



The MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions

26th Annual

AIME
American Invitational
Mathematics Examination
TEACHERS' MANUAL

Instructions and Reporting Forms for
School Contest Managers

Please read this booklet thoroughly

DO NOT OPEN the sealed package of exams until the date of the exam.
Your Principal or Vice Principal must be present at that time.

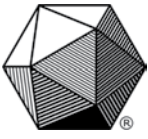
DATES TO REMEMBER

AIME - Tuesday, March 18, 2008

Alternate AIME -Wednesday, April 2, 2008

PLEASE NOTE:

USAMO TEACHERS MANUAL ON FLIP SIDE



The MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

Prof. Steven R. Dunbar
Director

To All AIME Managers:

Please read this manual regarding the AIME rules and regulations thoroughly. Note our policy for the official administration of the AIME. The AIME I can only be given officially on TUESDAY, March 18th, and the AIME II can only be given officially on WEDNESDAY, April 2, 2008. The contest should be scheduled from 9AM to 12 noon if possible.

All students who are in the top 5% of all American Mathematics Contest 12 (AMC 12) participants or score at least 100 points on the AMC 12 and those with a score in the top 1% of the American Mathematics Contest 10 (AMC 10) participants or score at least 120 points, are invited to take the AIME. The AIME is the second in the sequence of mathematics contests which leads to participation in the USA Mathematical Olympiad and the designation of Winners of the USAMO. The AIME is a 15-question, 3-hour examination. All answers are integers ranging from 000 to 999, inclusive. The score is the number of correct answers. There is neither partial credit nor a penalty for wrong answers.

I call your attention to several items found in this manual which answer inquiries I frequently receive from AIME Managers:

1. All administrations of the AIME must take place on the scheduled dates.

There will be a expedited handling fee for the second AIME as follows: 1-10 students = \$25, 11+ students = \$50. We will need your payment before the answer forms can be graded. A special envelope and payment form will be included with your AIME material, if you have AIME qualifiers. All AIME answer forms must be returned by "express mail" so that they arrive in the AMC office by April 4th, 2008.

2. Email requests for the second AIME may be sent to: AIMEQUAL@AMC.UNL.EDU

Or, you can call the AMC office at 1-800/527-3690. Please have your school identification number (CEEB) and charge card information available before calling. E-Mail requests should include the school's CEEB number, and complete mailing address.

3. Under no circumstances can a student take both AIMEs.

4. In section V you will find a discussion of how and when you will receive results.

5. On the reverse side of this Manual you will find a section relating to the USAMO. In this section you will find all the information and forms relevant to the taking of the USAMO. If you have had students qualify for the USAMO in the past, or you are anticipating this to be so this year, please look over this new material. REGARDLESS, do not throw this Manual away until you have received the results of this year's AIME. You may need the USAMO forms.

6. The second AIME on Wednesday, April 2, 2008 is a new set of 15 questions, completely different from, but comparable in difficulty to the AIME on March 18, 2008.

7. Selection of USAMO participants will depend on the AMC 10/12 score, AIME score, and the grade level of the student.

We feel that the 2008 AIME will be interesting and challenging to your student(s). On behalf of the Committee on the American Mathematics Competitions, I send my appreciation to you for assisting with the implementation of this examination.

Very sincerely,

Steven Dunbar, Director

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I. List of the Sponsors of the American Mathematics Competitions

The American Mathematics Competitions are sponsored by

The Mathematical Association of America — MAA	www.maa.org/
The Akamai Foundation	www.akamai.com/
<i>Contributors</i>	
American Mathematical Association of Two-Year Colleges – AMATYC	www.amatyc.org/
American Mathematical Society — AMS	www.ams.org/
American Society of Pension Actuaries — ASPA	www.aspa.org/
American Statistical Association — ASA	www.amstat.org/
Art of Problem Solving — AoPS	www.artofproblemsolving.com/
Awesome Math	www.awesomemath.org/
Canada/USA Mathcamp — C/USA MC	www.mathcamp.org/
Canada/USA Mathpath — C/USA MP	www.mathpath.org/
Casualty Actuarial Society — CAS	www.casact.org/
Clay Mathematics Institute — CMI	www.claymath.org/
IDEA Math	www.ideamath.org/
Institute for Operations Research and the Management Sciences — INFORMS	www.informs.org/
L. G. Balfour Company	www.balfour.com/
Mu Alpha Theta — MAT	www.mualphatheta.org/
National Assessment & Testing	www.natassessment.com/
National Council of Teachers of Mathematics — NCTM	www.nctm.org/
Pedagogy Software Inc.	www.peda.com/
Pi Mu Epsilon — PME	www.pme-math.org/
Society of Actuaries — SOA	www.soa.org/
U. S. A. Math Talent Search — USAMTS	www.usamts.org/
W. H. Freeman and Company	www.whfreeman.com/
Wolfram Research Inc.	www.wolfram.com/

II. Contents of Package

You will find enclosed, in addition to this Teachers' Manual, the following material:

- A list of your qualified AIME students.
- A sealed envelope containing one AIME for each of your qualified students and an extra copy for you. The envelope must NOT be opened until just before the examination is given on the authorized date.
- AIME Answer Forms. One for each participating student.
- One AIME School Identification Form.
- One AIME Report Envelope.
- One Alternate AIME Payment Form (page 10 in this Manual) and Return Envelope

We will mail the Solutions for the AIME with your AIME school report.

