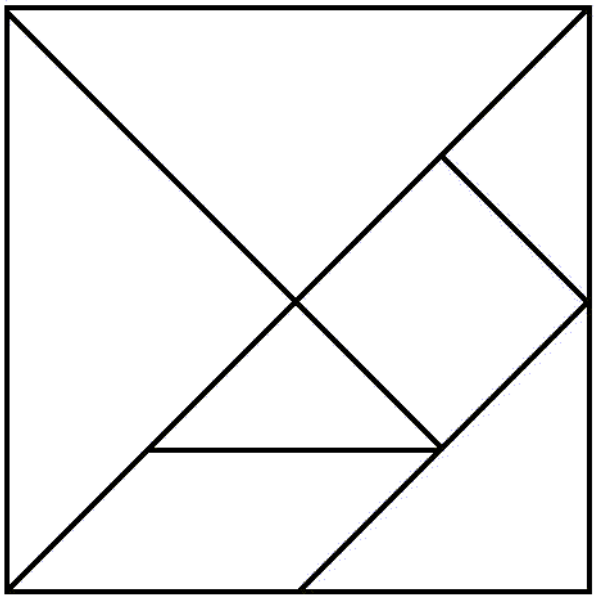
Alain Touwaide (President, Washington Academy of Sciences Historian, Smithsonian Institution):
Leafing through History: An Imaginary Walk through an Ancient Library.

This will be an overview of ancient scientific books, their writing, their illustration, their production, and their history, with a presentation of some important mathematical collections. The talk will be illustrated with images of ancient books, from papyrus to 16th and early 17th-century printed works.

Florence Fasanelli (American Association for the Advancement of Science): *Some Frontispieces in the Michalowicz Collection at American University.*

These frontispieces dating from 1647 to 1792 played an important role in the design of the mathematics texts that accompanied them by portraying images of the works that followed. Who made them and why as well as where and how will be discussed as we look carefully at the pages.

Peggy Aldrich Kidwell (National Museum of American History, Smithsonian Institution): *The Chinese Tangram, Mathematical Recreations, and Mathematics Education in the United States*



The tangram is a flat, seven-piece puzzle consisting of a square, a rhombus, and 5 isosceles right triangles of differing sizes. Invented in China in the late eighteenth century, the tangram may well have been the first mathematical recreation that could be described as a global fad. The puzzles reached the United States from Europe in 1817. In this same period, there was considerable emphasis on teaching arithmetic and even the elements of geometry to young children. At mid-century, Unitarian minister and textbook author Thomas Hill proposed using simplified tangrams in introductory geometry, and designed a set of puzzle pieces and pattern cards for school use. Hill's cards were only modestly successful, but fit nicely with the general growth in apparatus for elementary mathematics teaching at the time, as well as kindergarten apparatus being developed in Germany by Friedrich Froebel. Tangrams have been a mathematical recreation, sometimes used in mathematics teaching, ever since.

Betty Mayfield and **Kimber Tysdal** (Hood College): *Women and Mathematics in the Time of Euler: Undergraduate Research in the History of Mathematics*.

Last summer we supervised a group of four undergraduate students in research that focused on women and mathematics in 18th century Europe. We structured our program like a traditional REU, with speakers and field trips and collaboration as well as individual research projects. We will report on our experiences and on what our students learned about the history of mathematics.

Maryam Voulis (University of New Haven): *History of Arab Cryptanalysis.*

This presentation will discuss the contributions of medieval Muslim scientists to cryptography and cryptanalysis.

Ilhan Izmirli (American University): *Necessary Are Proofs in Mathematics? A Brief History*.

This talk will trace the idea of proof from Babylonian to Greek, medieval, and modern mathematics to Lakatosian ideas. It will also address the role of proof in mathematical pedagogy.

Bob Stein (California State University, San Bernardino): *The Math Wars and Culture.*

In recent decades, attempts to improve mathematics education in the US have been marked by fundamental differences that flare up as “math wars”. We will consider those differences in terms of Hofstede’s dimensions of

culture and their implications for mathematics education. The talk will be illustrated with personal experiences, notably with the California Mathematics Framework revision of 1997.

Next year HPM Americas Section will again meet at the MAA Carriage House in Washington, DC, either on the weekend of March 13-14 or March 20-21.

Bob Stein
San Bernardino, US

IVth HTEM

The IVth HTEM – Workshop on history and technology in the teaching of mathematic was held in Rio the Janeiro, Brazil, on the campus of the Federal University of Rio de Janeiro (UFRJ), May 5th to 9th, 2008.

The focus of the workshop was continued education of mathematics teachers, celebrating the 200th anniversary of the arrival of the Portuguese Cortes in Brazil, the first step for independence of Brazil from Portugal. The transfer of the ruling house of the Portuguese empire to its colony signaled widespread changes in culture, education and the sciences in Brazil.

The main speakers of the workshop were

David Tall (UK, University of Warwick): *Historical and Individual Development of Mathematical Thinking: Ideas that are Set-Before and Met-Before*

Evelyne Barbin (France, Université de Nantes): *The notion of Magnitude in History and in Teaching*

Fidel Oteiza (Chile, Universidade de Santiago de Chile: *La Aplicación de Un Modelo para el Aprendizaje Matemático y sus Implicaciones para la Formación Inicial y Continua de Docentes*

Gert Schubring (Germany, Universität Bielefeld): *Gauss e a Tábua dos Logaritmos*

Masami Isoda (Japan, University of Tsukuba): *Challenges for Developing Pedagogical Content Knowledge Through Lesson Study – using historical sources for teacher education.*

Michelle Artigue (France, Université Paris VII): *Digital technologies and mathematics education: reflecting on twenty years of research in that area*

René Grothmann (Germany, Katolischen Universität Eichstatt-Ingolstadt): *Past, Present and Future of Computers in Schools*

See http://limc.ufrj.br/htem_en/index.php/Program for the full program.

Joao Bosco Pitombeira
PUC do Rio de Janeiro, Brazil

The editors welcome reports from conferences.

Work in progress

We encourage young researchers in fields related to *HPM* to send us a brief description of their work in progress or a brief description of their dissertation.

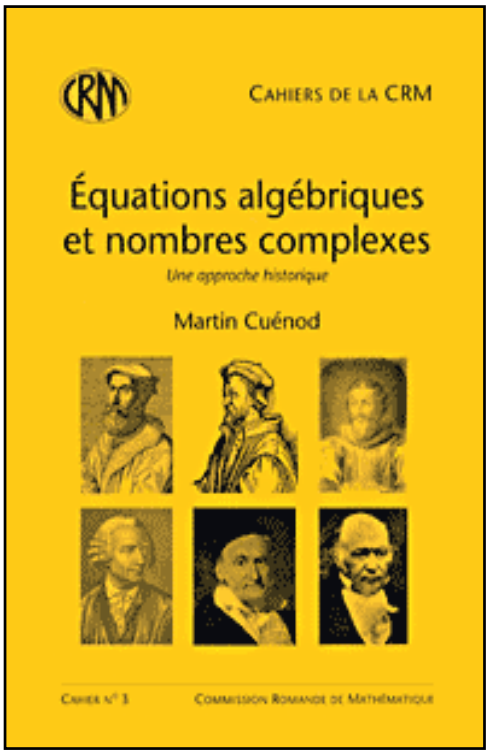
New Books

Équations algébriques et nombres complexes - Une approche historique,

Cahier CRM N° 3.
Martin Cuénod
Commission Romande de Mathématiques, 2006.

