



The MATHEMATICAL ASSOCIATION OF AMERICA  
**American Mathematics Competitions**  
www.unl.edu/amc amcinfo@maa.org 800/527-3690

**AMC 10A / AMC 12A**

9th / 59th Annual Contests

MEDITATIONES

PROBLEMATUM NAUTICARUM

AB ILLUSTRISSIMO MATH.

TENTAMEN  
NOVAE THEORIAE  
MUSICAE

CERTISSIMAE

HARMONICAE

LEONH.

PRINCIPES

# Teachers' Manual

**Tuesday, February 12, 2008**

Instructions and Reporting Forms  
for School Contest Managers

Please read this booklet completely upon receipt  
Exams must be administered over a continuous 75-minute period to all students at the same time

#### **2008 EXAMINATIONS**

AMC10/AMC12 - Tuesday, February 12, 2008

&/or Wednesday, February 27, 2008

AIME - Tuesday, March 18, 2008 or Wednesday, April 2, 2008

USAMO - Tuesday & Wednesday, April 29 & 30, 2008

AMC 8 - Tuesday, November 18, 2008



The MATHEMATICAL ASSOCIATION OF AMERICA  
**American Mathematics Competitions**

Steven Dunbar  
AMC Director

To all Contest Managers:

I am very pleased you will be providing the opportunity for your students to participate in the 59th annual American Mathematics Contest 12 (AMC 12) or its sister contest, the 9th annual American Mathematics Contest 10 (AMC 10). I believe that you and your students will find these contests to be both interesting and challenging. Again this year the Committee on the American Mathematics Competitions and panelists under the leadership of Committee Chairs LeRoy Wenstrom for the AMC 10 and Professor David Wells for the AMC 12 developed a total of 4 contests (two AMC 10 and two AMC 12). I extend special thanks to both of them for the effort involved in developing these excellent contests.

This year, we continue to include in this manual several handouts, included in Section XIV:

- \* Worksheets for contest preparation (page 14)
- \* A handout for parents on why mathematics is important (page 22)
- \* A sample Press Release (page 23)
- \* A Certificate of Participation (pages 24 & 25)

You may reproduce these pages for your students.

Very sincerely,

Steven Dunbar  
*Director*

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# Important Procedures

## Format

There are two official dates for the AMC 10 & AMC 12. Give the AMC 10 and AMC 12 at the same time within each participating school on TUESDAY, February 12, 2008 (AMC 10-A & AMC 12-A), or WEDNESDAY, February 27, 2008 (AMC 10-B & AMC 12-B) in a convenient 75-minute interval, preferably in the morning. All four contests consist of 25 questions.

**NOTE:** Each correct answer scores 6 points, a blank scores 1.5 points and an incorrect answer scores 0 points.

The AMC 10 and AMC 12 have several questions in common. The students in grades 10 and below should choose between the AMC 10 and AMC 12. Students in grades 11 and 12 may only take the AMC 12. All rules and awards apply to both contests for all schools and students. Any student who missed the exam may take it unofficially, and we will be happy to grade it. Students may take the contest booklets home with them the day of the contest.

## Answer Forms

The AMC 10 and AMC 12 each have their own color coded answer form (AMC 10-orange and AMC 12-red). Contest B answer forms have instructions in black ink. Please be careful to match the correct answer form with the appropriate contest when passing out the papers.

## AIME Qualification

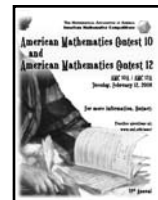
Students who score 100 or above or finish in the top 5% on this AMC 12 or students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 26th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 18, 2008 or Wednesday, April 2, 2008.

## Results

We will send your school's results by email (if available) and first class mail as soon as the answer forms are scored. If you have not received your results from our office within 30 days after the contest, please contact us to verify that your answer forms were in fact received. All student Answer Forms are held for 90 days after the exam date, then they are recycled.

## School Flier

Last year we received several requests from teachers to provide a flier they could use to promote the contests within their school. This year we have designed and included the appropriate flier with this package. It has space left for you to add school pertinent information, such as the location of the contests, and who to contact locally for more information. If you need both the A and B versions, or would like a color version, visit our web page to download a pdf version of the flier you need: [www.unl.edu/amc/](http://www.unl.edu/amc/)



### I. Preliminary Instructions for Administering the AMC 10/AMC 12

1. Inform students far in advance about the date for the AMC 10/AMC 12 and obtain a supply of No. 2 lead pencils.
2. Hand out the student Answer Forms and have the students complete the non-answer sections on the front and back. Have students pay special attention to marking their name and address accurately. Remind them that student names listed in the National Summary come from this form. The AMC Office will not do any editing of the information.
3. The name of the school, city, state and postal code must be stamped or written on each student answer form. This is very important because there is no other school identification on the answer forms. Please do not use stick-on labels to provide this information.
4. Announce that the students may use scratch paper, graph paper, ruler, compass, and/or a protractor. **CALCULATORS ARE NOT PERMITTED STARTING WITH THE 2008 CONTESTS.** No problems on the contest will require the use of a calculator.
5. Review past tests and solutions or inform your students
6. Encourage participation by students who have not taken the contest before, especially younger students, but make sure students know what to expect. Let them know about typical scores at your school last year at their grade level. Show students the national statistics in our National Summary of Results and Awards from last year. Tell them to set appropriate goals for themselves.
7. Remind students the day before the contest about the time and place of the AMC 10/AMC 12. Also tell them your plan if the school should suddenly close. All students must take the contest at the same time, either in one group or in separate classrooms under the supervision of a teacher.
8. Make sure you have arranged to follow all the rules and procedures in this manual. **EARLY ADMINISTRATION OF THE CONTESTS IS NEVER PERMITTED**, and will lead to disqualification. To assure the validity of the results we report, we take our rules very seriously.

## II. Instructions For The Day of The AMC 10/AMC 12

If today is not yet Tuesday, February 12, 2008, STOP EVERYTHING. Under no circumstances is the contest to be given to anyone before the official day, nor should the contest package(s) be opened before that date.

1. Take the unopened contest package(s) and Certification Form to your Principal (or equivalent) to certify that the package was opened within an hour before the contest. Have the Principal sign the Certification Form at that time.
2. When the students arrive, seat them so they are separated by an empty space, if possible.
3. Hand out the Answer Forms which have been partially completed by the students (AMC 10-orange/AMC 12-red).
4. As you hand out the papers, tell the students not to open the contest booklet. They should then read the entire front cover. Give them 5 minutes to do this.
5. Inform the students to, "Carefully read instructions 3 and 4 on the contest cover." (see [pages 26 and 27](#) of this manual). The AMC 10/AMC 12 has a unique scoring system which has important consequences for guessing. Unless you are fairly sure of the answer, it is better to leave a question unanswered than to guess. Six points are given for a correct answer, 1.5 points for a blank answer and 0 points for an incorrect answer. If a student can reduce the problem to three possible answers, it is advantageous to guess one of the three possible answers. If a student can only reduce to 4 possible answers by eliminating 1 of the possibilities, then it is not advantageous to guess.
6. Inform the participants they may not talk or ask any questions during the contest, and they must do their own work.
7. Remind students they have 75 minutes, then tell them to BEGIN. (See page 6 for student disabilities policy)
8. Students who finish the contest early may be dismissed provided they will be under the supervision of a teacher during the remainder of the contest period.
9. You (and other teachers, if there are many participants) should proctor continually as you would for any important contest. Students whose eyes wander should be warned; students caught copying answers or collaborating must be disqualified. Try to provide as quiet an environment as possible.
10. Announce when there are 30 minutes remaining and when there are 5 minutes remaining.
11. When time is up, tell the students to STOP and have them sign their name in the space provided on the Student Answer Form. Collect the Answer Forms as quickly as possible.
12. Please do not grade the answer forms. They are to be sent to the AMC office for grading. Students may circle their answers on the contest booklet. However, the official answers will be the ones blackened on the answer form.
13. Fan the forms, making sure none are stuck together, place the School ID Form on top. There is only one ID FORM to be used with all the AMC 10/AMC 12 answer forms.
14. YOUR SCHOOL'S CEEB NUMBER IS THE NUMBER

WRITTEN ON THE FRONT OF THE AMC 10/AMC 12 REPORT ENVELOPE.

