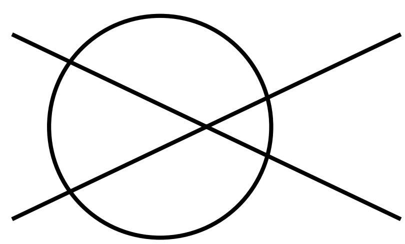
XVIII. AMC 8 Practice Questions

- A circle and two distinct lines are drawn on a sheet of paper. What is the largest possible number of points of intersection of these figures?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

2002 AMC 8, Problem #1—"Draw some figures"

- **Solution (D)** Two distinct lines can intersect in one point whereas a line can intersect a circle in two points. The maximum number 5 can be achieved if the lines and circle are arranged as shown. Note that the lines could also meet outside the circle for the same result. (Other arrangements of the lines and circle can produce 0, 1, 2, 3, or 4 points of intersection.)



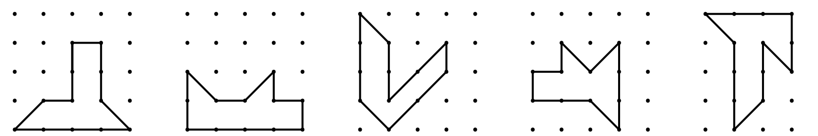
Difficulty: Easy

NCTM Standard: 6-8: Geometry: Use visualization, spatial reasoning, and geometric modeling to solve problems, draw geometric objects with specified properties, such as side lengths or angle measures.

Mathworld.com Classification:
Geometry > Line Geometry > Lines > Circle-Line Intersection;
Geometry > Plane Geometry > Circles > Circle-Line Intersection

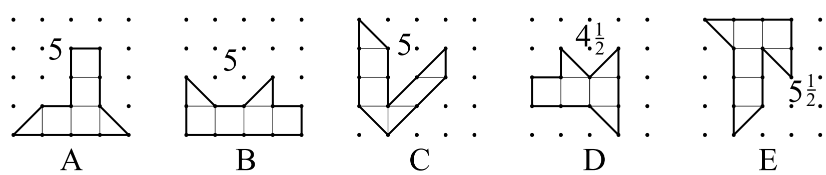
AMC 8 Practice Questions Continued

- Which of the following polygons has the largest area?



2002 AMC 8, Problem #15— “Divide in triangles and squares”

- **Solution (E)** Areas may be found by dividing each polygon into triangles and squares as shown.



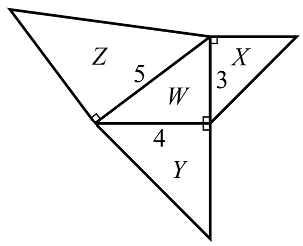
Note: Pick’s Theorem may be used to find areas of geoboard polygons. If I is the number of dots inside the figure, B is the number of dots on the boundary and A is the area, then $A = I + \frac{B}{2} - 1$. Geoboard figures in this problem have no interior points, so the formula simplifies to $A = \frac{B}{2} - 1$. For example, in polygon D the number of boundary points is 11 and $\frac{11}{2} - 1 = 4\frac{1}{2}$.

Difficulty: Easy

NCTM Standard: Grades 6-8 Geometry: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. Middle-grades students should explore a variety of geometric shapes and examine their characteristics. Students can conduct these explorations using materials such as geoboards, dot paper, . . .

Mathworld.com Classification:
Discrete Mathematics > Combinatorics > Lattice Paths and Polygons > Lattice Polygons > Pick’s Theorem;
Geometry > Computational Geometry > Triangulation > Triangulation

- Right isosceles triangles are constructed on the sides of a 3-4-5 right triangle, as shown. A capital letter represents the area of each triangle. Which one of the following is true?



(A) $X + Z = W + Y$ **(B)** $W + X = Z$ **(C)** $3X + 4Y = 5Z$
(D) $X + W = \frac{1}{2}(Y + Z)$ **(E)** $X + Y = Z$

2002 AMC 8, Number #16— "A variant of the Pythagorean Theorem"

- **Solution (E)** The areas are $W = \frac{1}{2}(3)(4) = 6$, $X = \frac{1}{2}(3)(3) = 4\frac{1}{2}$, $Y = \frac{1}{2}(4)(4) = 8$ and $Z = \frac{1}{2}(5)(5) = 12\frac{1}{2}$. Therefore, (E) is correct. $X + Y = 4\frac{1}{2} + 8 = 12\frac{1}{2} = Z$. The other choices are incorrect.

