- Find the number of ordered pairs of real numbers (a, b) such that $(a + bi)^{2002} = a - bi$.

(A) 1001 **(B)** 1002 **(C)** 2001 **(D)** 2002 **(E)** 2004

2002 AMC 12 A, Number #24— "Use the complex modulus"

- **Solution (E)** Let $z = a + bi$, $\overline{z} = a - bi$, and $|z| = \sqrt{a^2 + b^2}$. The given relation becomes $z^{2002} = \overline{z}$. Note that

$$|z|^{2002} = \left|z^{2002}\right| = |\overline{z}| = |z|,$$

from which it follows that

$$|z| \left(|z|^{2001} - 1\right) = 0.$$

Hence $|z| = 0$, and $(a, b) = (0, 0)$, or $|z| = 1$. In the case $|z| = 1$, we have $z^{2002} = \overline{z}$, which is equivalent to $z^{2003} = \overline{z} \cdot z = |z|^2 = 1$. Since the equation $z^{2003} = 1$ has 2003 distinct solutions, there are altogether $1 + 2003 = 2004$ ordered pairs that meet the required conditions.

This problem is similar to problem #2 on the 1999 AIME

Difficulty: Hard

NCTM Standard: Number and Operations Standard for Grades 9–12: Understand complex numbers as solutions to quadratic equations that do not have real solutions.

Mathworld.com Classification:
Calculus and Analysis > Complex Analysis > Complex Numbers