XXII. AMC 12 Student Practice Questions

- The arithmetic mean of the nine numbers in the set $\{9, 99, 999, 9999, \ldots, 999999999\}$ is a 9-digit number M, all of whose digits are distinct. The number M does not contain the digit

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

2002 AMC 12 B, Number #1— "Definition of mean, then factor"

- **Solution (A)** The number M is equal to

$$\frac{1}{9}(9 + 99 + 999 + \ldots + 999{,}999{,}999) = 1 + 11 + 111 + \ldots + 111{,}111{,}111$$
$$= 123{,}456{,}789.$$

The number M does not contain the digit 0.

Difficulty: Easy

NCTM Standard: Data Analysis and Probability Standard for Grades 9–12: For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics.

Mathworld.com Classification:
Calculus and Analysis > Special Functions > Means > Arithmetic Mean

- For how many integers n is $\frac{n}{20-n}$ the square of an integer?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 10

2002 AMC 12 B, Number #12— "Re-express, then use factors"

- **Solution (D)** If $\frac{n}{20-n} = k^2$, for some $k \geq 0$, then $n = \frac{20k^2}{k^2+1}$. Since k^2 and k^2+1 have no common factors and n is an integer, k^2+1 must be a factor of 20. This occurs only when $k = 0, 1, 2,$ or 3. The corresponding values of n are $0, 10, 16,$ and 18.

Difficulty: Hard

NCTM Standard: Number and Operations Standard for Grades 9–12: Use number-theory arguments to justify relationships involving whole numbers.

Mathworld.com Classification:
Number Theory > Diophantine Equations > Diophantine Equation

- A square and an equilateral triangle have the same perimeter. Let A be the area of the circle circumscribed about the square and B be the area of the circle circumscribed about the triangle. Find A/B.

(A) $\frac{9}{16}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{27}{32}$ **(D)** $\frac{3\sqrt{6}}{8}$ **(E)** 1

2003 AMC 12 A, Number #11— "Draw the figure, find the radius"

- **Solution (C)** Let the common perimeter be 1. Then the side length of the square is $1/4$, and the side length of the triangle is $1/3$. The radius of the circle cicumscribed about the square is half the diagonal length or $\sqrt{2}/8$. The area $A = \pi(\sqrt{2}/8)^2 = \pi/32$. The radius of the circle circumscribed about the triangle is $(2/3)(\sqrt{3}/6) = \sqrt{3}/9$. The area $B = \pi(\sqrt{3}/9)^2 = \pi/27$. Then the ratio $A/B = 27/32$.

Difficulty: Medium

NCTM Standard: Geometry Standard for Grades 9–12: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Mathworld.com Classification:
Geometry > Plane Geometry > Triangles > General Triangles > Triangle Circumscribing

- How many non-congruent triangles with perimeter 7 have integer side lengths?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2003 AMC 12 B, Number #7— "Use integer partitions and the triangle inequality"

- **Solution (B)** The perimeter 7 could possibly be written as the sums $5+1+1$, $4+2+1$, $3+3+1$, and $3+2+2$. The sum of the two shorter sidelengths must be greater than the third side length, so only $3+3+1$ and $3+2+2$ are possible triangles.

Difficulty: Medium-easy

NCTM Standard: Geometry Standard for 9-12: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.

Mathworld.com Classification:
Discrete Mathematics > Combinatorics > Enumeration > Triangle Counting;
Geometry > Plane Geometry > Triangles > General Triangles > Triangle Counting