- Let a, b, and c be real numbers such that $a-7b+8c = 4$ and $8a + 4b - c = 7$. Then $a^2 - b^2 + c^2$ is

(A) 0 **(B)** 1 **(C)** 4 **(D)** 7 **(E)** 8

2002 AMC 10 B Number #20— "Rearrange, square, rearrange again"

- **Solution (B)** We have $a + 8c = 4 + 7b$ and $8a - c = 7 - 4b$. Squaring both equations and adding the results yields

$$(a + 8c)^2 + (8a - c)^2 = (4 + 7b)^2 + (7 - 4b)^2.$$

Expanding gives $65(a^2 + c^2) = 65(1 + b^2)$. So $a^2 + c^2 = 1 + b^2$, and $a^2 - b^2 + c^2 = 1$.

Difficulty: Hard

NCTM Standard: Algebra Standard for Grades 9–12: Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency.

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Algebra > Algebraic Identities > Quarter Squares Rule