15. Complete the Certification Form (only one form is needed) and place it on top of the School ID Form and answer forms and place all in the Report Envelope. Seal and send it by First Class (trackable recommended) within 24 hours or as soon as possible. Please affix the proper postage before mailing.
16. **Please note:** After the Answer Forms have been delivered to the school office to be mailed, you may discuss the contest with your students under the following conditions which take into consideration the fact that there will be schools taking the contest in other locations at different times.
  - a. Inform the students that the contest may not be discussed with anyone outside of your school either orally, via email, www, copier or media of any type until after the contest period.
  - b. Students may keep the contest booklets and take them home.

## III. Eligibility

Any student who is officially enrolled in high school (or below) and is taking at least one course at the high school level, and has not graduated, is eligible to take the AMC 10 or AMC 12 or AIME (with qualifying score). *Please note: students in grades 11 & 12 can not take the AMC 10.* However, students in grades 9 & 10 may choose which contest they take.

### Home School Students

Home School Students age 19.5 and under are eligible for AMC 12 and AIME (with qualifying score) and age 17.5 and under are eligible for AMC 10 or AMC 12 and AIME (with qualifying score).

**International Students & Non-Citizens in USA Schools**  
US and Canadian Citizens and International Students residing in the United States (with qualifying scores) are eligible to take the USAMO.

Students learning "English as a Second Language" (ESL) may use a book or electronic dual-language nontechnical dictionary between their native language and English. A student may use the dictionary only the first time that he/she takes the AMC 10/AMC 12. The dictionary must be given to the school contest manager to examine and retain for the 24-hour period preceding the contest. The proctor must announce to other students that the student(s) has/have been given special permission to use the dictionary during the contest.

## IV. Team Score Identification

TO RECEIVE OFFICIAL TEAM STATUS AND AWARDS, A SCHOOL MUST HAVE AT LEAST THREE PARTICIPANTS ON A CONTEST DATE. The team score for a school is the sum of its three highest student scores and will be determined by the AMC Office. The score of USA and Canadian teams is used to determine National School awards. In addition, the team score is used to select the top 60 schools to identify teachers who are eligible for the Edyth May Sliffe Award for Distinguished High School Teaching.



## V. School Results

The AMC office will send results by email (if available) and first class mail as soon as the answer forms are scored. If you have not received your results from our office within 30 days after the AMC 10/AMC 12 please contact us to verify that your answer forms were in fact received.

If you would like to receive your results by e-Mail, and have not previously sent us your email address, send a message, including your name, school name, address, and CEEB # to:

hstran@amc.unl.edu

Results are not official until you receive the postal mailed paper copy of your report, and that should be no longer than 30 days after AMC 10/AMC 12.

## VI. Policy Statements

### Early Administration

Administration on an earlier date is NEVER permitted and will lead to disqualification. Such an administration would jeopardize the validity of all scores from other participating schools.

### Official Administration

The AMC 10-A/AMC 12-A will be given officially on Tuesday February 12, 2008. The AMC 10-B/AMC 12-B will be given officially on Wednesday, February 27, 2008. Only official participants, their school and their teacher are eligible for National Awards. In addition, official participants are eligible for all intramural awards and for participation in the AIME.

### Unofficial Administration

If you are unable to give Contest A on Tuesday, February 12, 2008 because:

- your school is closed,
- your school has an academic conflict,
- the class periods have been shortened due to an assembly or other reason,

then you may give the second version of the contests (AMC 10-B/AMC 12-B) on the second official day, Wednesday, February 27, 2008 (See Contest B Registration Form on [page 13](#)). You may still take either exam unofficially on later dates, but those contests will not be eligible for state and national awards and will not be eligible for participation in the AIME. Unofficial participants are still eligible for intramural awards.

It is important to note that the only days eligible for official participation are the two official Contest days: February 12, 2008, and February 27, 2008.

### One Contest per Date

A student may take only one exam on a given day but can participate on both contest dates if the school registers for both contests. The higher score will be used for individual awards.

### Students with Visual or Learning Disabilities

The AMC 10/AMC 12 time limit set by the CAMC for students who are visually impaired or learning disabled is 120 minutes. A teacher or a school administrator may read the questions to the student and mark the answers as directed by the student. The cost of a Braille or Large Print exam is \$7.00 shipping and handling plus \$1.60 per exam for the AMC 12 and \$1.40 per exam for the AMC 10. **They are mailed separately and must be ordered no later than three weeks before the test.**

## Sickness and Other Special Situations

A student who is sick or on a field trip on the first contest day may register and take the alternate Contest B on Wednesday, February 27, 2008. **YOU MUST REGISTER FOR CONTEST B** if you have not already done so. (see [page 13](#) for a Registration Form).

### Questionable Scores

If it is clear to the Contest Manager from personal observation that a student has cheated, then the Manager must disqualify the student. If the Contest Manager receives an accusation of cheating, or obtains other indirect evidence of cheating, then the Manager must hold back the student's paper and immediately report all the facts of the situation to the AMC Director, who in conjunction with the Chair of the CAMC, will determine what to do. **UNDER NO CIRCUMSTANCES** may the school decide on its own to accept a questionable score, nor should a school retest the student in question before receiving instructions from the AMC Office.

### Follow-up Inquiries and Reexamination

The results of this contest helps to identify students with unusual mathematical ability. To assure that this purpose is served, the CAMC reserves the right to retest students before deciding whether to grant official status to individual or team scores. Reexamination will be requested when, after an inquiry, there is a reasonable basis to disbelieve a score. Official status will not be granted if a student or school does not agree to a requested retesting.

### Policy for Changes

The CAMC may, from time to time, change the program rules, regulations, awards and conditions of participation in whole or in part. Whenever possible you will be notified of these changes ahead of time.

### Refund/Credit Policy

If your school is unable to take the contests, please use the materials as practice sets for the next year. Do not return them. **WE CAN NOT GIVE REFUNDS OR CREDITS AFTER THE CONTEST MATERIALS ARE SHIPPED.**

### Request for Student Names Policy

The following statement appears on the student answer forms for the AMC 10 and AMC 12:

*The American Mathematics Competitions (AMC) receives requests from educational institutions and organizations for the names, addresses and grade levels of high scoring AMC 12 (or AMC 10) students. The optional personal data on ethnic origin and gender is used for recruiting and academic purposes..*

*Blacken this circle if you give the AMC permission to release this information to these organizations. (Your score will not be affected if you do not blacken the circle.)*

Receiving information is an "opt-in" decision for each individual student.