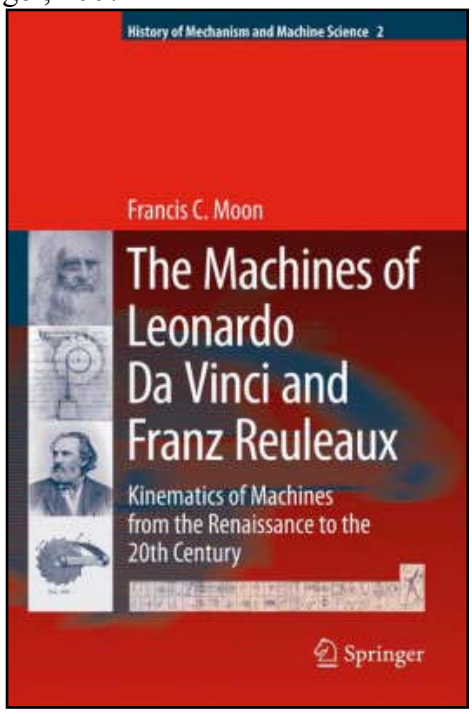
Table des matières: Introduction. Avertissement. Remerciements. Brève histoire de la résolution des équations du deuxième et troisième degree. L'algèbre mésopotamienne. L'algèbre grecque. Un système d'équation du 2e degree. Une équation du 3e degree. La période arabe. Les équations du 2e degree. Les équations du 3e degree. L'algèbre au temps de la Renaissance. La lente naissance des « quantités imaginaires ». Rafaele BOMBELLI. François VIÈTE. Abraham DE MOIVRE. Leonhard EULER. Des « quantités imaginaires » aux « nombres complexes ». Le travail de Jean-Robert ARGAND. La synthèse de Cari

Friederich GAUSS. La présentation axiomatique de William Rowan HAMILTON. Bibliographie



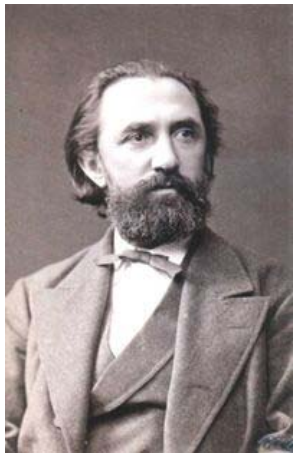
The Machines of Leonardo Da Vinci and Franz Reuleaux

Kinematics of Machines from the Renaissance to the 20th Century,
by Francis C. Moon,
Springer, 2007



Synopsis:

This book examines the evolution of machine design methodology from the Renaissance to the Age of Machines in the 19th century. This premise is based in part on the scholarly work of da Vinci scholar Ladislo Reti who translated the last discovered work of Leonardo da Vinci in 1967. In the Codex Madrid, Reti found evidence that Leonardo planned to write a book on basic machine elements and compared the great artist-engineer's drawings to the work of 19th C. machine theorist Franz Reuleaux of Berlin. Reuleaux is credited with classifying the basic elements of machine design and also enumerating six basic classes of mechanisms to change motion from one form to another.



Moon's book carries Reti's thesis further and provides detailed analysis, comparing design concepts of engineers of the 15th century Renaissance and the 19th century age of machines from a workshop tradition to the rational scientific discipline used today. The technical design ideas for machines of Leonardo and Reuleaux are placed in the historical, economic and social context of their times. There is also an appendix with a short description of the famous “theatre of machines” books of the 15th to the 18th centuries. This book makes use of the unique collection of 230 kinematic models of Reuleaux at Cornell University. Detailed comparisons of 20 basic machine mechanisms such as the slider crank and four-bar linkages in both Leonardo's drawings and Reuleaux’s models are made. These models illustrate the elegance and aesthetics of machine design in the 19th century pioneered by Franz Reuleaux.

The book hopes to convince the reader that the development of a rational design methodology for machines that grew from the time of Leonardo to the early 20th century was as great a feat as the invention of the machines themselves.

Have you read these?

Ben-Miled , Marouane: Mesure le Continu, dans la Tradition Arabe des Libres V et X des Éléments, Arabic Sciences and Philosophy, Vol. 18 (01), 2008, pp 1-18.

Boag, Elizabeth: Lattice multiplication, BSHM Bulletin: Journal of the British Society for the History of Mathematics, Vol. 22 (3), 2007, p.182.

Burn, Bob: Geometric progressions, BSHM Bulletin: Journal of the British Society for the History of Mathematics, Vol. 22 (3), 2007, pp.183-184.

Cellucci, Carlo: The nature of mathematical explanation, Studies in History and Philosophy of Science Part A, 39(2), 2008, pp. 202-210.

Coates, John: Euler's work on Zeta and L-functions and their special values, BSHM Bulletin: Journal of the British Society for the History of Mathematics, Vol. 23(1), 2008, pp. 37-41.

Damerow, Peter. "The material culture of calculation: a theoretical framework for a historical epistemology of the concept of number." In: *Mathematisation and demathematisation: social, philosophical and educational ramifications*, eds.: Gellert, Uwe; Jablonka, Eva. Rotterdam: Sense Publ., 2006 (publ. 2007).

Dauben, Joseph W.: 算數書 Suan Shu Shu A Book on Numbers and Computations, Archive for History of Exact Sciences, Vol. 62 (2), 2008, pp. 91-178.

Folina, Janet: Intuition Between the Analytic-Continental Divide: Hermann Weyl's Philosophy of the Continuum, *Philosophia Mathematica,* 16, 2008, pp. 25-55.

Gray, Jeremy: A short life of Euler, BSHM Bulletin: Journal of the British Society for the History of Mathematics, Vol. 23(1), 2008, pp. 1-12.

Gregory, Lavers: Carnap, Formalism, and Informal Rigour, *Philosophia Mathematica,* 16, 2008, pp. 4-24.

Hartshorne, Robin: Publication history of von Staudt's Geometrie der Lage, Archive for History of Exact Sciences, Vol. 62 (3), 2008, pp. 297-299.

Kent, Deborah: The Mathematical Miscellany and The Cambridge Miscellany of Mathematics: Closely connected attempts to introduce research-level mathematics in America, 1836–1843, *Historia Mathematica,*Vol. 35(2), 2008, pp. 102-122.

Mallion, Roger: A contemporary Eulerian walk over the bridges of Kaliningrad, BSHM Bulletin: Journal of the British Society for the History of Mathematics, Vol. 23(1), 2008, pp. 24-36.

Mason, John: Bartering problems in arithmetic books 1450-1890, BSHM Bulletin: Journal of the British Society for the History of Mathematics, Vol. 22 (3), 2007, pp.160-181.

Mormann, T.: Idealization in Cassirer's Philosophy of Mathematics, *Philosophia Mathematica*, 16(2), 2008, pp. 151-181.

Nabonnand, Philippe: La théorie des Würfe de von Staudt – Une irruption de l'algèbre dans la géométrie pure, Archive for History of Exact Sciences, Vol. 62 (3), 2008, pp. 201-242.

Navarro-Loidi, Juan & Llombart, José: The introduction of logarithms into Spain, *Historia Mathematica,*Vol. 35(2), 2008, pp. 83-101.

Palmieri, Paolo: Breaking the circle: the emergence of Archimedean mechanics in the late Renaissance, Archive for History of Exact Sciences, Vol. 62 (3), 2008, pp. 301-346.

Peckhaus, Volker: Logic and Metaphysics: Heinrich Scholz and the Scientific World View, *Philosophia Mathematica,* 16, 2008, pp. 78-90.

Polo-Blanco, Irene: Alicia Boole Stott, a geometer in higher dimension, *Historia Mathematica,* Vol. 35(2), 2008, pp.123-139.

Schappacher, Norbert: Diophantus of Alexandria : a Text and its History, April 2005, http://www-irma.u-strasbg.fr/~schappa/Dioph.pdf

Stedall, Jacqueline: Notes made by Thomas Harriot on the treatises of François Viète, Archive for History of Exact Sciences, Vol. 62 (2), 2008, pp. 179-200.

Voelke, Jean-Daniel: Le théorème fondamental de la géométrie projective: évolution de sa preuve entre 1847 et 1900, Archive for History of Exact Sciences, Vol. 62 (3), 2008, pp. 243-296.

Wardhaugh, Benjamin: Poor Robin and Merry Andrew: mathematical humour in Restoration England, BSHM Bulletin: Journal of the British Society for the History of Mathematics, Vol. 22 (3), 2007, pp. 151-159.

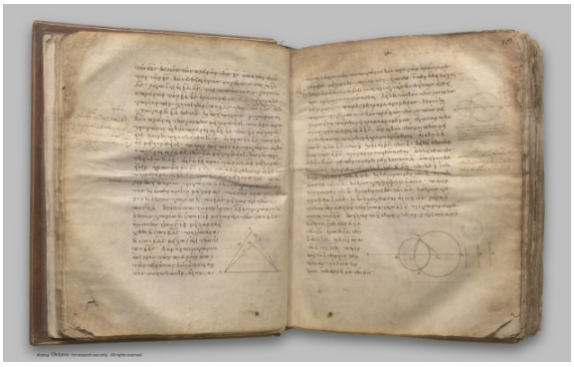
Wilson, Robin: Euler's combinatorial mathematics, BSHM Bulletin: Journal of the British Society for the History of Mathematics, Vol. 23(1), 2008, pp. 13-23.

Have you been here?

