Exploring Geometric Transformations in a Dynamic Environment
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Overview
The GeoGebra documents allow exploration of four geometric transformations taught in middle and high school level geometry classes. The design of the applets provide flexibility of content and grade level with hidden elements that can be revealed to create additional topics for investigating reflections, rotations, translations, and glide-reflections. Suggested activities and exercises are included but can be altered or omitted to accommodate various levels of mathematical understanding. The interactive sketches were initially created for use with pre-service teachers in content mathematics education courses at the university level but can also be adapted for middle and high school students.

Intended audience: Grades 6-12, Mathematics Teacher Education Courses

Instruction & Activities

General Instructions
- The use of show buttons are identified in the directions with red font to correlate with the red letters on the button.
- Calculations and coordinates are identified in the directions with green font to correlate with the green letters on the button.
- Words in capital letters indicate selections from the menu at the top of the GeoGebra document.

Activity 1: Exploring Reflections

Part I – Identifying Reflections
Directions:
- Move the mirror line by clicking and dragging point K or L.
- Click on the Show Reflection button. Note the location of the reflection.
- Hide the reflection and repeat steps 1 and 2. Attempt to “guess” the location of the transformation.
- Note: You may need to use the zoom in/out tool to see the entire reflection. (Select this function from the last icon on the toolbar.)
- Investigate the connection between the location of the mirror line and the figures by dragging point K or L with the reflection visible.

Questions:
1) Where is the mirror line located when the reflection and original figure intersect at a point?

2) Where is the mirror line located when the reflection and original figure overlap?

3) Can the mirror line be moved in such a way that the reflection and original are the same figure (identical, overlapped)?
Part II – Mirror Lines & Distance

Directions:
- Click on the Preimage to Mirror and Image to Mirror buttons.
- Drag point K or L and observe the changes for distance measurement.

Questions:
1) What do you notice about the distance from the mirror line to reflection and the mirror line to the original figure?

2) Why do you think this relationship exists? Provide a thorough explanation.

Part III – Reflections & Coordinate Geometry

Directions:
- Select AXES from the VIEW menu.
- Click on the Preimage Coordinates and Image Coordinates buttons.
- Drag points K and L in order to make the reflection line on the y-axis.
- Observe changes in the coordinates; answer questions #1 and #2 below.
- Drag points K and L in order to make the reflection line on the x-axis.
- Observe changes in the coordinates; answer questions #3 and #4 below.

Questions:
1) State the connection between the x and y coordinates for the preimage and reflection when the reflection line is the y-axis.

<table>
<thead>
<tr>
<th>Preimage</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x, y)</td>
<td></td>
</tr>
</tbody>
</table>

2) What conjecture(s) can you make regarding the coordinates of a figure reflected over the y-axis?

3) State the connection between the x and y coordinates for the preimage figure and reflection when the reflection line is the x-axis.

<table>
<thead>
<tr>
<th>Preimage</th>
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</tr>
</thead>
<tbody>
<tr>
<td>(x, y)</td>
<td></td>
</tr>
</tbody>
</table>

4) What conjecture(s) can you make regarding coordinates of a figure reflected over the x-axis?
Activity 2: Exploring Rotations

Part I – Identifying Rotations

Directions:
- Explore the rotation of the given figure about point O. Change the angle of rotation (α) by clicking and dragging on the slider.
- Click on the Show Rotation button. Note the location of the rotation.
- Hide the rotation and repeat steps 1 and 2. Attempt to “guess” the location of the transformation.

Questions:
1) At what degree measure(s) is the rotated figure identical to the preimage?

2) Describe the result of the following combinations:
   a) A 90° rotation followed by another 90° rotation.
   b) A 180° rotation followed by another 180° rotation.
   c) A 270° rotation followed by a 180° rotation.

Part II – Rotations & Coordinate Geometry

Directions:
- Select AXES from the VIEW menu.
- Click on the Preimage Coordinates and Image Coordinates buttons.
- Click and drag the slider to make \( m \tilde{\alpha} = 90° \)
- Observe changes in the coordinates and answer question #1 below.
- Click and drag the slider to make \( m \tilde{\alpha} = 180° \)
- Observe changes in the coordinates and answer question #2 below.
- Click and drag the slider to make \( m \tilde{\alpha} = 270° \)
- Observe changes in the coordinates and answer question #3 below.

Questions:
1) What conjecture can you make regarding the coordinates of a figure rotated 90°?

2) What conjecture can you make regarding the coordinates of a figure rotated 180°?

