# $11^{\text {th }}$ Asian Pacific Mathematical Olympiad 

March, 1999

1. Find the smallest positive integer $n$ with the following property: there does not exist an arithmetic progression of 1999 real numbers containing exactly $n$ integers.
2. Let $a_{1}, a_{2}, \ldots$ be a sequence of real numbers satisfying $a_{i+j} \leq a_{i}+a_{j}$ for all $i, j=1,2, \ldots$. Prove that

$$
a_{1}+\frac{a_{2}}{2}+\frac{a_{3}}{3}+\cdots+\frac{a_{n}}{n} \geq a_{n}
$$

for each positive integer $n$.
3. Let $\Gamma_{1}$ and $\Gamma_{2}$ be two circles intersecting at $P$ and $Q$. The common tangent, closer to $P$, of $\Gamma_{1}$ and $\Gamma_{2}$ touches $\Gamma_{1}$ at $A$ and $\Gamma_{2}$ at $B$. The tangent of $\Gamma_{1}$ at $P$ meets $\Gamma_{2}$ at $C$, which is different from $P$, and the extension of $A P$ meets $B C$ at $R$. Prove that the circumcircle of triangle $P Q R$ is tangent to $B P$ and $B R$.
4. Determine all pairs $(a, b)$ of integers with the property that the numbers $a^{2}+4 b$ and $b^{2}+4 a$ are both perfect squares.
5. Let $S$ be a set of $2 n+1$ points in the plane such that no three are collinear and no four concyclic. A circle will be called good if it has 3 points of $S$ on its circumference, $n-1$ points in its interior and $n-1$ points in its exterior. Prove that the number of good circles has the same parity as $n$.