New links in this issue

Digitization of the oldest extant manuscript of Euclid's *Elements*

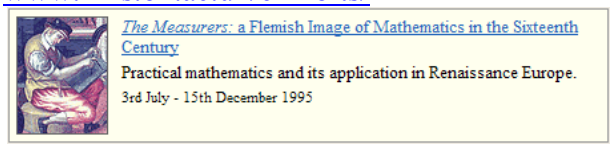
A complete digital edition of the manuscript of Euclid's Elements copied by Stephen the Clerk in Constantinople in the year 888 AD is available in
http://librarieswithoutwalls.org/bookviewer/



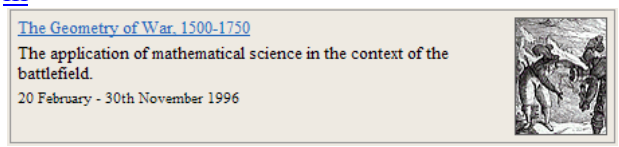
The digitization of this manuscript was a joint project of the Bodleian Library, the Clay Mathematics Institute, and Octavo.com.

Oxford Museum of the History of Science

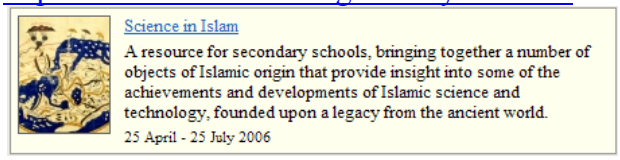
www.mhs.ox.ac.uk/exhibits/



http://www.mhs.ox.ac.uk/measurer/text/title.htm



http://www.mhs.ox.ac.uk/geometry/title.htm



http://www.mhs.ox.ac.uk/scienceislam/

Societies and organisations

Commission on the History of Mathematics in Africa (including newsletter)
http://www.math.buffalo.edu/mad/AMU/amuchma_online.html

Association des Professeurs de Mathematiques de l'Enseignement Public [APMEP] History site:
http://www.apmep.asso.fr/BMhist.html

British Society for the History of Mathematics [BSHM]
http://www.bshm.org

HOMSIGMAA - History of Mathematics Special Interest Group of the MAA
http://www.maa.org/sigmaa/hom

HPM Americas
http://www.hpm-americas.org/

Italian Society of History of Mathematics
http://www.dm.unito.it/sism/indexeng.html

Association pour la Recherche en Didactique des Mathématiques:
http://www.ardm.asso.fr/

Commission Française pour l'Enseignement des Mathématiques: http://www.cfem.asso.fr/

Instituts de Recherche sur l'Enseignement des Mathématiques (IREM):
http://www.univ-irem.fr/

Canadian Society for History and Philosophy of Mathematics
http://www.cshpm.org

Sociedade Brasileira de História da Matemática SBHMat

Brazilian Society for History of Mathematics
http://www.sbhmat.com.br

***Nuncius* Newsletter**
http://brunelleschi.imss.fi.it/nuncius/inln.asp?c=5302

International History, Philosophy and Science Teaching Group
www.ihpst.org

Centre for the History of the Mathematical Sciences.
The Open University, UK
http://purematths.open.ac.uk/pmd_research/CHMS/index.html

Topics and Resources

MATHS for EUROPE: The history of some aspects of mathematics like: history of mathematical persons, symbols, algorithms...
http://mathsforeurope.digibel.be/index.html
http://mathsforeurope.digibel.be/list.htm
http://mathsforeurope.digibel.be/olvp.htm
http://mathsforeurope.digibel.be/olvp2.htm
http://mathsforeurope.digibel.be/olvp3.htm

Ethnomathematics on the Web
http://www.rpi.edu/%7Eeglash/isgem.dir/links.htm

About Medieval Arabic Numbers
http://www.geocities.com/rmlyra/Numbers.html
http://www.geocities.com/rmlyra/arabic.html

Annotated Bibliography on Proof in Mathematics Education
http://fcis.oise.utoronto.ca/~ghanna/educationabstracts.html

BibM@th
http://www.bibmath.net/dico/index.php3?action=rub&quoi=0

Centro Virtual de Divulgación de las Matemáticas, esta siendo desarrollada por la Comisión de Divulgación de la *Real Sociedad Matemática Española (R.S.M.E.)*
http://www.divulgamat.net/index.asp

History of Statistics
http://www.stat.ucla.edu/history/

Images of Lobachevsky's context
http://www.ksu.ru/eng/museum/page0.htm

Images of Mathematicians on Postage Stamps
http://members.tripod.com/jeff560/index.html

Photos of Mathematicians
http://www.math.uni-hamburg.de/home/grothkopf/fotos/math-ges/

Numdam-Digitization of ancient mathematics documents
http://www.numdam.org/en/ressnum.php

The Montana Mathematics Enthusiast (journal)
http://www.montanamath.org/TMME/

Convergence: an online magazine of the MAA providing resources to teach mathematics through its history
http://convergence.mathdl.org/

International Journal for Mathematics Teaching and Learning,
http://www.cimt.plymouth.ac.uk/journal/default.htm

Homepage of International Journal for the History of Mathematics Education
http://www.tc.edu/centers/ijhmt/index.asp?Id=Journal+Home

Documents for the History of the teaching of mathematics in Italy
http://www.dm.unito.it/mathesis/documents.html

Ethnomathematics Digital Library
http://www.ethnomath.org/

Some Japanese Mathematical Landscapes*:* The results of wandering in a beautiful country, with a mathematical eye, aided by a digital camera, by A. Arcavi
http://math.criced.tsukuba.ac.jp/museum/arcavi/arcavi_english/index.html

Wann-Sheng Horng's webpage
with HPM related materials in Chinese.
http://math.ntnu.edu.tw/~horng/

Fred Rickey's History of Mathematics Page
http://www.dean.usma.edu/math/people/rickey/hm/default.htm

Culture*MATH*. Ressources pour les enseignants de Mathématiques
www.dma.ens.fr/culturemath/actu/livres.htm

The French INRP (National Institute for Pedagogical Research) is developing a website on questions related to mathematics teaching: EducMath
http://educmath.inrp.fr

Homepage of Albrecht Heeffer
http://logica.ugent.be/albrecht/

Homepage of Jens Høyrup
http://www.akira.ruc.dk/~jensh/

***L'Enseignement Mathématique*, Archive**
http://retro.seals.ch/digbib/vollist?UID=ensmat-001

Homepage of Prof. Leo Corry
http://www.tau.ac.il/~corry/

Opera Mathematica of Christoph Clavius
http://mathematics.library.nd.edu/clavius/

Geometrical books and instruments from 15th to 18th century
http://www.geometricum.com/

David Henderson' s Home Page [Educational and Historical Topics on Geometry]
http://www.math.cornell.edu/~dwh/

Archimedes Project [Some famous mathematical books of the Renaissance period are available on line, i.e. Pacioli's *Summa*]
http://archimedes2.mpiwg-berlin.mpg.de/archimedes_templates

Simon Stevin's *De Meetdaet* [The Practice of Measuring]
http://www.math.leidenuniv.nl/~wiskonst/meetdaet/index.html
and *The Principal Works of Simon Stevin*
http://www.historyofscience.nl/works_detail.cfm?RecordId=2702

Mathematicians Gallery
http://www.math.uconn.edu/MathLinks/mathematicians_gallery.php?Rendition=printerfriendly

History of Mathematics
http://www.otterbein.edu/resources/library/libpages/subject/mathhis.htm

The Garden of Archimedes. A museum for Mathematics
http://web.math.unifi.it/archimede/archimede_NEW_inglese/

Mathematical instruments
http://brunelleschi.imss.fi.it/museum/esim.asp?c=500164
and
http://web.mat.bham.ac.uk/C.J.Sangwin/Sliderules/sliderules.html
and
http://www.mhs.ox.ac.uk/epact/catalogue.php?ENumber=52265

Homepage of Eleanor Robson
http://www.hps.cam.ac.uk/dept/robson.html

We would like to provide a more comprehensive list of websites containing resources useful to researchers and students (not necessarily in English). If there are any you use, or you know are useful for students or researchers, please send your recommendations to the editors.

Notices

The Mathematical Education of Greek Women during the 19th century

The education of women in Greek territory, namely Greek communities of the Ottoman Empire and Greek state, first appears at the beginning of the 19th century with the operation of a small number of non-systematically organized schools. Women's systematic education begins in 1830s and 1840s for the state and communities respectively, with the operation of a) communal, self-sustained and "national" educational institutions in the communities' territory, b) state institutions – under the responsibility of municipalities – in the Greek state, and c) private schools in both of the aforementioned spaces of Hellenism.