III. Preparation Instructions

1. Please study the contents of this Teachers' Manual.
2. Reserve a room for 3 1/2 hours from 8:45 AM – 12:15 PM to administer the AIME on Tuesday, March 18, 2008 from 9AM – 12 noon. This allows 15 minutes for pre- and post-examination activities and 3 hours for the examination itself. In extreme cases where you cannot administer the AIME between 7:00 am and 3:00 pm, please complete the academic integrity pledge form on page 11.
3. Inform the students well in advance of the time and place. Urge them to prepare for the AIME by studying past AIME exams and solutions and other challenging problems. Refer to Order Form C-AIME ORDER FORM FOR PRIOR YEAR EXAMS on Page 9 in this manual for ordering information relating to prior year AIME exams.
4. Review the Examination Rules printed on the sealed envelope which contains the examination.
5. Obtain a supply of:
 - a. Number 2 lead pencils with good erasers
 - b. Unmarked scratch paper
 - c. Rulers, compasses, protractors, and graph paper.
6. Calculators and Computers are NOT permitted.
7. Complete the AIME School Identification Form using the instructions found on the Form.
8. Please study the AIME STUDENT ANSWER FORM. Have each student complete Side 1 of the Form using the instructions found on the Form. Please carefully follow the marking instructions printed on both the identification and student answer forms, and double check to see that all the items are filled out correctly. Students may not complete the personal information after the time allowed for the test. Only the information which has been properly blackened will be read by the OpScan reader. Be sure to mark the AMC 10 or AMC 12 score in the appropriate area, including decimals if needed.
9. The AIME Answer Form is now ready for distribution to your qualified students on Tuesday, March 18th.

10. Make arrangements to mail your AIME Report Envelope by First Class Mail (or Express Mail if taking the alternate date). Please note that this is an oversized envelope and thus requires special postage even if it weighs one ounce or less.

IV. Day of the AIME Instructions

1. Arrange for the Principal or Vice Principal to be present when you open the sealed envelope which contains the AIME(s).
2. Seat participants in alternate seats.
3. Ensure that each participant has the supplies listed on Item 5 under Section III.
4. Inform the students that they are not to open the exam booklet until instructed to do so.
5. Pass out the examination and have the students read the instructions on the front cover.
6. Next, distribute the AIME Answer Forms (which you or your students previously marked) to the students. Inform them that their answers should be marked with a pencil on SIDE 2 of the Form and that each problem has only one correct answer, an integer between 000 and 999 inclusive.
7. Read aloud the following instructions for recording answers to the problems:
 - a. For each question you answer, USING A #2 PENCIL, for your reference write your answer at the top of the problem answer columns using 3 digits, then in each of these columns blacken the one circle which is labeled with the digit you have written at the top. PLEASE NOTE: A single digit answer, such as 7, should be written and blackened as 007 and a two digit answer, such as 43, should be written and blackened as 043.
 - b. DO NOT BLACKEN ANSWERS UNTIL YOU ARE CERTAIN OF YOUR FINAL CHOICE. AVOID, IF POSSIBLE, ERASURES IN THE CIRCLES.
 - c. The results will be graded by computer. Only the blackened circles will be graded.
8. Ask the students if they have any questions about using the AIME Answer Form or about the instructions for the examination.
9. Start the Examination.
10. Check to see if each student is marking his/her Answer Form properly.
11. Allow exactly 3 hours working time.
12. If there is an urgent need to use a bathroom, inform the students that only one student may leave at one time and the AIME answer form and exam should be retained by you during the departure period. If there is a health or medical emergency a backup exam manager should be available for such a contingency.
13. At the end of 3 hours, instruct the students to:
 - a. STOP.
 - b. Put aside the booklet and scratch paper and look

again to be sure the AIME Answer Form has been filled out correctly.