The AMC handles requests from institutions and organizations on a case-by-case basis and evaluates each individually for appropriateness. We provide legitimate educational institutions of all levels, both secondary and collegiate/university level, with one-time use of selected names and addresses for postal mailings. We also provide professional and scholarly organizations such as those listed as contributors to the AMC with one-time

use of names and addresses for postal mailings, generally for professional or career information.

The only information we provide is the name, address, city, state, and zip code necessary for a postal mailing. We do not list individual scores or awards.

## VII. AIME Instructions

The 26th annual American Invitational Mathematics Examination (AIME) will be held on TUESDAY, March 18, 2008 with a second alternate exam given on the alternate date of WEDNESDAY, April 2, 2008. These are the only days the exam may be taken officially. You may give the exam for practice (unofficially), after the official dates. We will be pleased to grade it for you but your students will not be eligible to take the USAMO. The contest is provided free of charge to all those taking the exam on the first date, however those taking the exam on the second alternate date will be charged a processing fee to cover expedited delivery.

### AIME Rules for AMC 10/AMC 12

Students who score 100 or above or finish in the top 5% on this AMC 12 or students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 26th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 18, 2008 or Wednesday, April 2, 2008. PLEASE read the following participation rules to your students as soon as you receive the AMC 10/AMC 12 package so potential AIME students will be able to plan accordingly.

### AIME School Manager

1. The AMC office will include all materials relating to the examination (including instructions for the exam) with your AMC 10/AMC 12 results.
2. All questions or problems concerning the AIME should be directed to the AMC office (800-527-3690).
3. The AIME is a three-hour examination. Each of its 15 questions requires a three digit integer answer and each correct answer will receive one (1) point.
4. Calculators are not allowed.
5. The AIME Answer Forms are sent directly to the AMC office for grading and processing.
6. Each participating school will receive a report of their results, an AIME solution pamphlet, and a list of students who qualify for the USAMO.
7. All AMC 10/AMC 12 procedures for disqualification, follow-up inquiries and reexamination apply to the AIME as appropriate.
8. If you have students who you feel may qualify for the AIME please order prior year AIME exams and solutions for practice now. This way you will have these practice materials on hand when you receive your AMC 10/AMC 12 results.

### Second AIME Testing Date

Situations in which a student may take a second version of the AIME to be held on WEDNESDAY, April 2, 2008, keeping their USAMO eligibility open are:

1. School is closed on March 18, 2008 (i.e. spring break, weather).

2. Student is out of school the entire day due to attendance at an academic/school related event.
3. Student is ill and can not attend school on March 18, 2008.

There will be a processing fee for the second AIME as follows: 1-10 students = \$25, 11+ students = \$50. We will need your payment before the answer forms can be graded. A special envelope and payment form will be included with your AIME material, if you have AIME qualifiers. All AIME answer forms must arrive in the AMC office by **April 4, 2008**.

Email requests for the second AIME may be sent to:

AIMEQUAL@AMC.UNL.EDU

or, you can call the AMC office at 1-800/527-3690. Please have your school identification number (CEEB) and charge card information available before calling. E-Mail requests should include the school's CEEB number, and complete mailing address.

Under no circumstances can a student take both AIME's.

## VIII. USAMO Participant Selection

The USA Mathematical Olympiad (USAMO) is a two day, nine-hour, six-question, essay-proof examination. Selection for the USAMO will be explained in the AIME Teacher Manual. The goal is to select about 500 of the top scorers from the prior AIME and AMC 12A, AMC 12B, AMC 10A and AMC 10B contests to participate in the USAMO.

The USAMO is scheduled for Tuesday and Wednesday, April 29 & 30, 2008 at your school. If you feel you may have a qualifier, please arrange for a space and proctor for these dates.

The top 12 scoring students on the USAMO will be invited to attend an award ceremony held in Washington, D.C., **on June 8, 9, 2008**.

## IX. The MOSP Program

The Mathematical Olympiad Summer Program (MOSP) is a 3-week, academic challenge designed to broaden participants' view of mathematics while fostering excitement toward further math study. It is held each year at the University of Nebraska-Lincoln in June-July. Invited students include the top 12 USAMO winners, 12-18 high-scoring USAMO participants, who are current juniors and below, and an additional 30 ninth-grade USAMO participants, grant funding permitting.

Watch for further details to be announced in the 2008 AIME/USAMO Teachers' Manual and on the AMC website at [www.unl.edu/amc](http://www.unl.edu/amc).

## X. Regions of the AMC 10/AMC 12

The USA and Canada are partitioned into the following regions. National Awards are given to a minimum of 10 high scoring students and 5 schools (based on the team score) in each of these regions.

Region

- 0 Connecticut, Maine, Massachusetts, New Hampshire, Pennsylvania, Rhode Island, Vermont
- 1 New Jersey, New York
- 2 Delaware, District of Columbia, Maryland, North Carolina, South Carolina, Virginia, West Virginia

- 3 Alabama, American Embassy and APO/FPO Schools, Florida, Georgia, Puerto Rico, Virgin Islands
- 4 Indiana, Michigan, Ohio
- 5 Arkansas, Iowa, Kansas, Minnesota, Nebraska, North Dakota, Oklahoma, South Dakota, Wisconsin
- 6 Illinois, Kentucky, Missouri, Tennessee
- 7 Louisiana, Mississippi, Texas
- 8 Alaska, Arizona, Colorado, Guam, Hawaii, Idaho, Montana, Nevada, New Mexico, Oregon, Utah, Washington, Wyoming
- 9 California
- 10 Canada

## **XI. Intramural and National Awards**

### **Intramural Awards**

The AMC Office will send you Intramural awards, along with your school results. Your registration fee entitles you to pins, medals, certificates, and a copy of the National Summary of Results and Awards. An order form for additional intramural awards will be included with your results.

1. Winner Pin — given to the top scoring student in your school for both the AMC 10 and AMC 12. Medals will be given for consecutive wins in both contests mentioned above. Details of how medals are awarded will be included with your school's results.
2. Certificate of Distinction — awarded to all students who qualify for the AIME.
3. Honor Roll of Distinction Pin — given to the top 1% of the AMC 12 and to the top 1% of AMC 10 participants.
4. AMC 12 Certificate of Achievement — given to students in grade 10 and below who score 90 or above on the AMC 12 Contests.
5. AMC 10 Certificate of Achievement — given to students in grade 8 and below who score 90 or above on the AMC 10 Contests.
6. School Certificate of Honor — awarded to schools with a team score (AMC 12) of 400 or greater.
7. School Certificate of Merit — awarded to schools with a team score (AMC 12) between 300 and 399, inclusive.

### **State/Provincial Awards**

Plaques — Top Scoring Student Plaque awarded to a student in each state or province.

Some State/Provincial Directors offer various awards or scholarships in addition to the plaques.

### **National Awards**

In each of the eleven regions the AMC divides the United States and Canada into, the five schools with the highest team scores (sum of the highest three scores by participants) are recognized by Donor or Committee Awards.

The nationally highest scoring team will receive the Charles T. Salkind Memorial Silver Cup, a silver Trophy Cup awarded to the school with the highest team score over all, donated by the L.G. Balfour Company, Attleboro, MA.

The second nationally highest scoring team will receive the William H. Fagerstrom Memorial Silver Cup, a silver Trophy

Cup awarded to the school with the second highest team score over all, also donated by the L.G. Balfour Company, Attleboro, MA.

In each of the regions in which a silver cup is not awarded the Committee on the American Mathematics Competitions provides a Committee Bronze Cup to the school with the highest team score.