Women's education constitutes a distinct educational system, separate from that of men, not only as regards its structure, the courses provided and the duration of studies, but also as regards the school curricula (taught subjects and their content). This differentiation results from the fact that female education was organized on the basis of the "different nature" and the "different social destination" of women – gendered perceptions that were further reinforced during 19th century by the socio-biological theories (Social Darwinism) that had a great impact on Greek scholars and pedagogues.

With respect to the mathematic education of women, judging by the Math's teaching goals and textbooks, the school curricula, the timetables and the educators' published opinions with regard to the role of Mathematics and Sciences in female's education, it emerges that their Mathematics education was limited at both range and type of content. For example, Algebra was rarely taught, Geometry when appeared in the syllabuses embodied only very elementary units, whereas girls were taught from Arithmetic only the four basic skills, i.e. addition, subtraction, multiplication, and division, and in exceptional cases women's instruction included the teaching of decimal numbers. Also, their mathematical education was practically oriented towards their future social destination and the needs that women would be called to meet in the private sphere as wives, mothers and house-keepers. The shrunken provision of mathematical knowledge was founded on 'scientific' stances of the era concerning the limited intellectual capabilities of women, along with the damage that intellectual strain causes to their reproductive (!) ability.

This overall attitude towards the mathematical education of women was differentiated to some extent on the basis of:

a) *The time period of the schools' operation*. More particularly, up to 1830 (for the areas that comprised the first dominion of the Greek state) or to 1840 (for the Greek

communities remained within the dominion of the Ottoman Empire) Mathematics wasn't always among the courses taught in the schools for girls, as the primary purpose of those few, short-lived first girls' schools was the systematical instruction of the "female duties and works". During the first period of girls' systematic education, namely 1840 up to decade of 1860, Mathematics was included in all schools' syllabuses and their relevant education was shaped wider and more enhanced in perspective to the following period (1870 -1900), due to the influence of innovatory principles regarding women's education of Greek Enlightenment. Practical [Elementary] and Theoretical Arithmetic, Geometry, Cosmography were included in the schools' syllabuses and the hours devoted to girls' mathematical education were 8,3% to 10,5% of the weekly total teaching hours. Thus from 1870 onwards despite the completion of girls' educational system and the extension of their studies duration, mathematical education was reduced in regard with the taught units and teaching hours devoted to it. Algebra was excluded from the syllabuses, Theoretical Arithmetic was taught in few only schools (usually in the self sustained schools) and Geometry was limited to Descriptive and Plane Geometry, meanwhile it continued to be taught only in one school year as it used to during the previous educational period. Also the hours in the school timetable devoted to Mathematics were decreased to 6% as the subject stopped being included in the syllabus of all school years.

b) *The geographical region where schools operated*. In the area of Smyrna (Izmir, in Turkish) for instance, the competitive functioning of Greek schools towards the missionary and western schools, that were first founded at the area, the strong impact of the New Greek Enlightenment's tenets and especially of some celebrated Greek Enlighteners who took action at the area (foundation of innovative schools or restructuring of the curricula of existing establishments), as well as the endorsement and financial support to "the new ideas and new knowledge" by the emerging middle class lead to more radical educational attitudes towards girls education in general and their mathematical education particularly. As soon as the first Greek schools for girls were founded their curricula provided an ample knowledge in Mathematics, (including Algebra, major number of units in Geometry and in one case Trigonometry) as well as in Sciences. It is worth to be mentioned that the necessity of the Sciences' and the Mathematics' instruction in the schools for girls is frequently underlined by women mostly pedagogues in their public speeches and their published opinions.

c) *The administrative type of schools*, which was related to their social class orientation. Regardless of the geographical region and – to some extent – the educational period, in the self-sustained schools, where girls of the superior social class studied, not only the selection of the provided knowledge but also the teaching methodology and the media of instruction were more innovative and wider. The syllabuses of these schools exceeded the mathematical and Sciences knowledge provided – apart from the stereotypical mathematical units taught in all schools for girls – to the instruction of a greater number of units in Geometry, to Algebra, Astronomy, Trigonometry and – very rarely – to the instruction of Nautical Theory.

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Dividing and Composing the Squares

The Islamic craftsman, faced with a belief system that restricted forms of visual display, demonstrated his celebration of Islam by creating attractive abstract geometrical patterns. These very patterns show the role of geometry in Islamic arts. Numbers also played an important part in association with geometry, giving a personality to a shape; the number four, associated with the square, represented stability.

Abu l-Wafa (4th century hejira, 10th century AD) describes innovative cut and paste constructions in his famous book on applied geometry: O*n those parts of geometry needed by craftsman.* In part 9 of the book he explains methods for dividing and re-assembling squares that do not rely on Pythagoras' Theorem (also known as the Bride's Chair theorem by Arabic writers). He also worked with artisans and provided solutions to their questions. There is evidence to suggest that this approach to geometry was not restricted to 4th century (10th century AD) Baghdad but was common throughout the Islamic world. The examples explained by Abu'l Wafa provide descriptive proofs that make a link between theoretical and applied geometry, particularly useful for craftsmen. In addition, he seems well informed of links between number theory and geometry.

The main purpose of my study was to investigate the applied methods by Abu l-Wafa for dividing and composing squares. In addition, I have attempted to explain such patterns related to squares in attractive tiling and wooden arts in Isfahan as a workshop for my students. Then students could see applied geometry in Islamic arts clearly.

Educationally, since geometry is so appealing, we can use the subject to motivate students to take more interest in mathematics and particularly geometry learning. This is very important for increasing and fostering the creative power of students. In addition, searching historical fundamentals through manuscripts and Islamic constructions increased their self-dependence.

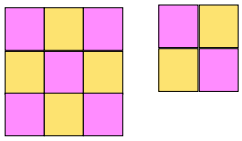
Methods for Dividing and Composing Squares

Abu l-Wafa classified numbers into two classes, *squares* and *non-squares*. In addition, numbers were classified into two classes: first those that are equal to the sum of two other square numbers, such as, 13 = 4 + 9, or 41 = 16 + 25; second those that are not equal to the sum of two squares such as seven. Where numbers are squares or equal to two squares, changing it to smaller squares is very simple.

Here, I explain Abu l-Wafa's methods through activities suitable for the classroom:

Activity 1) Divide squares with two and three lengths side to some congruent squares. (Figure 1)

Activity 2) Compose 16 congruent squares to a square. (Figure 2)



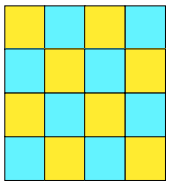


Figure 1: *dividing squares.*
Figure 2: *composing 16* congruent *squares.*

Activity 3) For squares that are equal to the sum of two squares, make a square.

a) If two squares are equal:

Activity 3-1) we want make a square from two 1 × 1 squares (figure 3).

(Since $2 = 1^2 + 1^2$ we can divide any 1 × 1 square in half along a diagonal; these diagonals become the sides of the new square.)

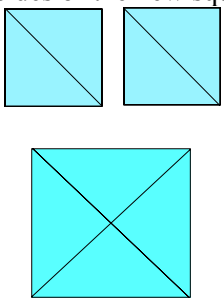


Figure 3: *composing a square from two* 1 × 1 *squares.*

Activity 3-2) we want make a square from two 2 × 2 squares (figure 4).

(Since $8 = 2^2 + 2^2$ we can divide any 2 × 2 square in half along a diagonal; these diagonals become the sides of the new square.)