- c. Be sure the written answers on the top of the AIME Answer Form are in agreement with the blackened circles' coded answers.
 - d. Sign their name on Side 2 of the answer form.
 - e. Hand in the AIME Answer Form.
 - f. Keep the examination booklet for their reference in conjunction with your review of the Solutions. Inform the students that they may now leave the examination room.
14. Complete the Certification Form A and USAMO Information Form B found on pages 7 and 8.
 15. The following material should be sealed in the AIME Report Envelope, postmarked NO LATER THAN 24 hours after the examination is given, and sent to the AMC Office:
 - a. Signed AIME Certification Form A and USAMO Information Form B.
 - b. AIME School Identification Form.
 - c. One AIME Answer Form for each participating student. Please do not paper clip or staple the forms together. Do not fold the forms.

APO/FPO AND USA EMBASSY SCHOOLS must return materials by AIR EXPRESS MAIL (DHL, Federal Express, etc.).
 16. Please note that if you have requested the "Alternate" AIME you will need to send your payment separately and in advance of the Wednesday, April 2nd test date.

V. Report of the AIME Results

The AIME score of your student(s), Certificates of Participation, an AIME Solutions Pamphlet and the list of USAMO selectees will be sent to you. No AIME results will be given out by telephone until after April 13th.

Please retain this report for future reference, either in your files or with the student counselor's office. Many students cite these scores on college applications.

VI. Eligibility

Any student who is officially enrolled in high school (or below) and is taking at least one course at the school, and has not graduated, is eligible to take the AIME (with qualifying score). US and Canadian Citizens and students residing in the United States (with qualifying scores) are eligible to take the USAMO.

Home School Students age 19.5 and under are eligible for the AIME (with qualifying score)

Students learning "English as a Second Language" (ESL) may use a book or electronic dual-language non-technical dictionary between their native language and English. A student may use the dictionary only the first time that he/she takes the AIME. The dictionary must be given to the school contest manager to examine and retain for the 24-hour period preceding the contest. The proctor must announce to other students that the

student(s) has/have been given special permission to use the dictionary during the contest.

VII. Policy on AIME Administration

You must give the AIME on the official date (March 18th) to all students in the same place and at the same time; invited students who cannot be there miss this opportunity. Be advised that if students miss the exam, Tuesday, March 18, 2008, they may take the alternate sitting of the exam, Wednesday, April 2, 2008. The second AIME on Wednesday, April 2, 2008 is a new set of 15 questions completely different from, but comparable to the AIME on March 18, 2008.

If you need to have a student or students sit for the second exam on April 2nd, read below for the applicable details and then contact the AMC office if you have questions. There is a minimal charge for the second exam due to compressed shipping considerations. These two dates will be the only dates acceptable for qualifying for the USAMO.

There is a processing fee for the second AIME as follows: 1-10 students = \$25, 11+ students = \$50. We need your payment before we grade the answer forms. A special envelope and payment form has been included with your AIME material. All Alternate AIME (AIME II) answer forms must be returned by "express mail" so that they arrive in the AMC office by April 4, 2008.

Email requests for the second AIME may be sent to:

AIMEQUAL@AMC.UNL.EDU

Or, you can call the AMC office at 800/527-3690. Please have your school identification number (CEEB) available before calling. E-mail requests should include the school's CEEB number and complete mailing address, teacher name, number of students taking the AIME II, and method of payment. The AMC office will send a confirmation of registration for the AIME II. Please note that you will be billed for all AIME II shipped, unless you cancel your order before March 18, 2008.

Under no circumstances can a student take both AIME's.

VIII. Visually Impaired and Learning Disabled Students

If one of your AIME qualified students is visually impaired and/or learning disabled please call the AMC office and we will discuss the options available to you. We do not have copies available in braille. The time allowance for students with learning disabilities is 4.5 hours.

IX. Request for Student Names

The following statement appears on the student answer forms for the AMC 10 and AMC 12:

1. The American Mathematics Competitions (AMC) receives requests from educational institutions and organizations for the names, addresses and grade levels of high scoring students. This information is used for recruiting and academic purposes.

2. Blacken this circle if you give the AMC permission to release this information to these organizations. (Your score will not be affected if you do not blacken the circle.)

Receiving information is an “opt-in” decision for each individual student.

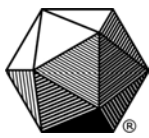
The AMC handles requests from institutions and organizations on a case-by-case basis and evaluates each individually for appropriateness. We provide legitimate educational institutions of all levels, both

secondary and collegiate/university level, with one-time use of selected names and addresses for postal mailings. We also provide professional and scholarly organizations such as those listed as contributors to the AMC with one-time use of names and addresses for postal mailings, generally for professional or career information.

The only information we provide is the name, address, city, state, and zip code necessary for a postal mailing. We do not list individual scores or awards.

