- For how many integers $n$ is $\frac{n}{20-n}$ the square of an integer?

**(A)** $1$ **(B)** $2$ **(C)** $3$ **(D)** $4$ **(E)** $10$

## 2002 AMC 12 B, Number #12— "Re-express, then use factors"

- **Solution (D)** If $\frac{n}{20-n} = k^2$, for some $k \geq 0$, then $n = \frac{20k^2}{k^2+1}$. Since $k^2$ and $k^2+1$ have no common factors and $n$ is an integer, $k^2+1$ must be a factor of 20. This occurs only when $k = 0, 1, 2,$ or $3$. The corresponding values of $n$ are $0, 10, 16,$ and $18$.

**Difficulty:** Hard

**NCTM Standard:** Number and Operations Standard for Grades 9–12: Use number-theory arguments to justify relationships involving whole numbers.

**Mathworld.com Classification:**
Number Theory > Diophantine Equations > Diophantine Equation