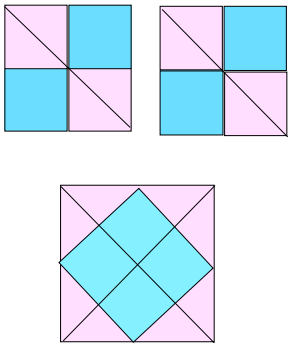


Figure 4: *composing a square from two 2 × 2 squares.*

b) If two squares are not equal:

Activity 3-3) $13 = 4 + 9 = 2^2 + 3^2$

(Since 13 = 6 + 6 + 1 first, divide into two 2 × 3 rectangles and a unit square, then divide the rectangles along their diagonal; arrange the resulting triangles around the unit square.) (Figure 5)

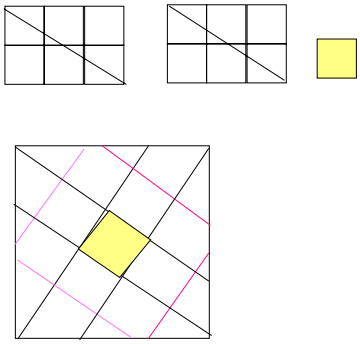


Figure 5: *composing* 2 × 2 *and* 3 × 3 *squares.*

Activity 3-4) 20 = 16 + 4

(Since 16 = 8 + 8 + 4 first, divide into two 2 × 4 rectangles and a 2 × 2 square, then divide the rectangles along their diagonals, then arrange resulting triangles around the square.) (Figure 6)

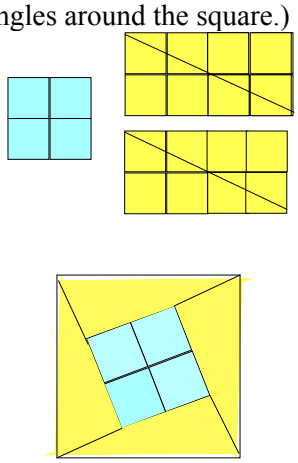


Figure 6: *composing squares.*

The following patterns can result from the above figures:

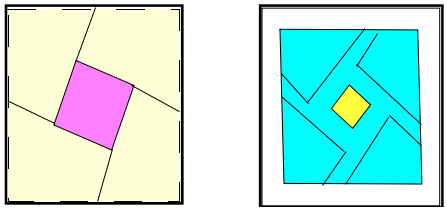


Figure 7: *square patterns.*

Activity 4) Using a non-square number of equal squares to make a square, for example making a square from three equal squares. In this section Abu l-Wafa explained that artisans made errors when applying this problem. He investigated their methods then rejects some solutions, and provided better solutions for this problem.

First, we cut two squares in half along their diagonals. Then we will lay any part on the sides of the third square as in figure 8:

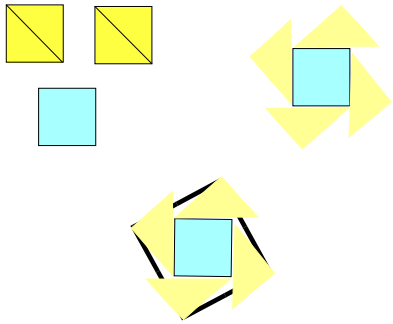
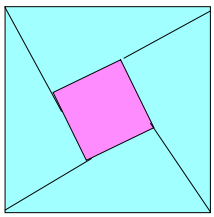


Figure 8: *composing three square to make a 'square'.*

Activity 5) Using some *unequal* squares to make a square. This method applies generally when we use two squares and to generate others. First, we will lay the smaller square on the other square. Then cut increased parts from larger square. We will cut a new small square and two rectangles and divide along their diagonal, and then cut those and arrange the resulted part as figure 9:



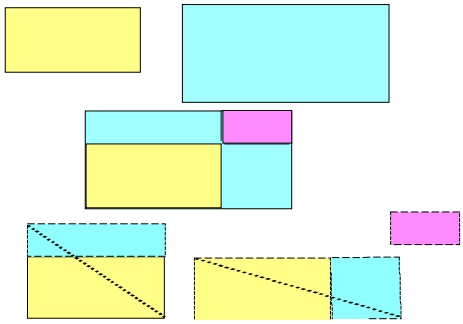


Figure 9: *composing two non-congruent squares to make a square.*

Activity 6) We want to cut a square with certain side length from another square. First, draw semi-circles with square side length and then by drawing arcs with radius equal to the desired square side mark points on the arcs. Then we will join these points. The resulting square is smaller than the desired square. Now, we will cut these parts and make a rectangle from any two triangles and then cut small square from one rectangle and arrange those as figure 10:

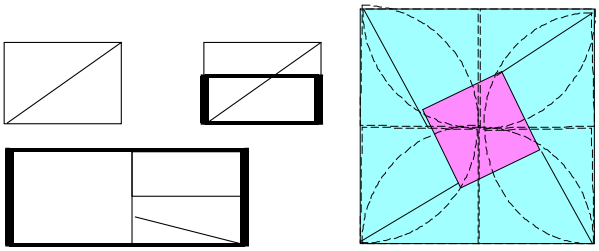


Figure 10: *cutting a square from other square.*

The following pattern is from an *applied geometry* book. It is from an appendix written several centuries after Buzjani (Jazbi believes the writer is Koobnaani, but Alpay believes the author is unknown). Consider squares and rectangles.

Activity 7) In this section, students have provided some artful samples by following pattern. By survey tiling, wooden arts and other Islamic arts in Isfahan we found very attractive samples.

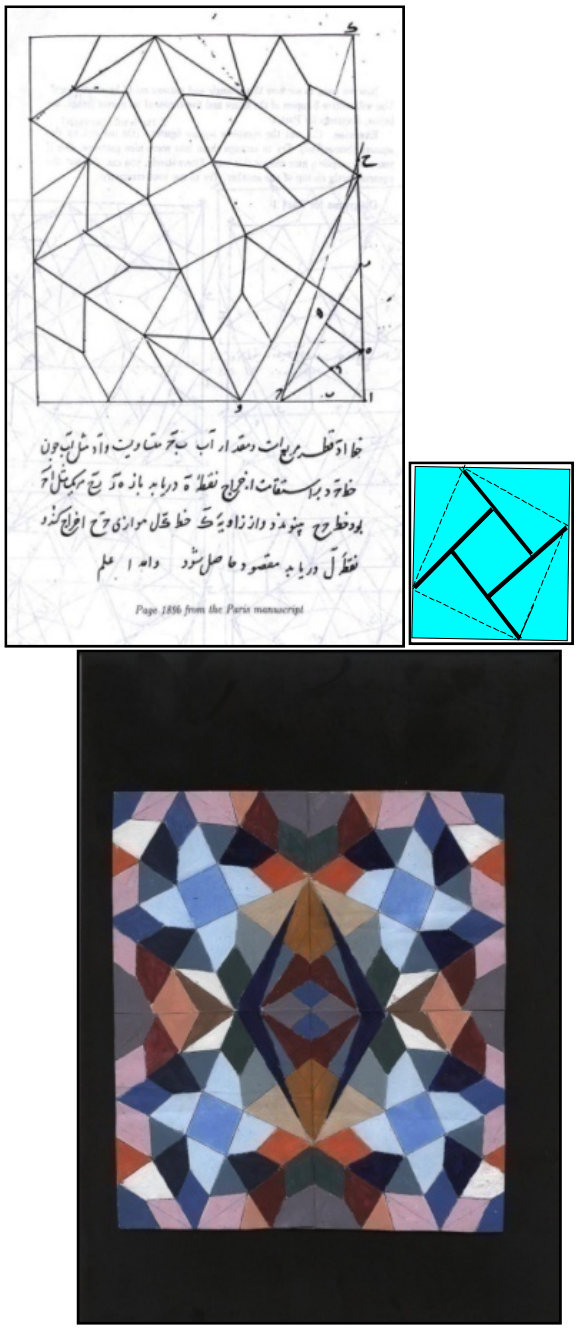


Figure 11: *a problem from unknown author and samples of student works.*

Samples of geometrical patterns in Isfahan constructions

Finally, here are some illustrations of applied square patterns that the students are able to see around them in Isfahan.

Isfahan Friday mosque: the date of building construction referred to 4th to 11th hejira (AD 10th-17th) century.

Figure 12*: North Iwan, four symmetric divided square.*

These tilings have applied pattern figure 8.

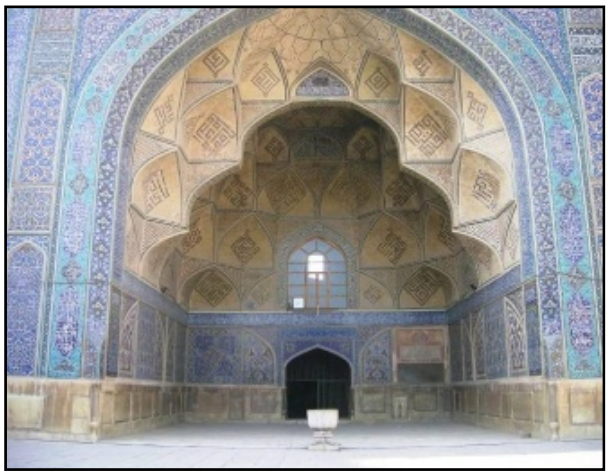


Figure 13: *An attractive square tiling, South Iwan.*
Figure 14*: South Iwan perspective.*

Here are some more four squares tilings.

