XIV. Classroom Accessories
AMC 10 Student Practice Questions

- Brianna is using part of the money she earned on her weekend job to buy several equally-priced CDs. She used one fifth of her money to buy one third of the CDs. What fraction of her money will she have left after she buys all the CDs?

(A) $\dfrac{1}{5}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{2}{5}$ (D) $\dfrac{2}{3}$ (E) $\dfrac{4}{5}$

2005 AMC 10 B, Problem #5— "If she used one fifth of her money to buy one third of the CDs then what fraction of her money has she spent to buy three thirds of the CDs?"

- **Solution (C)** The number of CDs that Brianna will finally buy is three times the number she has already bought. The fraction of her money that will be required for all the purchases is $(3)(1/5) = 3/5$. The fraction she will have left is $1 - 3/5 = 2/5$.

Difficulty: Easy
NCTM Standard: Problem Solving Standard: Solve problems that arise in mathematics and in other contexts
Mathworld.com Classification:
Number Theory > Arithmetic > Fractions > Fraction

- Three tiles are marked X and two other tiles are marked O. The five tiles are randomly arranged in a row. What is the probability that the arrangement reads XOXOX?

(A) $\frac{1}{12}$ (B) $\frac{1}{10}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

2005 AMC 10 A, Problem #9— "What is the likelihood that the first spot will be an X..."

- **Solution (B)** There are three X's and two O's, and the tiles are selected without replacement, so the probability is

$$\frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{1}{10}.$$

OR

The three tiles marked X are equally likely to lie in any of $\binom{5}{3} = 10$ positions, so the probability of this arrangement is $1/10$.

Difficulty: Medium

NCTM Standard: Data Analysis and Probability Standard: Understand and apply basic concepts of probability

Mathworld.com Classification:
Probability and Statistics > Probability > Probability

- How many three-digit numbers satisfy the property that the middle digit is the average of the first and the last digits?

(A) 41   **(B)** 42   **(C)** 43   **(D)** 44   **(E)** 45

2005 AMC 10 A, Problem #14— **"What are the conditions for the middle digit to be an integer?"**

- **Solution (E)** The first and last digits must be both odd or both even for their average to be an integer. There are $5 \cdot 5 = 25$ odd-odd combinations for the first and last digits. There are $4 \cdot 5 = 20$ even-even combinations that do not use zero as the first digit. Hence the total is 45.

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Mathworld.com Classification:

Number Theory > Arithmetic > Number Bases > Digit

- An equiangular octagon has four sides of length 1 and four sides of length $\frac{\sqrt{2}}{2}$, arranged so that no two consecutive sides have the same length. What is the area of the octagon?

(A) $\frac{7}{2}$　　　　(B) $\frac{7\sqrt{2}}{2}$　　　　(C) $\frac{5+4\sqrt{2}}{2}$　　　　(D) $\frac{4+5\sqrt{2}}{2}$　　　　(E) 7

2005 AMC 10 A, Problem #20— **"What can we break the octagon into?"**

- **Solution (A)** The octagon can be partitioned into five squares and four half squares, each with side length $\sqrt{2}/2$, so its area is
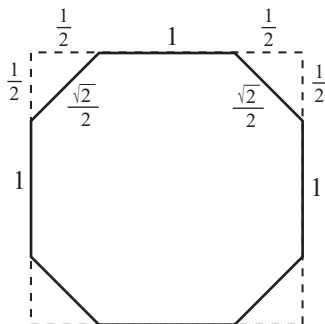
$$\left(5 + 4 \cdot \frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{7}{2}.$$
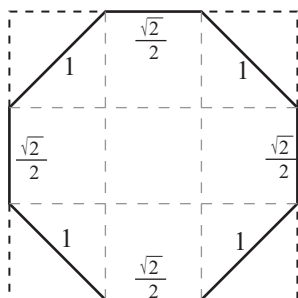
OR

The octagon can be obtained by removing four isosceles right triangles with legs of length $1/2$ from a square with sides of length 2. Thus its area is

$$2^2 - 4 \cdot \frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{7}{2}.$$

Difficulty: Hard

NCTM Standard: Geometry Standard: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Mathworld.com Classification:

Geometry > General Geometry > Euclidean Geometry