### **Awards for Schools not receiving a Trophy Cup**

The four remaining schools in each region will receive one of these awards. Distribution of awards is arranged so that schools do not receive duplicates of awards made to them in prior years.

1. CAMC Mathematics Books — in each of the eleven regions, five books are donated by the Committee on the American Mathematics Competitions to one school having a high team score.
2. W. H. Freeman Books — in each of the eleven regions, a set of books, donated by W. H. Freeman Company, San Francisco, CA is awarded to one school having a high team score.
3. Mathematics Magazine — in each of the eleven regions, a one-year subscription is donated by the Committee on the American Mathematics Competitions to two or more schools having a high team score.

### **Most Improved Team Score Award**

1. Pedagoguery Award — awarded to one school in each of the 50 states, US Territories, Military Schools abroad, and Canada with the "Most Improved" Team Score on the AMC 10 and AMC 12.



# XII. Contest A Certification

The AMC 10 and AMC 12 must be administered by a teacher or an adult not associated with or related to any of the participants. The administration of the contest must take place in a public building (e.g. school, library, church). Please send all Answer Forms from your school or group at one time.

The Contest Manager and the Principal, Vice Principal, or Headmaster must sign this form which is to be returned with your student Answer Forms.

### Certification by the Principal, official or person with comparable title:

- (a) I certify that the exam package(s) were retained in their sealed condition within an hour of the start of the contest.
- (b) I accept for our school the rules and procedures described on this page and pages 4-7, and accept that failure to follow these rules and procedures may result in DISQUALIFICATION from official standing of all scores from our school.

Signature \_\_\_\_\_ Time \_\_\_\_\_

Title \_\_\_\_\_ Date \_\_\_\_\_

### Certification by the Contest Manager:

I certify that the following statements are true or that, if there are any exceptions, I have checked the box at the bottom of this page and have listed them on a separate page. I understand that the absence of either signature from this form, and a consideration of the exceptions may result in DISQUALIFICATION of all scores from our school.

1. The contest was held on TUESDAY, February 12, 2008.
2. The AMC 10-A/AMC 12-A were given at the same time.
3. The participants were continually monitored during the contest, and they were separated by an empty space, if possible.
4. No aids were permitted other than scratch paper, graph paper, ruler, compass, and protractor. NO CALCULATORS WERE USED - (see Section I. Item 4).
5. Participants had exactly 75 minutes working time. (See page 6 for Student Disabilities Policy)
6. No students were permitted to proctor or grade the contest.
7. The instructions relating to the opening of the "Complimentary Solutions Envelope" and/or Solution Packets were followed.
8. After the contest, the answer forms were kept secure and no changes were made in the answers.
9. No parent assisted in the administration of the contest.
10. I have followed all the rules as stated in this Teachers' Manual.

Signature \_\_\_\_\_ E-mail (please print) \_\_\_\_\_

Day & Date Test was given \_\_\_\_\_

Name of School \_\_\_\_\_

City \_\_\_\_\_ State/Province \_\_\_\_\_

Telephone ( \_\_\_\_\_ ) \_\_\_\_\_ School CEEB Number \_\_\_\_\_

PLEASE INDICATE THE EXACT NUMBER OF AMC 10-A/AMC 12-A ANSWER FORMS RETURNED FOR GRADING. \_\_\_\_\_

EXCEPTIONS

# Service Questionnaire

This years questions have to do with the math curriculum at your school. Answer to the best of your ability, but if you don't know give us a "best estimate".

1. What is the average number of students in your school's math classes?
 

Ⓐ 1-10	Ⓐ 11-20	Ⓐ 21-25	Ⓐ 26-30	Ⓐ 31-35	Ⓐ 36+
--------	---------	---------	---------	---------	-------
2. What grade levels are covered in your school? Check all that apply.
 

Ⓐ pre-k	Ⓐ K-4	Ⓐ 5-6	Ⓐ 7-8	Ⓐ 9-10	Ⓐ 11-13
---------	-------	-------	-------	--------	---------
3. How many students attend your school?
 

Ⓐ 1-99	Ⓐ 100-399	Ⓐ 400-799	Ⓐ 800-1,199	Ⓐ 1,200-1,999	Ⓐ 2,000+
--------	-----------	-----------	-------------	---------------	----------
4. How many "High School" programs (schools) are there in your school district?
 

Ⓐ 1	Ⓐ 2-5	Ⓐ 6-10	Ⓐ 11-15	Ⓐ 16-19	Ⓐ 20+
-----	-------	--------	---------	---------	-------
5. Who makes the decision in your school on what math curriculum to follow? Check all that apply.
 

Ⓐ Individual Teacher makes their choice	Ⓐ School District Math Supervisor
Ⓐ School Committee of teachers who teach the classes	Ⓐ State Mandated Curricula/Guidelines
Ⓐ School District Committee--teachers from the various schools	Ⓐ Other _____
6. Which of the following curricula are used at your school? Check only the item most used.
 

Ⓐ <b>University of Chicago School Mathematics Project (UCSMP)</b>
Ⓐ <b>Contemporary Mathematics in Context (CORE-Plus)</b>
Ⓐ <b>Interactive Mathematics Program (IMP)</b>
Ⓐ <b>Saxon Math</b> - Secondary, for grades 9-12
Ⓐ <b>Mathematics: Modeling Our World (ARISE)</b>
Ⓐ Other _____
7. Most comprehensive math programs provide more than just the text book. Which of the supplemental materials do you use?
 

Ⓐ Supplemental Worksheets	Ⓐ Pre-designed unit tests
Ⓐ Teacher study guides	Ⓐ On-line help for students
Ⓐ Pre-designed lesson plans	Ⓐ Other _____
8. On the whole, are you happy with the formal materials you are using?
 

Ⓐ Definitely	Ⓐ Unhappy
Ⓐ Somewhat happy	Ⓐ A Disaster
Ⓐ Indifferent	
9. Do you feel the need to supplement the established curricula with additional material?
 

Ⓐ No - No need	Ⓐ Yes - 2-3 times a week
Ⓐ No - do not have time	Ⓐ Yes - daily
Ⓐ No - District/supervisor frowns on deviating from established curricula	
Ⓐ Yes - limited, 1x a week	
10. From where do you draw your supplemental material?
 

Ⓐ Teacher workshops	Ⓐ Other companies formal curricula
Ⓐ Supplemental math problem books	Ⓐ Networking with other teachers
Ⓐ On-line materials	Ⓐ Other _____

# XIII. Additional Forms used - (if the form you need is not here, please see our web site)

## Additional Bundles Form

Please fill in the information below and FAX your order. The administrator or authorized person of the school agrees to pay the American Mathematics Competitions for the following materials:

School Name \_\_\_\_\_ CEEB # \_\_\_\_\_

Address \_\_\_\_\_

City \_\_\_\_\_ State \_\_\_\_\_ Zip \_\_\_\_\_

Teacher placing the order \_\_\_\_\_

**AMC 10** Contest A Bundles of ten ..... # \_\_\_\_\_ @ \$14/bundle = .....\$ \_\_\_\_\_  
 A Solutions Sets of ten (optional) ..... # \_\_\_\_\_ @ \$ 6/set = .....\$ \_\_\_\_\_

**AMC 12** Contest A Bundles of ten ..... # \_\_\_\_\_ @ \$16/bundle = .....\$ \_\_\_\_\_  
 A Solutions Sets of ten (optional) ..... # \_\_\_\_\_ @ \$ 6/set = .....\$ \_\_\_\_\_