Figure 15: *West Iwan perspective.*
Figure 16*: West Iwan, and square tiling.*

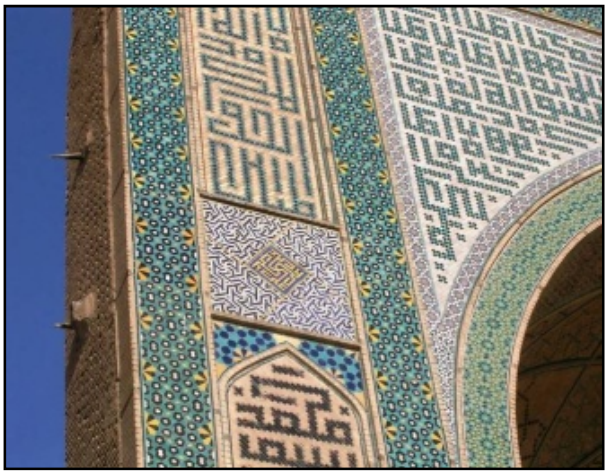


Figure 17: *West Iwan, and square tiling.*
Figure 18: *West Iwan, and square tiling that covered by Ali name.*

In the next picture, you can see a geometrical shape that covered by poem that known as *katibeh khate bannaee* and consist of a poem about Imam Ali:

Figure 19: *katibeh in West Iwan.*

Sheikh lotf ol lah mosque: this construction built in 11th hejira (AD 17th century) by order of *Shah Abbas Safavi*. One of archaeologists said: '*hardly, I can say that this building has been constructed with human hands*'.

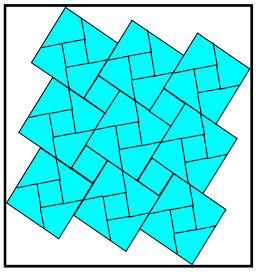


Figure 20: *repeated pattern.*

Figure 21: *wooden door covered by repeated pattern.*

Khan mosque: the date of building khan mosque referred to 1090 hejira (AD 17th century) by order of *Shah Soleiman Safavi.* The brick tiling constructions in khan mosque is very attractive and unique.

Figure 22: *brick art repeated pattern.*

Jame'e Abbasi mosque: building mosque started in *Safavi age* (1020 hejira/ 17 AD century).

Figure 23: *square tiling in jame'e Abbasi mosque.*

Figure 24: *symmetric square tiling in west and east side of jame'e Abbasi mosque.*

Chahar bagh madrasa: the beautiful building in Shah *Soltan Hosein Safavi* (1116-1126 hejira/AD 17th century).

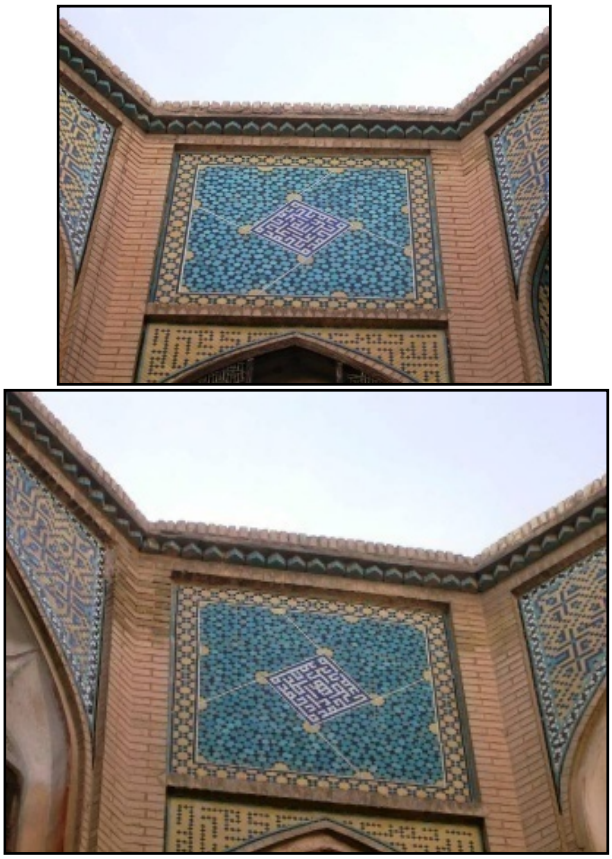


Figure 25: *symmetric square tiling in chahar bagh madrasa.*

Conclusion

The results of this research indicate, by implication, that with a historical background

in mathematics education, particularly geometry, the learner can make links between the arts and mathematics. Our students generally ask where is mathematics used, then we can state and analyse such valuable samples to improve their mathematical thinking, and so foster their creativity power. By using these patterns, we will teach abstract subjects as applied subjects. In addition, by using the geometrical patterns, students could have a better understanding about geometrical concepts such as rotation, symmetry, reflection, transformation, mapping, translation and dilation.

References

[1] S.A. Jazbi, *Iranian Geometry, Applied Geometry.* Tehran*:* Soroush Publication. (date of original manuscript 4th hejira). 1990.

[2] A. Soltanzade. *Isfahan Friday mosque in Ale bouye age.* Tehran. 1970.

[3] A. Gorbani. *Boodzjani nameh.* Tehran. 1970.

[4] Ozdural, Alpay. *ABU'L WAFA, ARTISANS, AND ORNAMENTAL ARTS.* The Genius of Buzdjan Seminar. Iran.1995.

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Narges Assarzadegan, a high school teacher from Isfahan, Iran has written before (Newsletter 62) about the Persian manuscript *Lubb Al-Hisab* (Book of Reckoning). Here she describes some cut and paste activities she designed for her class (Grade 11, age 14), based on a book by Abu'l- Wafa Buzdjani (4th Hejira/ AD 10th century).

An introduction to wasan, native Japanese mathematics

Japanese culture is much indebted to the Chinese one, including its mathematics. However, at the beginning of the seventeenth century, under the Tokugawa shogunate, Japan entered a period of *sakuko*, seclusion from the outside world. Japanese were not allowed to go abroad and foreigners were either forbidden in Japan or very restricted in their movement. The Dutch were limited in number and confined to a small island before Nagasaki. Remaining Chinese and Koreans were placed under strict supervision. Despite this seclusion, but thanks to this period of peace and security, Japan enjoyed a period of development of arts and crafts. It is in this era that *wasan* developed as a native kind of mathematics distinct from its Chinese heritage. Some philosophers have raised the question: could mathematics develop different from the way it did in history? In a way, because of its isolated development, the study of *wasan* provides an almost experimental situation to explore this question. There were many schools in the major cities of Japan teaching *wasan* in a way somewhat similar to European *abbaco* schools or *rechenmeister* in Germany and Flanders. The best known schools are those of Seki and later Aida. Contrary to the European context however, *wasan* served little practical purpose. It was a sort of mathematics mainly pursued for its intellectual and esthetic aspects. Good examples of this are the so-called *sangaku* problems. These are problems of geometrical nature which were displayed on wooden tablets at the entrance of Shinto shrines and Buddhist temples. An impressive collection of them including a mathematical analysis was published by Fukugawa and Pedoe (1989). See figure 1 for an example.

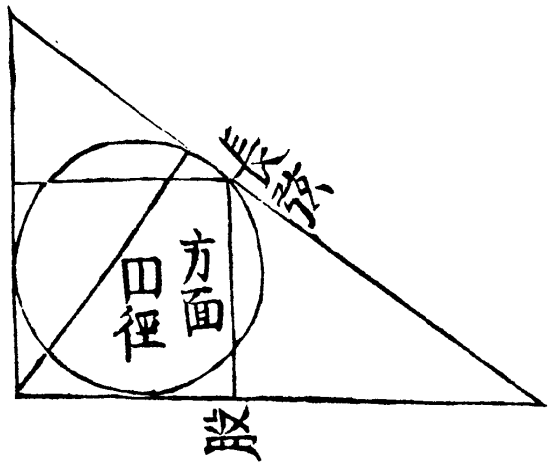

FIGURE 1: A SIMPLE SANGAKU PROBLEM. GIVEN THE SUM OF THE LENGTH OF THE BASE AND THE LONGER PART OF THE HYPOTHENUSE, AND GIVEN THE SUM OF THE LENGTH OF THE LEG, THE DIAMETER AND THE SIDE OF THE SQUARE, FIND THESE INDIVIDUAL VALUES (PROBLEM 40 OF SEKI'S *KENKI SANPŌ* 1683).