3) What conjecture can you make regarding the coordinates of a figure rotated 270°?
Activity 3: Exploring Translations

Part I – Identifying Translations

Directions:
- Change the distance of the translation by clicking and dragging point Q.
- Click on the Show Translation button. Note the location of the translation.
- Hide the translation and repeat steps 1 and 2. Attempt to “guess” the location of the transformation.
- Click on the Show Length of Vector and Distance from Preimage to Image button.
- Drag point Q and observe the changes for distance measurement.

Questions:
1) How does the distance between points P and Q compare to the distance between the points in the preimage and transformation? Explain your observation.

2) At what distance is the translated figure identical to the preimage?

3) What happens to points P and Q in the case of question #2?

Part II – Translations and Coordinate Geometry

Directions:
- Select AXES from the VIEW menu.
- Click on the Coordinates of Preimage and Coordinates of Image buttons.
- Drag point Q to the origin of the coordinate plane.
- Observe changes in the coordinates and answer question #1 below.
- Drag point Q to the coordinate (-6, -6).
- Observe changes in the coordinates and answer question #2 below.

Questions:
1) What change do you notice happens to the coordinates of the preimage and the figure translated horizontally?

2) What change do you notice happens to the coordinates of the preimage and the figure translated vertically?
Activity 4: Exploring Glide-Reflections

Part I – Identifying Glide-Reflections
Directions:
• Change the distance of vector PQ by clicking and dragging point Q.
• Click on the Show Translation and Show Glide-Reflection buttons. Note the location of the transformation.
• Hide the translation and glide-reflection. Repeat steps 1 and 2; attempt to “guess” the location of the transformation.
• Click on the Show Length of Vector button.
• Also click on Distance Preimage, Distance Translation, and Distance Image buttons.
• Observe the distance measurements.

Questions:
1) State the two transformations used to create a glide-reflection.

2) What transformation occurs when point P and Q overlap?

3) What conjecture can you make regarding the distance between the vertices of the figure and vector PQ?

Part II – Glide-Reflections and Coordinate Geometry
Directions:
• Select AXES from the VIEW menu.
• Click on Coordinates Preimage, Coordinates Translation, and Coordinates Image buttons.
• Drag point Q to the origin of the coordinate plane.
• Observe the coordinates.

Questions:
1) What conjecture can you make regarding the coordinates of the three figures?
Activity 5: Exploring Two Reflections with Parallel Lines

Directions:
• Before showing the reflections, answer question #1 below.
• Select the Show Reflection #1 and Show Reflection #2 buttons.
• Answer question #2 below.
• Drag point L to change the slope of the parallel lines.
• Answer question #3 below.

Questions:
1) What do you believe will be the final product of the two reflections?

2) What is the composition of two reflections through parallel lines?

3) Do your answers to questions #1 and 2 hold true after changing the slope of the lines? Explain your response.

Activity 6: Exploring Three Reflections with Parallel Lines

Directions:
• Before showing the reflections, answer question #1 below.
• Select the Show Reflection #1 and Show Reflection #2 and Show Reflection #3 buttons.
• Answer question #2 below.
• Drag point L to change the slope of the parallel lines.
• Answer questions #3 and 4 below.

Questions:
1) What do you believe will be the final product of the three reflections?

2) What is the composition of three reflections through parallel lines?

3) Do your answers to questions #1 and 2 hold true after changing the slope of the lines? Explain your response.

4) Make a conjecture about the number of parallel lines and the composition of reflections.
**Activity 7: Exploring Reflections with Intersecting Lines**

*Directions:*
- Before showing the reflections, answer question #1.
- Select the **Show Reflection #1** and **Show Reflection #2** buttons.
- Answer question #2 below.
- Drag point L to change the slope of the mirror lines.
- Answer question #3 below.

*Questions:*
1) What do you believe will be the final product of the two reflections?

2) What is the composition of two reflections through intersecting lines?

3) Do your answers to questions #1 and 2 hold true after changing the slope of the lines? Explain your response.

**Activity 8: Exploring Reflections with Concurrent Lines**

*Directions:*
- Before showing the reflections, answer question #1.
- Select the **Show Reflection #1** and **Show Reflection #2** and **Show Reflection #3** buttons.
- Answer question #2 below.
- Drag point L to change the slope of the mirror lines.
- Answer question #3 below.

*Questions:*
1) What do you believe will be the final product of the three reflections?

2) What is the composition of three reflections through concurrent lines?

3) Do your answers to questions #1 and 2 hold true after changing the slope of the lines? Explain your response.