To order either the 2007-2008 AMC 10/12 Math Club Package or the 21<sup>st</sup> Cenury CD with pdf's of all contests **2001-2007**, download the Publications Order Form from the AMC website at [www.unl.edu/amc/](http://www.unl.edu/amc/)

Postage/handling Fee (see chart below) .....\$ \_\_\_\_\_

Total..... \$ \_\_\_\_\_

P.O. Number \_\_\_\_\_

VISA/MC# : \_\_\_\_\_ Address: \_\_\_\_\_

Name (Please Print): \_\_\_\_\_

Exp. Date: \_\_\_\_\_

### AMC ORDERING -- TERMS

1. VISA & MasterCard accepted.

FAX 402-472-6087 or 1-800-527-3690

2. Make checks payable to:  
AMERICAN MATHEMATICS COMPETITIONS

3. PAYMENT IN U.S. FUNDS ONLY.

Please Send Your Order To:

4. U.S.A.:	<u>Order TOTAL</u>	<u>Shipping Charge*</u>
	\$10.00 -- \$40.00	\$7.00
	\$40.01 -- \$50.00	\$9.00
	\$50.01 -- \$75.00	\$12.00
	\$75.01 -- UP	\$15.00

American Mathematics Competitions  
 ATTN: AMC 10/12 Additional Bundles  
 P.O. Box 81606  
 Lincoln, NE 68501-1606

5. OUTSIDE U.S.A.: Add additional \$10 to U.S.A. shipping costs.

\*Orders after **February 1<sup>st</sup>** add \$10.00 additional for 1-day Service.

### Proof of Intent to Pay

This document is intended to be used in lieu of pre-payment when calling or faxing in an order. Please indicate if you wish to be billed or will be sending a "check in the mail" (to be received within 2 weeks of order or you will be billed). Mail orders not wishing to be billed should include a check when returning this form. The person who signs this form must be authorized to pay the order that is placed by the teacher.

BILLED

Name of Person Authorized to Pay (please print): \_\_\_\_\_

Signature: \_\_\_\_\_

Title: \_\_\_\_\_

Date: \_\_\_\_\_

# Rescoring Request Form

I would like to have the following student's answer form rescored. I understand that there is a \$5.00 charge for each student answer form rescored

\$ 5.00/each

Student Name \_\_\_\_\_ \$ \_\_\_\_\_  
Contest taken: AMC 10-A AMC 10-B AMC12-A AMC12-B

Student Name \_\_\_\_\_ \$ \_\_\_\_\_  
Contest taken: AMC 10-A AMC 10-B AMC12-A AMC12-B

Grand Total \_\_\_\_\_ \$ \_\_\_\_\_

Teacher's Name \_\_\_\_\_ CEEB # \_\_\_\_\_

School Name \_\_\_\_\_

Address \_\_\_\_\_

City \_\_\_\_\_ State \_\_\_\_\_ Zip \_\_\_\_\_

## Method of Payment:

Check (US funds only) made payable and mailed with this form to the:  
AMERICAN MATHEMATICS COMPETITIONS  
University of Nebraska-Lincoln  
P.O. Box 81606  
Lincoln, NE 68501-1606

Charge to Visa/Mastercard#: \_\_\_\_\_

Name on card (print): \_\_\_\_\_

Signed

Expiration Date: \_\_\_\_\_ Telephone: \_\_\_\_\_

FAX to: 402/472-6087



# 2008 REGISTRATION - B

## AMC 10B & 12B

### FORM B FOR 2/27/08

#### PLEASE PRINT

High School CEEB# (6 digits): \_\_\_\_\_

Contest Manager \_\_\_\_\_

School Name \_\_\_\_\_

School Street Address \_\_\_\_\_

City \_\_\_\_\_ State \_\_\_\_\_ Zip \_\_\_\_\_

( \_\_\_\_\_ )

School Phone # \_\_\_\_\_

#### BILLING ADDRESS (if different from school):

\_\_\_\_\_

City \_\_\_\_\_ State \_\_\_\_\_ Zip \_\_\_\_\_

( \_\_\_\_\_ )

Phone # \_\_\_\_\_

E-mail (for sending results) Please Print Clearly \_\_\_\_\_

(Circle appropriate responses, below)

Type of Group: Public School Private School Home School Other

Grades: PreK K 1 2 3 4 5 6 7 8 9 10 11 12 13

School Size: 0-200 201-400 401-1000 1001+

#### PAYMENT OPTIONS

Do not send payment alone. The Registration Form must be included with your payment option. **Checks sent without appropriate registration information cannot be processed and will be returned to sender.**

Check P.O. # \_\_\_\_\_ Visa Master Card

Terms - Payment in U.S. Funds only.

Make checks payable to: **American Math Competitions**

VISA/MC # \_\_\_\_\_

Exp. Date: \_\_\_\_\_

Name (Please Print) \_\_\_\_\_

(Please give an address for mailing the charge receipt in "Billing Address" above)

All orders **Non-Refundable** once shipped.  
Mail along with your payment or Purchase Order to

**MAA American Mathematics Competitions**  
ATTN: AMC 10/12 Registration  
P.O. Box 81606  
Lincoln, NE 68501-1606

or fax to  
402-472-6087

#### REGISTRATION

##### Step 1 - Pick either U.S. or International Registration

##### US REGISTRATION, Contiguous 48 states -- REQUIRED

- ♦ One fee covers U.S. 10B/12B Registration & shipping (choose 1)

Registration/Expedited Shipping ..... \$ 50.00

- ♦ REQUIRED for Short Shipping window
- ♦ 2-day shipping

OR

Registration/Overnight Shipping ..... \$ 60.00

- ♦ REQUIRED after January 27 (Jan. 28--Feb 20).
- ♦ Overnight shipping

U. S. REGISTRATION SUBTOTAL ..... \$ \_\_\_\_\_

##### OR CANADIAN/INT'L REGISTRATION

Canadian/Int'l Schools, Guam, Alaska, Puerto Rico, Hawaii Registration ..... \$ 50.00 +

+ Shipping (\$63 maximum)

Exam # bundles @ \$7.00/bundle = ..... \$ \_\_\_\_\_ =

CANADIAN/INT'L SUB-TOTAL ..... \$ \_\_\_\_\_ \$ \_\_\_\_\_

##### Step 2 - Indicate your selection of contests (and solutions)

##### CONTEST BUNDLES OF TEN --

##### ♦ AMC 10B -- Ten contests per bundle

English 10B contests=# @ \$14/bundle= ..... \$ \_\_\_\_\_ +

Spanish 10B contests=# @ \$14/bundle= ..... \$ \_\_\_\_\_ +

French 10B contests=# @ \$14/bundle= ..... \$ \_\_\_\_\_ +

(optional)

English 10B Solutions = # @ \$ 6/set= ..... \$ \_\_\_\_\_ +

##### ♦ AMC 12B -- Ten contests per bundle

English 12B contests=# @ \$16/bundle= ..... \$ \_\_\_\_\_ +

Spanish 12B contests=# @ \$16/bundle= ..... \$ \_\_\_\_\_ +

French 12B contests=# @ \$16/bundle= ..... \$ \_\_\_\_\_ +

(optional)

English 12B Solutions = # @ \$ 6/set= ..... \$ \_\_\_\_\_ =

CONTEST BUNDLES SUB-TOTAL ..... \$ \_\_\_\_\_ \$ \_\_\_\_\_

Braille & Large Print Contests are priced individually, and are not available for International Schools:

Braille AMC 10B # @ 1.40/each = ..... \$ \_\_\_\_\_ +

LgPrint AMC 10B # @ 1.40/each = ..... \$ \_\_\_\_\_ +

Braille AMC 12B # @ 1.60/each = ..... \$ \_\_\_\_\_ +

LgPrint AMC 12B # @ 1.60/each = ..... \$ \_\_\_\_\_ +

+ Shipping (sent separately) ..... \$ 7.00 =

BRILLE & LG PRINT SUB-TOTAL, order by Jan 22 \$ \_\_\_\_\_ \$ \_\_\_\_\_

##### Step 3 - Decide if you want to include a Math Club Package

##### AMC 10/12 MATH CLUB PACKAGE:

Study Guide, CD, Web materials, see brochure.