Wasan thrived until the second half of the nineteenth century. At the beginning of the eighteenth century under Yoshimune

Tokugawa,[1] who reigned from 1716 to 1744, the prohibition of foreign books became somewhat relaxed. The shogun himself showed an interest in Western astronomy and calendar composition and started building a library, of which later a catalogue as been found (Heeffer 2008). In the middle of the nineteenth century Kyō Uchida taught mathematics to several hundreds of students in Tokyo, adopting the Dutch word *Mathematische* for the name of his school. Any Dutch book that could be found on one of the ships in Nagasaki became a precious item and circulated within a small group of Japanese scholars called *rangakusha*. These consisted of medical doctors, astronomers, gunnery instructors and mathematicians of the samurai class in service of feudal lords. Early nineteenth century there even was a book dealer in Yedo (now Tokyo) who specialized in Dutch books. In 1857 the *Yōsan Yōhō* (洋算用法, *The method of Western arithmetic*) was published by Yanagawa Shunzō. It introduces Japan to the Western (Dutch) method of calculating. Shortly after the Meiji restoration of 1868 it was decided that only *yōzan* (Western mathematics) was to be taught at Japanese schools. In a matter of a few years *wasan* became completely neglected and *wasan* books were replaced by schoolbooks translated from French, English and German. Many *wasan* books were discarded and only used for their paper. However, some scholars collected these precious books and manuscripts, One of them being Tsuruichi Hayashi (1873 – 1935) who became the first historian to publish on *wasan* in a Western language.[2] He left an enormous collection of 14,470 *wasan* books and manuscripts, now the bulk part of the *wasan* collection at Tohoku University. Together with two other minor collections, Tohoku university library now holds the largest collection of *wasan* books in the world with 18,335 items. It is estimated that this accounts for two-thirds of the books produced on this subject. It gives us an idea of the spread of this tradition in Japanese society and the influence it must have had.

Hayashi published a well-received series of articles in the Dutch mathematical journal *Nieuw Archief voor Wiskunde*. His "Brief History of the Japanese Mathematics" (sic) is the first published account of the *wasan* tradition in a Western language (Hayashi 1905a, b). Mikami (1909) published a reaction to Hayashi correcting him on several points, later followed by a more extensive history on *wasan* (Mikami 1913). This publication was further augmented by Smith (1914) and turned into a popular book, at this time still the only English book on the subject. There are not so many Western publications on *wasan* since then.[3] The most extensive study to date is in French by Annick Horiuchi (1994) who primarily studied the works of Seki and his student Katahiro Takebe. While Martzloff (1997) writes on Chinese mathematics, he also provides valuable links to *wasan*, especially on its Chinese heritage. Despite its limited appearance in Western publications on the history of mathematics the study of *wasan* in Japan is blossoming more than ever since the Meiji restoration. There are several academic centers in Japan where scholars study the original sources and there is The Wasan Institute in Tokyo which publishes a dedicated journal and regular books with facsimiles and commentaries. In addition, there is a lively scene of amateurs, collecting and studying *wasan* literature.[4]

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[1] Contrary to Japanese custom, I will list Japanese names with their first name first.

[2] The parallel with Western book collectors and historians of mathematics is obvious. Guillaume Libri, who published the classic *History of mathematics in Italy* (Libri 1838) owned thousands of books and manuscripts, most of which were sold during his lifetime. At some point he was accused of having stolen them from libraries. The many appendices in this three-volume work contain transcriptions from books and manuscripts of his own library, including the *Trattato d'abaco* by Pierro della Francesca. The other example is David Eugene Smith who wrote the two volume work *History of mathematics*. According to an inventory made in 1940 he owned over 20,000 items on mathematics, including 11,000 rare books, numerous manuscripts, autographs and portraits, now part of the Columbia university library collection (Simons 1945).

[3] For a list of *wasan* publications in Western languages see http://logica.ugent.be/albrecht/wasanbib.htm.

[4] This combination of academic and amateur efforts may seem exceptional but there is also a similar

So what is now so special about *wasan*? What makes it different from Western mathematics? Let us first look at the subjects treated. They can be broadly divided into three categories: arithmetic and algebra, geometry and recreational problems. Typical arithmetical problems include merchant problems as we also know from Renaissance arithmetic. However, within *wasan* they are mostly related to the use of the *soroban*, the Japanese equivalent of the abacus. Calculation with Hindu-Arabic numerals was not generally known before the 1880's. I have seen the *soroban* still in use in today's high-tech Japan. Other recurring themes are the approximation of fractions, infinite series and Bernoulli numbers. Geometrical problems include the calculation of the area of triangles, polygons and rings. In the process of devising techniques for the calculation of the area under a curve, Seki became close to what developed as calculus in the West. Recreational problems concern mostly magic squares and circles of a intricate nature and the *mamakodate* problem, known in the West as the Josephus or Christians and Turks problem. Possibly this last one has been imported into Japan by Portuguese Jesuits but it appears that the problem was known in Japan already from the tenth century! And finally there is algebra which deserves our closer attention.

We can rephrase our question about an alternative mathematics with respect to algebra. Did algebraic solutions to arithmetical and geometrical problems within *wasan* develop into a symbolic algebra as it did in Europe? Opinions on this differ widely, also among Japanese scholars. Ken'Ichi Sato devoted an article to the comparison of *wasan* algebra with its Chinese predecessor and concludes that the Japanese version cannot be considered algebra at all and that it "should be recognized as a category independent on that of Western mathematics" (Sato 1995, 65). Chikara Sasaki, on the other hand, in an article comparing Japanese with Western algebra, concludes that precisely because of its symbolic characteristics "Seki's school of algebra can be compared with that of Viète" (Sasaki 1999, 19). We tend closer to the second position but the situation is more complex than that.

Japanese algebra is derived from the so-called *celestial element method* in China. It first appeared in a Chinese work of 1248 by Li Ye (or Li Shi), *Sea-mirror of the circle measurements*. More important for Japan however was a book written in 1299 and titled The *Introduction to mathematical studies* (*Suan xue qi meng* (SQ), 算学啓蒙) (Lam Lay-Yong, 1979). The Chinese edition of the *SQ* was lost for a long time until 1839 when a Korean reprint of 1660 was discovered. Though lost in China, it was translated into Japanese in 1658, printed in several editions and widely read. The original Chinese method solves higher degree equations by laying out and manipulating the coefficients of the unknown terms using colored rods, called *sangi*, on a grid. Black rods represent positive values, red ones negatives. In Japan the method was called *tengen jutsu*. Early Japanese texts follow the Chinese method but print the rods as Chinese characters for numbers. For example, the equation $11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$ is represented (using Hindu-Arabic numerals) as in table 1.

TABEL 1: THE REPRESENTATION OF AN EQUATION BY COEFFICIENTS ON A GRID

10000	1000	100	10	1		
1	1	5	2	0		実
	–	4	3	2	x	方
	–	2	3	6	x^2	廉
				4	x^3	偶
				1	x^4	

situation in Germany on the study of old arithmetic books, concentrated around the activities of the Adam Ries museum in Annaberg-Buchholz.

So, moving from a tangible form of performing algebra to a printed representation involves already a step towards symbolization. The rods, as material means, were replaced by Chinese characters. The red colored rods become replaced by a negative sign (a diagonal dash).

A further development introduced by Seki was the method of side writing. The idea probably evolved from the custom of annotating Chinese characters with hiragana (the Japanese phonetic script) to show how to pronounce them. At some point Seki started adding literals to the numerical coefficients in the grids to tackle more general problems. For example: given a rectangle with an area of A and the sum of its length and width being B, how to determine the length and width. Take x (the celestial element) to be the length, then the width will be $(B - x)$ and the area $x(B - x) = A$. The coefficients to be placed on the grid would then be -1, 1B and -1A respectively, representing the equation $-x^2 + Bx - A = 0$. While the method allowed expressing complex general problems in equations, it was kept secret within Seki's school. In de *Hatsubi Sanpō* of 1674 Seki solved 15 problems posed in an earlier book without revealing his method of side writing. It was his student Katahiro Takebe who would first explain the method in a commentary published in 1687.

It may be all too easy to draw similarities between Seki and Viète on basis of their use of generalized coefficients. Important differences withhold us to attribute the qualification of symbolic algebra to *wasan*. With Viète, European algebra evolved away from problem solving to the study of the structure of equations. Important discoveries such as the relations between de roots of an equation and its coefficients, the number of roots and the degree of an equation, and the relation between algebra and geometry are missing in *wasan*. By the end of the sixteenth century the foundations of European algebra became completely transformed as part of the humanist program of providing more solid foundations based on Greek roots. Algebra became a normative model for a *mathesis universalis*, a universal symbolic language to approach the laws of nature, and as such had a decisive influence on the development of Western science. In contrast, *wasan* contributed nothing to the understanding of nature. Despite its ability to solve complex problems in an efficient way, the *wasan* tradition failed to produce a single law on mechanics, hydraulics or kinematics. Even when Dutch books on Newtonian physics became available during the eighteenth century, Japanese scholars did not resort to *wasan* methods of algebra for its mathematical foundations and instead struggled with ways to represent Western symbols (Ravina 1993, 213). The failure of embedding *wasan* within other intellectual traditions made it possible to replace it by Western mathematics in a very short period during the Meiji restoration.

