

4. Level Surfaces

It is difficult to draw many interesting level surfaces by hand, so I generally have my students use CalcPlot3D to do most of the work for this type of exercise. There are actually two ways to enter and graph the level surface equations for a particular function of three variables in CalcPlot3D:

- (1) Solve each equation for z in terms of x , y , and C and enter the level surface using one or two functions of x and y , or
- (2) Graph the level surface equation by **Adding an Implicit Surface** from the **Graph** menu and entering the equation for the level surface in the dialog box there.

If the surfaces are complicated enough, you may not have a choice. If you are not able to solve for z , you will need to use the Implicit Surface option.

Here is an example I use in class shown both ways.

$$f(x, y, z) = z^2 - x^2 + y^2$$

Setting $f(x, y, z) = z^2 - x^2 + y^2 = C$, we obtain the following equations if we solve for z .

$$z = \sqrt{C + x^2 - y^2}$$

$$z = -\sqrt{C + x^2 - y^2}$$

For $C = 2$:

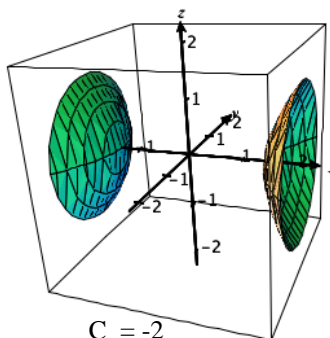
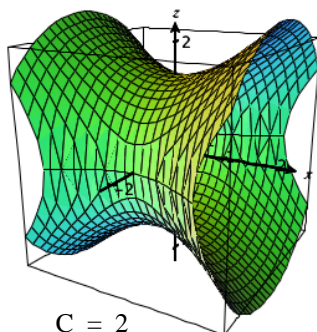
$$z = \sqrt{2 + x^2 - y^2}$$

$$z = -\sqrt{2 + x^2 - y^2}$$

For $C = -2$:

$$z = \sqrt{-2 + x^2 - y^2}$$

$$z = -\sqrt{-2 + x^2 - y^2}$$



We can obtain the following graphs of these surfaces by graphing the implicit equations.

$$z^2 - x^2 + y^2 = 2 \quad \text{and} \quad z^2 - x^2 + y^2 = -2$$

These equations will be entered as:

$$z^2 - x^2 + y^2 = 2$$

$$z^2 - x^2 + y^2 = -2$$

