- Let $\{a_k\}$ be a sequence of integers such that $a_1 = 1$ and $a_{m+n} = a_m + a_n + mn$, for all positive integers m and n. Then a_{12} is

(A) 45 **(B)** 56 **(C)** 67 **(D)** 78 **(E)** 89

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- **Solution (D)** By setting $n = 1$ in the given recursive equation, we obtain $a_{m+1} = a_m + a_1 + m$, for all positive integers m. So $a_{m+1} - a_m = m + 1$ for each $m = 1, 2, 3, \ldots$. Hence,

$$a_{12} - a_{11} = 12,\ a_{11} - a_{10} = 11,\ \ldots,\ a_2 - a_1 = 2.$$

Summing these equalities yields $a_{12} - a_1 = 12 + 11 + \cdots + 2$. So

$$a_{12} = 12 + 11 + \cdots + 2 + 1 = \frac{12(12+1)}{2} = 78.$$

Difficulty: Hard

NCTM Standard: Algebra Standard for Grades 9–12: Generalize patterns using explicitly defined and recursively defined functions.

Mathworld.com Classification:
Discrete Mathematics > Computer Science > Algorithms > Recursion > Recursive Sequence;
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