The current growth of interest in wasan is not only from a historical perspective. The rich history of *wasan* and its enormous legacy of woodblock printed books provide ample opportunities of using historical material in the classroom. Being an integral part of Japanese culture and its close relation with religion (*sangaku*) makes *wasan* an excellent example of the use of history for mathematics eduction. Several projects have been started along this line (Leung e.a. 2006). Specific lessons have been developed by specialized institutes as the CRICED from Tsukuba University.[5] Placing traditional *wasan* methods of problem solving against their Western counterparts provides students with a better understanding of the development of mathematics and the way mathematics is part of our social and cultural legacy.

References

Fukagawa, H. and D. Pedoe (1989) *Japanese Temple Geometry Problems*. Winnipeg, Canada, Charles Babbage Research Center.

[5] For a lesson example of teaching algebra by means of *sangi*, see the website of the CRICED at Tsukuba. http://math-info.criced.tsukuba.ac.jp/Forall/project/history/2003/sangi/sangi_index.html

Hayashi, T. (1905a) "A brief history of the Japanese mathematics (part 1)", *Nieuw Archief voor Wiskunde* Serie 2, 6: 296-361.

Hayashi, T. (1905b) "A brief history of the Japanese mathematics (part 2)", *Nieuw Archief voor Wiskunde* Serie 2, 7: 105-163.

Horiuchi, A. (1994) *Les mathématiques japonaises à l'époque d'Edo (1600-1868). Une étude des travaux de Seki Takakazu (?-1708) et de Takebe Katahiro (1664-1739)*, Librairie Philosophique J. Vrin.

Heeffer A. (2008) "Dutch arithmetic and algebra in Japan before the Meiji Restoration", forthcoming.

Lam Lay-yong (1979) Chu shih-chieh's Suan hsüeh ch'i-meng, *Archive for the history of sciences*, 21, pp. 1-31.

Leung, Frederick K. S., Klaus-D. Graf and Francis J. Lopez-Real (2006) *Mathematics Education in Different Cultural Traditions. A Comparative Study of East Asia and the West*, The 13th ICMI Study, Dordrecht: Springer.

Martzloff, J.-C. (1997) *A History of Chinese Mathematics*. New York, Springer.

Mikami, Y. (1909) "Remarks on T. Hayashi's 'Brief History of Japanese Mathematics'" *Nieuw Archief voor Wiskunde* Serie 2, 9: 373-386.

Mikami, Y. (1913) "The Development of Mathematics in China and Japan", *Zeitschrift für Mathematik und Physik. Abhandlungen zur Geschichte der Mathematischen Wissenschaften mit Einschluss ihrer Anwendungen begründet von Moritz Cantor*, 30: 1-332.

Ravina, M. (1993) "Wasan And The Physics That Wasnt, Mathematics In The Tokugawa-Period", *Monumenta Nipponica*, 48(2): 205-224.

Sasaki, C. (1999) "The French and Japanese schools of algebra in the seventeenth century: A comparative study", *Historia Scientiarum. International Journal of the History of Science Society of Japan*, 9(1): 17-26.

Sato, K. (1995) "Reevaluation of Tengenjutsu or Tianyuanshu: In the Context of Comparison between China and Japan", *Historia Scientiarum. International Journal of the History of Science Society of Japan*, 5(1): 57-68.

Smith, D. E. and Y. Mikami (1914). *A history of Japanese mathematics*, Chicago, Open Court publications.

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Announcements of events

ICME-11
July 6-13, 2008
Monterrey, Mexico
http://www.icme11.org.mx/icme11/

of special interest is the
TSG 23: The role of history of mathematics in mathematics education
See HPM Newsletter 67 or
http://tsg.icme11.org/tsg/show/24
for more information on this.

HPM 2008
History and Pedagogy of Mathematics
The HPM Satellite Meeting of ICME 11
July 14-18, 2008
Mexico City, Mexico
See First Announcement in Newsletter 66. The Second Announcement is available on the website: http://www.red-cimates.org.mx/HPM2008.htm
Plenary Activities:
The following plenary lectures will be given:

David Pengelley (New Mexico State University, USA): *Teaching mathematics with primary historical sources: Should it go mainstream? Can it?*

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[6] Fellow of the Research Foundation Flanders (FWO). This paper was written during a research visit at Center of Cross-cultural Studies of Kobe University, Japan. The author wishes to thank his host professor Nobuo Miura for the support and hospitality.

Rosa María Farfán (Instituto de Ciencia y Tecnología del DF, & Cinvestav IPN, México): *Socioepistemological approach to Fourier series. Mathematics, Physics and Social Practice*.

Michele Emmer (Università di Roma "La Sapienza", Italy): *Mathematics, Art, Architecture and Technology: the Idea of Space*.

Glen Van Brummelen (Quest University, Canada): *Crossing Cultures, Oceans, Religions, and the Cosmos: In Search of the Origins of Trigonometry*

Alejandro Garciadiego (FC, UNAM), Ricardo Quintero (DME – Cinvestav IPN), Alberto Camacho (Instituto Tecnológico de Chihuahua II), Francisco Cordero (DME – Cinvestav, IPN), Gisela Montiel (Prome – Cicata, IPN). *Title to be announced*

Evelyne Barbin (Université de Nantes, France): *Dialogism in mathematical writing: historical, philosophical and pedagogical issues*

Ubiratan d'Ambrosio (Pontificia Universidade, Catolica de Sao Paulo, Brazil): *The transmission and acquisition of Mathematics in Colonial and Early Independent Countries in the Americas*

Karen Hunger Parshall (University of Virginia, USA): *The Establishment of a Community of Mathematical Researchers in North America: 1636-1950*

Participants wishing to register after **June 15**, 2008, should pay on the spot **250**$ / **150**$ (for students)
HPM2008@red-cimates.org.mx

10th Iranian Mathematics Education Conference (IMEC-10)

August 12-15, 2008
Yazd, Iran
Aims: To expand the mathematical awareness in general and to exchange the mathematical education experiences in particular.
Scientific Programs

- Plenary talks
- Paper presentations
- Workshops
- Panels
- Exhibitions

Conference Themes

1- Theoretical foundations of mathematics education.
2- Professional development of mathematics teachers.

The Scientific Committee is pleased to announce that all the local expenses (except air fare) of foreign participants will be taken care by the local organizers of the 10th IMEC.

Important Dates:
Registration & submission of contributions (Papers, exhibition. etc): April 30, 2008
Website: http://www.imec10yazd.com

Models in Developing Mathematics Education (10th International Conference of The Mathematics Education into the 21st Century Project)

September 12-18, 2008
Dresden, Germany
For further information contact
arogerson@inetia.pl

Models in Developing Mathematics Education

September 11–17, 2009
Dresden, Germany
For all conference details and updates please email alan@rogerson.pol.pl .

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The Newsletter of HPM is primarily a tool for passing on information about forthcoming events, recent activities and publications, and current work and research in the broad field of history and pedagogy of mathematics. The Newsletter also publishes brief articles which they think may be of interest. Contributions from readers are welcome on the understanding that they may be shortened and edited to suit the compass of this publication.

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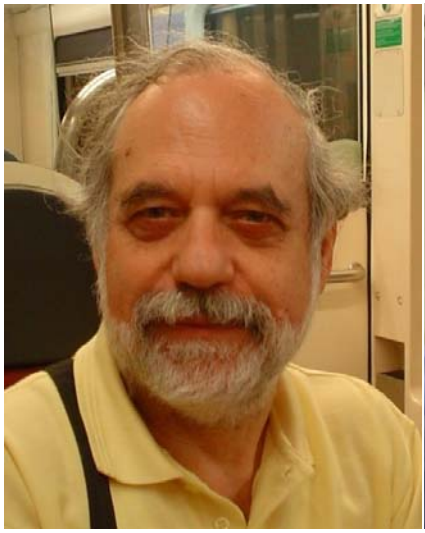
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The Newsletter appears three times a year with the following deadlines for next year.

No.	Deadline for material	Sent to distributors
69	12 October 2008	1 November 2008
70	12 February 2009	1 March 2009
71	12 June 2009	1 July 2009

The Newsletter is the communication of the International Study Group on the Relations between the History and Pedagogy of Mathematics, an affiliate of the International Commission on Mathematical Instruction.

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