$39^{\text {th }}$ International Mathematical Olympiad<br>Taipei, Taiwan<br>Day I<br>July 15, 1998

1. In the convex quadrilateral $A B C D$, the diagonals $A C$ and $B D$ are perpendicular and the opposite sides $A B$ and $D C$ are not parallel. Suppose that the point $P$, where the perpendicular bisectors of $A B$ and $D C$ meet, is inside $A B C D$. Prove that $A B C D$ is a cyclic quadrilateral if and only if the triangles $A B P$ and $C D P$ have equal areas.
2. In a competition, there are $a$ contestants and $b$ judges, where $b \geq 3$ is an odd integer. Each judge rates each contestant as either "pass" or "fail". Suppose $k$ is a number such that, for any two judges, their ratings coincide for at most $k$ contestants. Prove that $k / a \geq(b-1) /(2 b)$.
3. For any positive integer $n$, let $d(n)$ denote the number of positive divisors of $n$ (including 1 and $n$ itself). Determine all positive integers $k$ such that $d\left(n^{2}\right) / d(n)=k$ for some $n$.

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4. Determine all pairs $(a, b)$ of positive integers such that $a b^{2}+b+7$ divides $a^{2} b+a+b$.
5. Let $I$ be the incenter of triangle $A B C$. Let the incircle of $A B C$ touch the sides $B C, C A$, and $A B$ at $K, L$, and $M$, respectively. The line through $B$ parallel to $M K$ meets the lines $L M$ and $L K$ at $R$ and $S$, respectively. Prove that angle RIS is acute.
6. Consider all functions $f$ from the set $N$ of all positive integers into itself satisfying $f\left(t^{2} f(s)\right)=s(f(t))^{2}$ for all $s$ and $t$ in $N$. Determine the least possible value of $f(1998)$.

