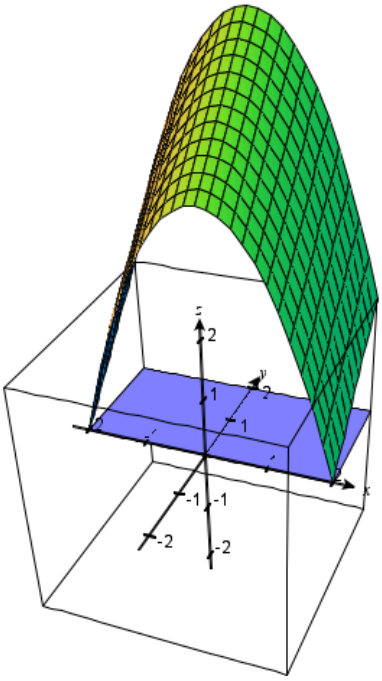
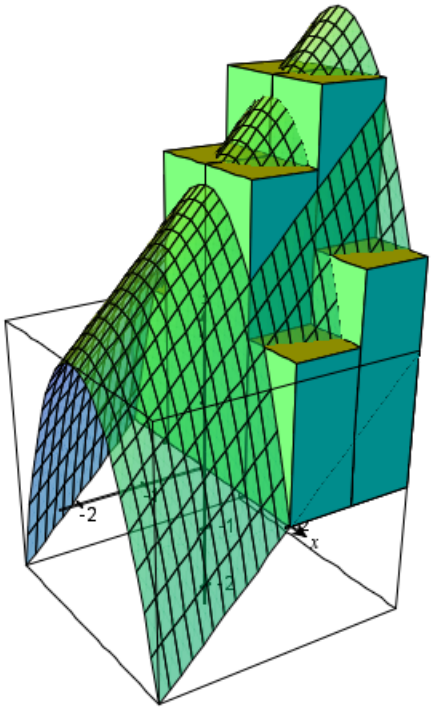
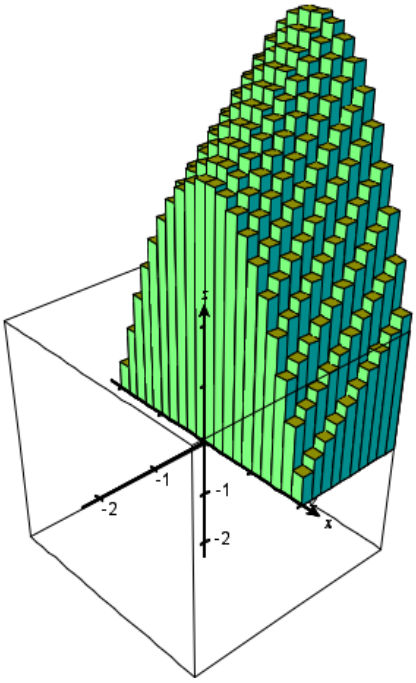
9. Riemann Sums of a Double Integral

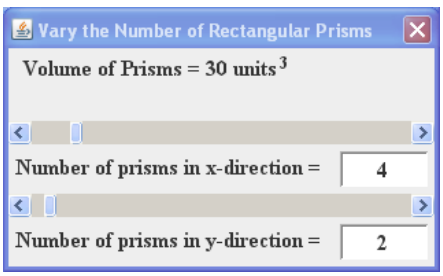
When introducing double integrals, I like to show the students how we can use rectangular prisms to approximate the volume of the solid region between the region of integration (in the *xy*-plane) and the surface. It is nice to connect these approximations to the value we obtain by evaluating the corresponding double integral. Here is an exercise students can do to improve their understanding of this connection.

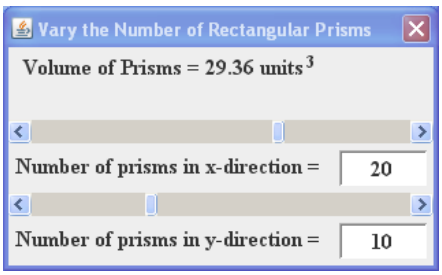


Exercise: Use 8 rectangular prisms to approximate the volume of the solid region between $f(x, y) = 4 - x^2 + y$ and the rectangular region R in the *xy*-plane given by $-2 \le x \le 2$ and $0 \le y \le 2$. Then use a double integral to find the exact volume of this solid region. Show all work on paper for both parts. Then use CalcPlot3D to create a graph of the region in the *xy*-plane along with the rectangular prisms you used and the surface above the region. Print a view of this graph that shows the surface and the rectangular prisms well. Use the **Show grid scrollbars** option on the **Create Bounded Region** dialog to vary the number of prisms in each direction. What do you notice happens to the volume of the prisms when you vary the number of prisms in the *y*-direction? Can you explain why this happens for this function? As you increase the number of prisms in the *x*-direction, does the volume of the prisms approach the exact volume you calculated? What is the volume of the prisms when there are 30 prisms in the *x*-direction?

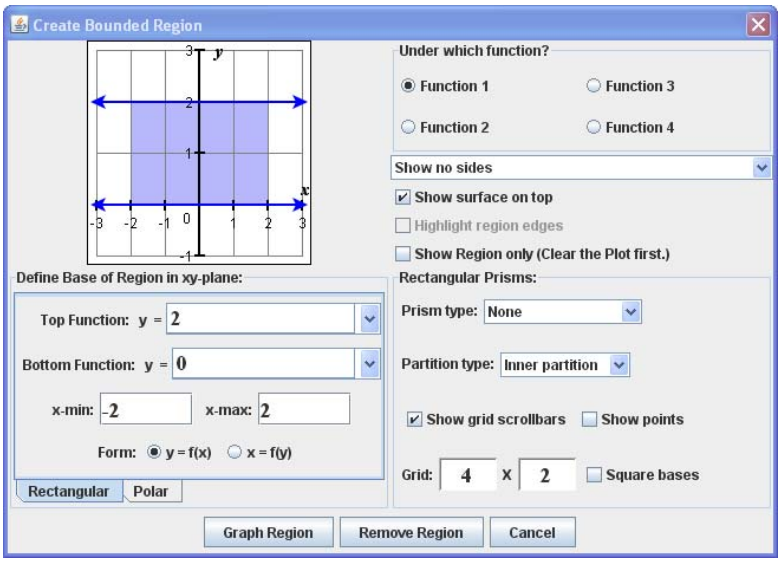








To create these graphs:



1. Graph the function z = 4 – x^2 + y.
2. Select **Add a Region** from the **Graph** menu.
3. The dialog shown at right will appear. Define the base of the region by entering y = 0 for the bottom function and y = 2 for the top function. Then enter -2 for x-min and 2 for x-max.
4. At the top-right, Select the function number where you just graphed the function in step 1.
5. Choose **Show no sides** from the drop-down menu just below the functions.
6. Select to **Show surface on top**.
7. Choose the **Prism type** to be **Midpoint** and then enter the number of prisms you want in the *x*- and *y*-directions next to the word **Grid**.
8. Select **Show grid scrollbars** and then click on the **Graph Region** button.
9. You can now use the scrollbars on the little dialog labeled **Vary the Number of Rectangular Prisms** to see what happens to the volume of the prisms as you vary their number in each direction. Note that the volume of the prisms is updated in this dialog as you do this.

The graphs below show another example: $f(x, y) = (x^2 + y^2)/2$ over the region in the *xy*-plane bounded by the graphs of $y = \frac{x^3}{4}$ and $y = 2\sqrt{\frac{x}{2}}$.