

## XXI. AMC 10 Student Practice Questions

You will find these and additional problems for the AMC 10 and AMC 12 on AMC's web site: <http://www.unl.edu/amc>, available from the 2004 AMC 10/12 Teacher Manual directory, (<http://www.unl.edu/amc/d-publication/d1-pubarchive/2003-4pub/04tm12/04amc1012tm.html>) or from our Problems page archives (<http://www.unl.edu/amc/a-activities/a7-problems/problem81012archive.html>).

- What is the value of

$$(3x - 2)(4x + 1) - (3x - 2)4x + 1$$

when  $x = 4$ ?

- (A) 0            (B) 1            (C) 10            (D) 11            (E) 12

**2002 AMC 10 B, Number #4;**  
**2002 AMC 12 B, Number #2—**  
**“Factor to make easier”**

- **Solution (D)** Since

$$\begin{aligned}(3x - 2)(4x + 1) - (3x - 2)4x + 1 &= (3x - 2)(4x + 1 - 4x) + 1 \\ &= (3x - 2) \cdot 1 + 1 = 3x - 1,\end{aligned}$$

when  $x = 4$  we have the value  $3 \cdot 4 - 1 = 11$ .

**Difficulty:** Easy

**NCTM Standard:** Algebra Standard for Grades 9–12: Understand the meaning of equivalent forms of expressions.

**Mathworld.com Classification:**

Algebra > Polynomials > Polynomial;

Algebra > Polynomials > Polynomial Factorization

**XXI. AMC 10 Student Practice Questions continued**

- Let  $\{a_k\}$  be a sequence of integers such that  $a_1 = 1$  and  $a_{m+n} = a_m + a_n + mn$ , for all positive integers  $m$  and  $n$ . Then  $a_{12}$  is

(A) 45            (B) 56            (C) 67            (D) 78            (E) 89

**2002 AMC 10 B, Number #23—**  
**“Create the sequence”**

- **Solution (D)** By setting  $n = 1$  in the given recursive equation, we obtain  $a_{m+1} = a_m + a_1 + m$ , for all positive integers  $m$ . So  $a_{m+1} - a_m = m + 1$  for each  $m = 1, 2, 3, \dots$ . Hence,

$$a_{12} - a_{11} = 12, \quad a_{11} - a_{10} = 11, \quad \dots, \quad a_2 - a_1 = 2.$$

Summing these equalities yields  $a_{12} - a_1 = 12 + 11 + \dots + 2$ . So

$$a_{12} = 12 + 11 + \dots + 2 + 1 = \frac{12(12+1)}{2} = 78.$$

**Difficulty:** Hard

**NCTM Standard:** Algebra Standard for Grades 9–12: Generalize patterns using explicitly defined and recursively defined functions.

**Mathworld.com Classification:**

Discrete Mathematics > Computer Science > Algorithms > Recursion > Recursive Sequence;

Discrete Mathematics > Recurrence Equations > Recursive Sequence

**XXI. AMC 10 Student Practice Questions continued**

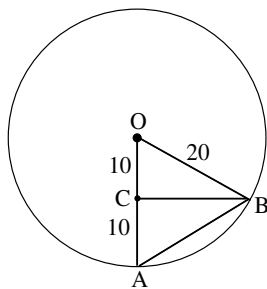
- Riders on a Ferris wheel travel in a circle in a vertical plane. A particular wheel has radius 20 feet and revolves at the constant rate of one revolution per minute. How many seconds does it take a rider to travel from the bottom of the wheel to a point 10 vertical feet above the bottom?

(A) 5                      (B) 6                      (C) 7.5                      (D) 10                      (E) 15

**2002 AMC 10 B, Problem #24—**  
**“Use 30-60-90 triangle geometry”**

- **Solution (D)** In the figure, the center of the wheel is at  $O$ , and the rider travels from  $A$  to  $B$ . Since  $AC = 10$  and  $OB = OA = 20$ , the point  $C$  is the midpoint of  $\overline{OA}$ . In the right  $\triangle OCB$ , we have  $OC$  half of the length of the hypotenuse  $OB$ , so  $m\angle COB = 60^\circ$ . Since the wheel turns through an angle of  $360^\circ$  in 60 seconds, the time required to turn through an angle of  $60^\circ$  is

$$60 \left( \frac{60}{360} \right) = 10 \text{ seconds.}$$



**Difficulty:** Hard

**NCTM Standard:** Geometry Standard for Grades 9–12: Use visualization, spatial reasoning, and geometric modeling to solve problems.

**Mathworld.com Classification:**

Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > 30-60-90 Triangle;

Geometry > Plane Geometry > Circles

**XXI. AMC 10 Student Practice Questions continued**

- Let  $d$  and  $e$  denote the solutions of  $2x^2 + 3x - 5 = 0$ . What is the value of  $(d - 1)(e - 1)$ ?

(A)  $-\frac{5}{2}$       (B) 0      (C) 3      (D) 5      (E) 6

**2003 AMC 10 A, Problem #5—**

**“Use the sum and product of the roots formulas”**

- **Solution (B)** If  $x = d$  and  $x = e$  are the roots of the quadratic equation  $ax^2 + bx + c = 0$ , then

$$de = \frac{c}{a} \quad \text{and} \quad d + e = -\frac{b}{a}.$$

For our equation this implies that

$$(d - 1)(e - 1) = de - (d + e) + 1 = -\frac{5}{2} - \left(-\frac{3}{2}\right) + 1 = 0.$$

One can also factor the quadratic directly, find the roots and evaluate the expression!

**Difficulty:** Easy

**NCTM Standard:** Algebra Standard for Grades 9–12: Represent and analyze mathematical situations and structures using algebraic symbols; understand the meaning of equivalent forms of expressions, equations, inequalities, and relations; write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency.

**Mathworld.com Classification:**

Algebra > Algebraic Equations > Quadratic Equations;

Algebra > Polynomials > Vieta's Formulas