- Let $f(x) = x^2 + 6x + 1$, and let R denote the set of points (x, y) in the coordinate plane such that

$$f(x) + f(y) \leq 0 \quad \text{and} \quad f(x) - f(y) \leq 0.$$

The area of R is closest to

(A) 21 **(B)** 22 **(C)** 23 **(D)** 24 **(E)** 25

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- **Solution (E)** Note that

$$f(x) + f(y) = x^2 + 6x + y^2 + 6y + 2 = (x + 3)^2 + (y + 3)^2 - 16$$

and

$$f(x) - f(y) = x^2 - y^2 + 6(x - y) = (x - y)(x + y + 6).$$

The given conditions can be written as

$$(x + 3)^2 + (y + 3)^2 \leq 16 \quad \text{and} \quad (x - y)(x + y + 6) \leq 0.$$

The first inequality describes the region on and inside the circle of radius 4 with center $(-3, -3)$. The second inequality can be rewritten as

$$(x - y \geq 0 \text{ and } x + y + 6 \leq 0) \quad \text{or} \quad (x - y \leq 0 \text{ and } x + y + 6 \geq 0).$$

Each of these inequalities describes a half-plane bounded by a line that passes through $(-3, -3)$ and has slope 1 or -1. Thus, the set R has half the area of the circle, which is $8\pi \approx 25.13$.

Difficulty: Hard

NCTM Standard: Algebra Standard for Grades 9–12: Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency.

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Algebra > Algebraic Geometry > Abstract Algebraic Curves > Algebraic Curve;

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