

Exploring Geometric Transformations in a Dynamic Environment

Cheryll E. Crowe, Ph.D.
Eastern Kentucky University

Overview

The GeoGebra documents allow exploration of four geometric transformations taught in middle and high school level geometry classes. The design of the applets provide flexibility of content and grade level with hidden elements that can be revealed to create additional topics for investigating reflections, rotations, translations, and glide-reflections. Suggested activities and exercises are included but can be altered or omitted to accommodate various levels of mathematical understanding. The interactive sketches were initially created for use with pre-service teachers in content mathematics education courses at the university level but can also be adapted for middle and high school students.

Intended audience: Grades 6-12, Mathematics Teacher Education Courses

Instruction & Activities

General Instructions

- The use of show buttons are identified in the directions with **red** font to correlate with the red letters on the button.
- Calculations and coordinates are identified in the directions with **green** font to correlate with the green letters on the button.
- Words in capital letters indicate selections from the menu at the top of the GeoGebra document.

Activity 1: Exploring Reflections

Part I – Identifying Reflections

Directions:

- Move the mirror line by clicking and dragging point K or L.
- Click on the **Show Reflection** button. Note the location of the reflection.
- Hide the reflection and repeat steps 1 and 2. Attempt to “guess” the location of the transformation.
- *Note:* You may need to use the zoom in/out tool to see the entire reflection. (Select this function from the last icon on the toolbar.)
- Investigate the connection between the location of the mirror line and the figures by dragging point K or L with the reflection visible.

Questions:

- 1) Where is the mirror line located when the reflection and original figure intersect at a point?
- 2) Where is the mirror line located when the reflection and original figure overlap?
- 3) Can the mirror line be moved in such a way that the reflection and original are the same figure (identical, overlapped)?

Part II – Mirror Lines & Distance

Directions:

- Click on the **Preimage to Mirror** and **Image to Mirror** buttons.
- Drag point K or L and observe the changes for distance measurement.

Questions:

- 1) What do you notice about the distance from the mirror line to reflection and the mirror line to the original figure?

- 2) Why do you think this relationship exists? Provide a thorough explanation.

Part III – Reflections & Coordinate Geometry

Directions:

- Select AXES from the VIEW menu.
- Click on the **Preimage Coordinates** and **Image Coordinates** buttons.
- Drag points K and L in order to make the reflection line on the **y-axis**.
- Observe changes in the coordinates; answer questions #1 and #2 below.
- Drag points K and L in order to make the reflection line on the **x-axis**.
- Observe changes in the coordinates; answer questions #3 and #4 below.

Questions:

- 1) State the connection between the x and y coordinates for the preimage and reflection when the reflection line is the **y-axis**.

Preimage	Image
(x, y)	

- 2) What conjecture(s) can you make regarding the coordinates of a figure reflected over the **y-axis**?

- 3) State the connection between the x and y coordinates for the preimage figure and reflection when the reflection line is the **x-axis**.

Preimage	Image
(x, y)	

- 4) What conjecture(s) can you make regarding coordinates of a figure reflected over the **x-axis**?

Activity 2: Exploring Rotations

Part I – Identifying Rotations

Directions:

- Explore the rotation of the given figure about point O. Change the angle of rotation (α) by clicking and dragging on the slider.
- Click on the **Show Rotation** button. Note the location of the rotation.
- Hide the rotation and repeat steps 1 and 2. Attempt to “guess” the location of the transformation.

Questions:

- 1) At what degree measure(s) is the rotated figure identical to the preimage?
- 2) Describe the result of the following combinations:
 - a) A 90° rotation followed by another 90° rotation.
 - b) A 180° rotation followed by another 180° rotation.
 - c) A 270° rotation followed by a 180° rotation.

Part II – Rotations & Coordinate Geometry

Directions:

- Select AXES from the VIEW menu.
- Click on the **Preimage Coordinates** and **Image Coordinates** buttons.
- Click and drag the slider to make $m \angle \alpha = 90^\circ$
- Observe changes in the coordinates and answer question #1 below.
- Click and drag the slider to make $m \angle \alpha = 180^\circ$
- Observe changes in the coordinates and answer question #2 below.
- Click and drag the slider to make $m \angle \alpha = 270^\circ$
- Observe changes in the coordinates and answer question #3 below.

Questions:

- 1) What conjecture can you make regarding the coordinates of a figure rotated 90° ?
- 2) What conjecture can you make regarding the coordinates of a figure rotated 180° ?
- 3) What conjecture can you make regarding the coordinates of a figure rotated 270° ?

Activity 3: Exploring Translations

Part I – Identifying Translations

Directions:

- Change the distance of the translation by clicking and dragging point Q.
- Click on the **Show Translation** button. Note the location of the translation.
- Hide the translation and repeat steps 1 and 2. Attempt to “guess” the location of the transformation.
- Click on the **Show Length of Vector** and **Distance from Preimage to Image** button.
- Drag point Q and observe the changes for distance measurement.

Questions:

- 1) How does the distance between points P and Q compare to the distance between the points in the preimage and transformation? Explain your observation.

- 2) At what distance is the translated figure identical to the preimage?

- 3) What happens to points P and Q in the case of question #2?

Part II – Translations and Coordinate Geometry

Directions:

- Select AXES from the VIEW menu.
- Click on the **Coordinates of Preimage** and **Coordinates of Image** buttons.
- Drag point Q to the origin of the coordinate plane.
- Observe changes in the coordinates and answer question #1 below.
- Drag point Q to the coordinate (-6, -6).
- Observe changes in the coordinates and answer question #2 below.

Questions:

- 1) What change do you notice happens to the coordinates of the preimage and the figure translated horizontally?

- 2) What change do you notice happens to the coordinates of the preimage and the figure translated vertically?

Activity 5: Exploring Two Reflections with Parallel Lines

Directions:

- Before showing the reflections, answer question #1 below.
- Select the **Show Reflection #1** and **Show Reflection #2** buttons.
- Answer question #2 below.
- Drag point L to change the slope of the parallel lines.
- Answer question #3 below.

Questions:

- 1) What do you believe will be the final product of the two reflections?

- 2) What is the composition of two reflections through parallel lines?

- 3) Do your answers to questions #1 and 2 hold true after changing the slope of the lines? Explain your response.

Activity 6: Exploring Three Reflections with Parallel Lines

Directions:

- Before showing the reflections, answer question #1 below.
- Select the **Show Reflection #1** and **Show Reflection #2** and **Show Reflection #3** buttons.
- Answer question #2 below.
- Drag point L to change the slope of the parallel lines.
- Answer questions #3 and 4 below.

Questions:

- 1) What do you believe will be the final product of the three reflections?

- 2) What is the composition of three reflections through parallel lines?

- 3) Do your answers to questions #1 and 2 hold true after changing the slope of the lines? Explain your response.

- 4) Make a conjecture about the number of parallel lines and the composition of reflections.