Sent separately; available Fall 2007

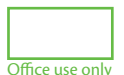
With Shipping for Contiguous US 48 states = ... \$ 25.00 = \$ \_\_\_\_\_

Additional postage required for International/Overseas Addresses, please email AMC office: amcinfo@maa.org

##### Step 4 - Add sub-totals for steps 1,2, and 3

TOTAL ORDER -- ..... \$ \_\_\_\_\_

MUST BE PAID IN US FUNDS





# XIV. Classroom Accessories

## AMC 10 Student Practice Questions

You will find these and additional problems for the AMC 10 and AMC 12 on AMC's web site: <http://www.unl.edu/amc>, available from the current and previous AMC 10/12 Teacher Manuals, (<http://www.unl.edu/amc/e-exams/e6-amc12/archive12.shtml>) or from our Problems page archives (<http://www.unl.edu/amc/a-activities/a7-problems/problem81012archive.shtml>).

The larger of two consecutive odd integers is three times the smaller. What is their sum?

- (A) 4    (B) 8    (C) 12    (D) 16    (E) 20

**2007 AMC 10 A, Problem #4—**

**2007 AMC 12 A, Problem #3—**

**“Set up an equation to represent the relation of the two integers.”**

**Solution**

**Answer (A):** Let the smaller of the integers be  $x$ . Then the larger is  $x + 2$ . So  $x + 2 = 3x$ , from which  $x = 1$ . Thus the two integers are 1 and 3, and their sum is 4.

**Difficulty:** Easy

**NCTM Standard:** Algebra Standard: represent and analyze mathematical situations and structures using algebraic symbols.

**Mathworld.com Classification:** Number Theory > Integers > Odd number

**AMC 10 Student Practice Questions continued**

The 2007 AMC 10 will be scored by awarding 6 points for each correct response, 0 points for each incorrect response, and 1.5 points for each problem left unanswered. After looking over the 25 problems, Sarah has decided to attempt the first 22 and leave only the last 3 unanswered. How many of the first 22 problems must she solve correctly in order to score at least 100 points?

- (A) 13    (B) 14    (C) 15    (D) 16    (E) 17

**2007 AMC 10 B, Problem #6—**

**2007 AMC 12 B, Problem #5—**

**“Sarah must earn at least 95.5 points on the first 22 problems.”**

**Solution**

**Answer (D):** Sarah will receive 4.5 points for the three questions she leaves unanswered, so she must earn at least  $100 - 4.5 = 95.5$  points on the first 22 problems. Because

$$15 < \frac{95.5}{6} < 16,$$

she must solve at least 16 of the first 22 problems correctly. This would give her a score of 100.5.

**Difficulty:** Medium-easy

**NCTM Standard:** Problem Solving Standard: solve problems that arise in mathematics and in other contexts.

**Mathworld.com Classification:** Calculus and Analysis > Inequalities  
Number Theory > Arithmetic > Addition and Subtraction

**AMC 10 Student Practice Questions continued**

The Dunbar family consists of a mother, a father, and some children. The average age of the members of the family is 20, the father is 48 years old, and the average age of the mother and children is 16. How many children are in the family?

- (A) 2    (B) 3    (C) 4    (D) 5    (E) 6

**2007 AMC 10 A, Problem #10—**

**“Set up equations with two variables, and use the conditions above to eliminate one. ”**

**Solution**

**Answer (E):** Let  $N$  represent the number of children in the family and  $T$  represent the sum of the ages of all the family members. The average age of the members of the family is 20, and the average age of the members when the 48-year-old father is not included is 16, so

$$20 = \frac{T}{N+2} \quad \text{and} \quad 16 = \frac{T-48}{N+1}.$$

This implies that

$$20N + 40 = T \quad \text{and} \quad 16N + 16 = T - 48,$$

so

$$20N + 40 = 16N + 64.$$

Hence  $4N = 24$  and  $N = 6$ .

**Difficulty:** Medium-hard

**NCTM Standard:** Data Analysis and Probability Standard: select and use appropriate statistical methods to analyze data.

**Mathworld.com Classification:** Algebra > Algebraic Equations > Linear Equation  
Calculus and Analysis > Special Functions > Means > Arithmetic Mean

## AMC 10 Student Practice Questions continued

How many pairs of positive integers  $(a, b)$  are there such that  $a$  and  $b$  have no common factors greater than 1 and

$$\frac{a}{b} + \frac{14b}{9a}$$

is an integer?

- (A) 4    (B) 6    (C) 9    (D) 12    (E) infinitely many

**2007 AMC 10 B, Problem #25—**

**2007 AMC 12 B, Problem #24—**

**“Play with the restriction:  $\frac{a}{b} + \frac{14b}{9a}$  is an integer.”**

### Solution

**Answer (A):** Let  $u = a/b$ . Then the problem is equivalent to finding all positive rational numbers  $u$  such that  $u + \frac{14}{9u} = k$ , for some integer  $k$ . This equation is equivalent to  $9u^2 - 9uk + 14 = 0$ , whose solutions are

$$u = \frac{9k \pm \sqrt{81k^2 - 504}}{18} = \frac{k}{2} \pm \frac{1}{6}\sqrt{9k^2 - 56}.$$

Hence  $u$  is rational if and only if  $\sqrt{9k^2 - 56}$  is rational, which is true if and only if  $9k^2 - 56$  is a perfect square. Suppose that  $9k^2 - 56 = s^2$  for some positive integer  $s$ . Then  $(3k - s)(3k + s) = 56$ . The only factors of 56 are 1, 2, 4, 7, 8, 14, 28, and 56, so  $(3k - s, 3k + s)$  is one of the ordered pairs (1, 56), (2, 28), (4, 14), or (7, 8). The cases (1, 56) and (7, 8) yield no integer solutions. The cases (2, 28) and (4, 14) yield  $k = 5$  and  $k = 3$ , respectively. If  $k = 5$ , then  $u = 1/3$  or  $u = 14/3$ . If  $k = 3$ , then  $u = 2/3$  or  $u = 7/3$ . Therefore there are four pairs  $(a, b)$  that satisfy the given conditions, namely (1, 3), (2, 3), (7, 3), and (14, 3).