OR

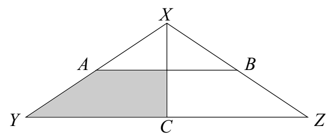
By the Pythagorean Theorem, if squares are constructed on each side of any right triangle, the sum of the areas of the squares on the legs equal the area of the square on the hypotenuse. So $2X+2Y = 2Z$, and $X+Y = Z$.

Difficulty: Hard

NCTM Standard: Grades 6-8 Geometry: Use visualization, spatial reasoning, and geometric modeling to solve problems. ...Eighth graders should be familiar with one of the many visual demonstrations of the Pythagorean relationship, the diagram showing three squares attached to the sides of a right triangle.

Mathworld.com Classification:
Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Right Triangles

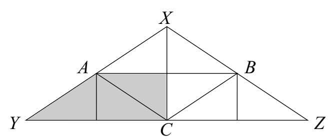
- The area of triangle XYZ is 8 square inches. Points A and B are mid points of congruent segments $\overline{XY}$ and $\overline{XZ}$. Altitude $\overline{XC}$ bisects $\overline{YZ}$. The area (in square inches) of the shaded region is



(A) $1\frac{1}{2}$ **(B)** 2 **(C)** $2\frac{1}{2}$ **(D)** 3 **(E)** $3\frac{1}{2}$

2002 AMC 8, Problem #20— "Divide into congruent triangles"

- **Solution (D)** Segments $\overline{AD}$ and $\overline{BE}$ are drawn perpendicular to $\overline{YZ}$. Segments $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ divide $\triangle XYZ$ into four congruent triangles. Vertical line segments $\overline{AD}$, $\overline{XC}$ and $\overline{BE}$ divide each of these in half. Three of the eight small triangles are shaded, or $\frac{3}{8}$ of $\triangle XYZ$. The shaded area is $\frac{3}{8}(8) = 3$.



OR

Segments $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ divide $\triangle XYZ$ into four congruent triangles, so the area of $\triangle XAB$ is one-fourth the area of $\triangle XYZ$. That makes the area of trapezoid $ABZY$ three-fourths the area of $\triangle XYZ$. The shaded area is one-half the area of trapezoid $ABZY$, or three-eighths the area of $\triangle XYZ$, and $\frac{3}{8}(8) = 3$.

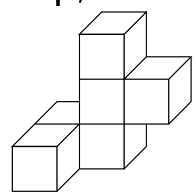
Difficulty: Medium-hard

NCTM Standard: Grades 6-8 Geometry : Use visualization, spatial reasoning, and geometric modeling to solve problems. Apply transformations and use symmetry to analyze mathematical situations describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling; examine the congruence, similarity, and line or rotational symmetry of objects using transformations.

Mathworld.com Classification:

Geometry > Plane Geometry > Geometric Similarity > Congruent

- Six cubes, each an inch on an edge, are fastened together, as shown. Find the total surface area in square inches. Include the top, bottom and sides.



(A) 18 **(B)** 24 **(C)** 26 **(D)** 30 **(E)** 36

2002 AMC 8, Problem #22— "Subtract hidden faces from all faces"

- **Solution (C)** Before the cubes were glued together, there were $6 \times 6 = 36$ faces exposed. Five pairs of faces were glued together, so $5 \times 2 = 10$ faces were no longer exposed. This leaves $36 - 10 = 26$ exposed faces.

Difficulty: Hard

NCTM Standard: Use visualization, spatial reasoning, and geometric modeling to solve problems; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume.

Mathworld.com Classification:

Geometry > Solid Geometry > Polyhedra > Cubes > Polycubes

- Loki, Moe, Nick and Ott are good friends. Ott had no money, but the others did. Moe gave Ott one-fifth of his money, Loki gave Ott one-fourth of his money and Nick gave Ott one-third of his money. Each gave Ott the same amount of money. What fractional part of the group's money does Ott now have?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{2}{5}$ **(E)** $\frac{1}{2}$

2002 AMC 8, Problem #25— "Make the numbers easy"

- **Solution (B)** Only the fraction of each friend's money is important, so we can assume any convenient amount is given to Ott. Suppose that each friend gave Ott \$1. If this is so, then Moe had \$5 originally and now has \$4, Loki had \$ 4 and now has \$3, and Nick had \$3, and now has \$2. The four friends now have $\$4 + \$3 + \$2 + \$3 = \$12$, so Ott has $\frac{3}{12} = \frac{1}{4}$ of the group's money. This same reasoning applies to any amount of money.

Difficulty: Hard

NCTM Standard: Grades 6-8 Problem Solving: Instructional programs from prekindergarten through grade 12 should enable all students to solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems;

Mathworld.com Classification:

Number Theory > Arithmetic > Fractions > Fraction