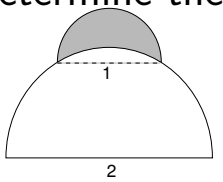
- A semicircle of diameter 1 sits at the top of a semicircle of diameter 2, as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a *lune*. Determine the area of this lune.



(A) $\frac{1}{6}\pi - \frac{\sqrt{3}}{4}$ **(B)** $\frac{\sqrt{3}}{4} - \frac{1}{12}\pi$ **(C)** $\frac{\sqrt{3}}{4} - \frac{1}{24}\pi$ **(D)** $\frac{\sqrt{3}}{4} + \frac{1}{24}\pi$ **(E)** $\frac{\sqrt{3}}{4} + \frac{1}{12}\pi$

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- **Solution (C)** First note that the area of the region determined by the triangle topped by the semicircle of diameter 1 is

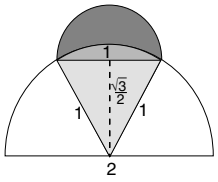
$$\frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2}\pi \left(\frac{1}{2}\right)^2 = \frac{\sqrt{3}}{4} + \frac{1}{8}\pi.$$

The area of the lune results from subtracting from this the area of the sector of the larger semicircle,

$$\frac{1}{6}\pi(1)^2 = \frac{1}{6}\pi.$$

So the area of the lune is

$$\frac{\sqrt{3}}{4} + \frac{1}{8}\pi - \frac{1}{6}\pi = \frac{\sqrt{3}}{4} - \frac{1}{24}\pi.$$



Note that the answer does not depend on the position of the lune on the semicircle.

Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 9–12: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Mathworld.com Classification:
Geometry > Plane Geometry > Arcs > Lune