OR

Rewrite the equation  $\frac{a}{b} + \frac{14b}{9a} = k$ , in two different forms. First, multiply both sides by  $b$  and subtract  $a$  to obtain

$$\frac{14b^2}{9a} = bk - a.$$

Because  $a$ ,  $b$ , and  $k$  are integers,  $14b^2$  must be a multiple of  $a$ , and because  $a$  and  $b$  have no common factors greater than 1, it follows that 14 is divisible by  $a$ . Next, multiply both sides of the original equation by  $9a$  and subtract  $14b$  to obtain

$$\frac{9a^2}{b} = 9ak - 14b.$$

This shows that  $9a^2$  is a multiple of  $b$ , so 9 must be divisible by  $b$ . Thus if  $(a, b)$  is a solution, then  $b = 1, 3, \text{ or } 9$ , and  $a = 1, 2, 7, \text{ or } 14$ . This gives a total of twelve possible solutions  $(a, b)$ , each of which can be checked quickly. The only such pairs for which

$$\frac{a}{b} + \frac{14b}{9a}$$

is an integer are when  $(a, b)$  is (1, 3), (2, 3), (7, 3), or (14, 3).

**Difficulty:** Hard

**NCTM Standard:** Algebra for Grades 9-12: Analyze change in various contexts .

**Mathworld.com Classification:** Number Theory > Prime Numbers > Prime Factorization > Factor

## AMC 12 Student Practice Questions

A sphere is inscribed in a cube that has a surface area of 24 square meters. A second cube is then inscribed within the sphere. What is the surface area in square meters of the inner cube?

- (A) 3    (B) 6    (C) 8    (D) 9    (E) 12

**2007 AMC 10 A, Problem #21—**

**2007 AMC 12 A, Problem #11—**

**“The sphere inscribed within the cube has diameter 2 meters, which is also the length of the diagonal of the cube inscribed in the sphere.”**

### Solution

**Answer (C):** Since the surface area of the original cube is 24 square meters, each face of the cube has a surface area of  $24/6 = 4$  square meters, and the side length of this cube is 2 meters. The sphere inscribed within the cube has diameter 2 meters, which is also the length of the diagonal of the cube inscribed in the sphere. Let  $l$  represent the side length of the inscribed cube. Applying the Pythagorean Theorem twice gives

$$l^2 + l^2 + l^2 = 2^2 = 4.$$

Hence each face has surface area

$$l^2 = \frac{4}{3} \text{ square meters.}$$

So the surface area of the inscribed cube is  $6 \cdot (4/3) = 8$  square meters.

**Difficulty:** Hard

**NCTM Standard:** Geometry Standard: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

**Mathworld.com Classification:** Geometry > Plane Geometry > Squares



**AMC 12 Student Practice Questions continued**

Let  $a, b, c, d,$  and  $e$  be distinct integers such that

$$(6 - a)(6 - b)(6 - c)(6 - d)(6 - e) = 45.$$

What is  $a + b + c + d + e$ ?

- (A) 5    (B) 17    (C) 25    (D) 27    (E) 30

**2007 AMC 12 A, Problem #14—**

**“Consider  $(6 - a), (6 - b), (6 - c), (6 - d), (6 - e)$  as five distinct integer factors of 45.”**

**Solution**

**Answer (C):** If 45 is expressed as a product of five distinct integer factors, the absolute value of the product of any four is at least  $|(-3)(-1)(1)(3)| = 9$ , so no factor can have an absolute value greater than 5. Thus the factors of the given expression are five of the integers  $\pm 1, \pm 3,$  and  $\pm 5$ . The product of all six of these is  $-225 = (-5)(45)$ , so the factors are  $-3, -1, 1, 3,$  and  $5$ . The corresponding values of  $a, b, c, d,$  and  $e$  are  $9, 7, 5, 3,$  and  $1$ , and their sum is 25.

**Difficulty:** Medium-hard

**NCTM Standard:** Algebra Standard: use mathematical models to represent and understand quantitative relationships.

**Mathworld.com Classification:** Number Theory > Prime Numbers > Prime Factorization > Factors

**AMC 12 Student Practice Questions continued**

Point  $P$  is inside equilateral  $\triangle ABC$ . Points  $Q$ ,  $R$ , and  $S$  are the feet of the perpendiculars from  $P$  to  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{CA}$ , respectively. Given that  $PQ = 1$ ,  $PR = 2$ , and  $PS = 3$ , what is  $AB$ ?

- (A) 4    (B)  $3\sqrt{3}$     (C) 6    (D)  $4\sqrt{3}$     (E) 9

**2007 AMC 10 B, Problem #17—**

**2007 AMC 12 B, Problem #14—**

**“Area of  $\triangle ABC$  equals the sum of areas of  $\triangle APB$ ,  $\triangle BPC$ , and  $\triangle CPA$ .”**

**Solution**

**Answer (D):** Let the side length of  $\triangle ABC$  be  $s$ . Then the areas of  $\triangle APB$ ,  $\triangle BPC$ , and  $\triangle CPA$  are, respectively,  $s/2$ ,  $s$ , and  $3s/2$ . The area of  $\triangle ABC$  is the sum of these, which is  $3s$ . The area of  $\triangle ABC$  may also be expressed as  $(\sqrt{3}/4)s^2$ , so  $3s = (\sqrt{3}/4)s^2$ . The unique positive solution for  $s$  is  $4\sqrt{3}$ .

**Difficulty:** Hard

**NCTM Standard:** Geometry Standard for Grades 9-12: explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.

**Mathworld.com Classification:** Geometry > Plane Geometry > Triangles > Special Triangles > Equilateral Triangles

**AMC 12 Student Practice Questions continued**

The first 2007 positive integers are each written in base 3. How many of these base-3 representations are palindromes? (A palindrome is a number that reads the same forward and backward.)

- (A) 100    (B) 101    (C) 102    (D) 103    (E) 104

**2007 AMC 12 B, Problem #21—**

**“ $3^6 = 729 < 2007 < 2187 = 3^7$ .”**

**Solution**

**Answer (A):** Because  $3^6 = 729 < 2007 < 2187 = 3^7$ , it is convenient to begin by counting the number of base-3 palindromes with at most 7 digits. There are two palindromes of length 1, namely 1 and 2. There are also two palindromes of length 2, namely 11 and 22. For  $n \geq 1$ , each palindrome of length  $2n+1$  is obtained by inserting one of the digits 0, 1, or 2 immediately after the  $n$ th digit in a palindrome of length  $2n$ . Each palindrome of length  $2n + 2$  is obtained by similarly inserting one of the strings 00, 11, or 22. Therefore there are 6 palindromes of each of the lengths 3 and 4, 18 of each of the lengths 5 and 6, and 54 of length 7. Because the base-3 representation of 2007 is 2202100, that integer is less than each of the palindromes 2210122, 2211122, 2212122, 2220222, 2221222, and 2222222. Thus the required total is  $2 + 2 + 6 + 6 + 18 + 18 + 54 - 6 = 100$ .

**Difficulty:** Hard

**NCTM Standard:** Algebra Standard for Grade 9-12: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

**Mathworld.com Classification:** Number Theory > Special Numbers > Palindromic Numbers > Palindrome

Number Theory > Arithmetic > Number Bases > Base



# The MATHEMATICAL ASSOCIATION OF AMERICA

## American Mathematics Competitions

Steven Dunbar  
Director

February, 2008

Dear Parent or Guardian:

On February 12, 2008 and/or February 27, 2008 your son or daughter will participate in the 59th annual American Mathematics Competitions contest. This contest has grown from a single city-wide competition in New York City in 1950, organized by the local section of The Mathematical Association of America to a sequence of contests. In 2007, over 250,000 students from over 4,200 US and international schools competed for school, regional, and national awards in this contest and found it fun and rewarding. Top 10, well-known U.S. universities and colleges, including internationally recognized U.S. technical institutions, ask for AMC scores on their application forms. Your students deserve the chance to list these scores on their applications!



Each year the AMC 10 and AMC 12 are on the **National Association of Secondary School Principals Advisory List of Contests and Activities**. The AMC Contests are sponsored by the Mathematical Association of America, and are considered to be such a valuable stimulus to student interest in mathematics that 23 professional societies and organizations, including the National Council of Teachers of Mathematics and those represented below, support the contests with financial contributions.



With these contests, there are awards in each school for the student with the highest score, certificates for high-scoring students in each school, state-wide awards, regional awards, and even national awards. These contests lead to other more selective math contests, even all the way to the USA team sent to the International Mathematical Olympiad, the premier international high school level problem solving contest.

But the real rewards come from challenging each student with mathematics that is new, different, and “outside of the box.” The problems on the contest are hard, but designed to be within reach. Just by participating in the contest your student should still feel accomplishment, because these problems are meant to be more challenging than routinely encountered in mathematics courses.

Mathematics is increasingly important in our technological and scientific age. Taking enough mathematics in high school is the gateway to jobs and careers of all kinds, even those that are not explicitly mathematical, scientific, or technological. We hope that by offering these contests, we can challenge and inspire students to learn more mathematics. We hope that your son or daughter enjoyed the contests, and will continue to take mathematics courses and competitions in high school and beyond.

Sincerely,

Dr. Steven R. Dunbar  
AMC Director

## Publicity

The sample Contest Announcement news release below, should be prepared and distributed to the newspapers, radio and television stations in your region. To make preparation of the news release easier, visit our web site, and download the text from the on-line copy of the AMC 10/12 Teachers' Manual. See the AMC website, or the 2007 Summary of Results for statistics and figures from the 2007 contest. Statistics and figures for the 2008 Contest will be available on our website in March 2008.

(School or School District)  
FOR IMMEDIATE RELEASE

### (School) STUDENTS PARTICIPATE IN NATIONAL MATHEMATICS COMPETITION

(#) students at (School) participated in the 59th annual American Mathematics Contest 12, and/or the 9th annual American Mathematics Contest 10. The contests were held on Tuesday, February 12, 2008, (or Wednesday, February 27, 2008). The students competed for local, regional and national student and school awards. The contest, which covers high school mathematics, is given in participating schools. Its purpose is to spur interest in mathematics and develop talent through the excitement of friendly competition at problem solving in a timed format. In 2007 over 250,000 students from 4200 schools participated in the AMC 10 & AMC 12 contest including (#) students from (#) schools in (State). Top scorers at (school) were (\_\_\_\_\_, \_\_\_\_\_, etc.).

According to Prof. Steven Dunbar, who serves as Director of the American Mathematics Competitions, the AMC 12 (first offered in 1950), and the AMC 10 (first offered in 2000), are part of a series of contests sponsored each year by The Mathematical Association of America, through their program, the American Mathematics Competitions. The AMC offers the only math competition series in the country leading to the United States of America Mathematical Olympiad (USAMO) and the Mathematical Olympiad Summer Program (MOSP). From this group of students, the AMC sends the highly competitive USA Team to the prestigious annual International Mathematical Olympiad. The AMC program includes :

American Mathematics Contest 8 (AMC8)	Grades 6- 8	November
American Mathematics Contest 10 (AMC 10)	Grades 10 & below	2 dates in February
American Mathematics Contest 12 (AMC 12)	Grades 12 & below	2 dates in February
American Invitational Mathematics Examination (AIME)	All who qualify	2 dates in March
USA Mathematical Olympiad (USAMO)	All who qualify	mid- to late-April
Mathematical Olympiad Summer Program (MOSP)	Qualify thru USAMO	June
International Mathematical Olympiad (IMO)	Top six from USAMO, MOSP	July

The AMC is located at the University of Nebraska - Lincoln. and receives support from the Akamai Foundation, American Mathematical Association of Two Year Colleges, American Mathematical Society, American Society of Pension Actuaries, American Statistical Association, Art of Problem Solving, Awesome Math, Canada/USA Mathpath, Canada/USA Mathcamp, Casualty Actuarial Society, Clay Mathematics Institute, Institute for Operations Research and the Management Sciences, Mu Alpha Theta, National Association of Mathematicians, National Council of Teachers of Mathematics, Pedagoguery, Inc., Pi Mu Epsilon, and the Society of Actuaries. The Contests are given across the U.S.A, Canada, and in many schools abroad.

Details concerning the 2008 AMC 10/12 contests for High School, as well as the rest of AMC's programs are available on the AMC web site: [www.unl.edu/amc/](http://www.unl.edu/amc/).

For further information contact the AMC -- telephone: 800/527-3690, email: [amcinfo@maa.org](mailto:amcinfo@maa.org).





The Mathematical Association of America  
**American Mathematics Competitions**

# CERTIFICATE

*Awarded to*

*for participating in the*

American Mathematics Contest 10  
(AMC 10)

*Steven R. Dunbar*

**2008**

*Tebay Wenstrom*

*Director*

*American Mathematics Competitions*

*Chair*

*AMC 10 Subcommittee*



The Mathematical Association of America  
**American Mathematics Competitions**

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*Awarded to*

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*Steven R. Dunbar*

**2008**

*Tebay Wenstrom*

*Director*

*American Mathematics Competitions*

*Chair*

*AMC 10 Subcommittee*



The Mathematical Association of America  
**American Mathematics Competitions**

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*for participating in the*

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(AMC 12)

*Steven R. Dunbar*

**2008**

*David M. Wells*

*Director*

*American Mathematics Competitions*

*Chair*

*AMC 12 Subcommittee*



The Mathematical Association of America  
**American Mathematics Competitions**

# CERTIFICATE

*Awarded to*

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(AMC 12)

*Steven R. Dunbar*

**2008**

*David M. Wells*

*Director*

*American Mathematics Competitions*

*Chair*

*AMC 12 Subcommittee*

# Facsimile of AMC 10 Front Cover

**Tuesday, FEBRUARY 12, 2008**

9<sup>th</sup> Annual American Mathematics Contest 10

## AMC 10 CONTEST A



THE MATHEMATICAL ASSOCIATION OF AMERICA  
American Mathematics Competitions

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR GIVES THE SIGNAL TO BEGIN.
2. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have 75 MINUTES to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

*Students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 26<sup>th</sup> annual American Invitational Mathematics Examination (AIME) on Tuesday, March 18, 2008 or Wednesday, April 2, 2008. More details about the AIME and other information are on the back page of this test booklet.*

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of the AMC 10 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules. After the contest period, permission to make copies of problems in paper or electronic form including posting on web-pages for educational use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear the copyright notice.

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# Facsimile of AMC 12 Front Cover

**Tuesday, FEBRUARY 12, 2008**

59<sup>th</sup> Annual American Mathematics Contest 12

## AMC 12 CONTEST A



THE MATHEMATICAL ASSOCIATION OF AMERICA  
American Mathematics Competitions

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR GIVES THE SIGNAL TO BEGIN.
2. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have 75 MINUTES to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

*Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 26<sup>th</sup> annual American Invitational Mathematics Examination (AIME) on Tuesday, March 18, 2008 or Wednesday, April 2, 2008. More details about the AIME and other information are on the back page of this test booklet.*

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**AMERICAN MATHEMATICS COMPETITIONS**