X. FASCIMILE OF THE MARCH 18, 2008 AIME FRONT COVER

THE MATHEMATICAL ASSOCIATION OF AMERICA
AMERICAN MATHEMATICS COMPETITIONS



26th Annual

**AMERICAN INVITATIONAL
MATHEMATICS EXAMINATION**

(AIME I)

Tuesday, March 18, 2008

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR GIVES THE SIGNAL TO BEGIN.
2. This is a 15-question, 3-hour examination. All answers are integers ranging from 000 to 999, inclusive. Your score will be the number of correct answers; i.e., there is neither partial credit nor a penalty for wrong answers.
3. No aids other than scratch paper, graph paper, ruler, compass, and protractor are permitted. In particular, calculators and computers are not permitted.
4. A combination of the AIME and the American Mathematics Contest 10 or the American Mathematics Contest 12 scores are used to determine eligibility for participation in the U.S.A. Mathematical Olympiad (USAMO). The USAMO will be given in your school on TUESDAY and WEDNESDAY, April 29-30, 2008.
5. Record all of your answers, and certain other information, on the AIME answer form. Only the answer form will be collected from you.

After the contest period, permission to make copies of individual problems in paper or electronic form including posting on web-pages for educational use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear the copyright notice.

Copyright © 2008, The Mathematical Association of America

AIME 2008 Teacher Manual
FORM A – Examination Certification 2008 AIME

The Examination Manager and the Principal, Vice Principal, or Headmaster must sign this form. Return it with your student Answer Forms.

Certification by the Principal, Official or person with comparable title:

- a) I certify that the exam package(s) were retained in their sealed condition within a half hour of the start of the examination;
- b) I accept for our school the rules and procedures described on this page and pages 4-5, and accept that failure to follow these rules and procedures may result in DISQUALIFICATION from official standing of all scores from our school.

Signature _____ Time _____

Title _____ Date _____

Certification by the Examination Manager:

I certify that the 2008 American Invitational Mathematics Examination was given today, Tuesday, March 18, 2008; that all students who took it did so at the same time and place; that only invited students took it; that the 3-hour time limit was strictly enforced; that the test envelope was sealed until the time of the test; that no student had access to the test questions in advance; that the students were continually supervised by a certified teacher while they were taking the test; that I am returning those materials as required in Section IV; and that all other rules for administering the test were followed.

YES NO If no, describe the exceptions on a separate sheet of paper.

I also certify that I understand and accept the following for our school. The Committee on the American Mathematics Competitions reserves the right to:

1. Disqualify all scores from our school if it is determined that the required security procedures were not followed.
2. Re-examine students if, after an inquiry, there is a reasonable basis to believe that the scores do not validly represent the ability of the students. (The procedures for disqualification, follow-up inquiries and re-examination are similar to those for the AMC 10 and AMC 12, as outlined in the AMC 10/12 Teachers' Manual.)

Signature of AIME School Manager _____

Date _____ School Telephone # _____

Name of School _____

State _____

School Identification Number (CEEB) _____

The AIME was administered on March 18, 2008 YES please circle NO

Starting Time _____ Ending Time _____

Please also complete the OLYMPIAD INFORMATION Form B, found on Page 8.

FORM B – USAMO Information Form

When a student qualifies for the United States of America Mathematical Olympiad (USAMO), both the student and the USAMO School Manager are notified at least one week before the USAMO date. Don't worry about an invitation getting lost in the mail. When a USAMO invitation is extended, both the student and the USAMO School Manager must complete and return a USAMO Response Form. If the Form is not returned, we do a follow-up.

Please provide the information requested below if you agree to administer the USAMO. There is no fee for participating in the USAMO.

AGREEMENT STATEMENT:

In the event that I have a qualifying student(s), I or my designee (indicated below) agree to:

1. Serve as the USAMO School Manager.
2. Administer the USAMO on Tuesday, April 29th , and Wednesday, April 30th, 2008, to the student(s) in my school who qualify and not to any other students.
3. Arrange for the exclusive use of a room for four and a half hours (from 12:30 p.m. to 5:00 p.m., eastern daylight time or your time zone equivalent, see below) on the designated days which will be used to administer the USAMO.
4. Have a proctor present in the room to supervise the student(s) during the entire nine hours duration of the examination.
5. Have the ability to fax the students answer sheets to the AMC fax number immediately, to be graded with the rest of the papers by the AMC.

I understand that results of my participating USAMO students could be voided if the rules and procedures associated with the administration of the USAMO are not followed.

Signature _____

Telephone Number _____ Date _____

Name of USAMO School Manager _____

eMail _____

School _____ CEEB#: _____

City _____ State _____ Zip _____

PLEASE NOTE THIS YEARS 4.5 HOURS PER DAY TIME LENGTH AND 2 DAYS LENGTH FOR THE 2008 USAMO:

TUESDAY, April 29, 2008

12:30–5:00 p.m., Eastern Daylight Time
 11:30–4:00 p.m., Central Daylight Time
 10:30–3:00 p.m., Mountain Daylight Time
 9:30–2:00 p.m., Pacific Daylight Time

&

WEDNESDAY, April 30, 2008

12:30–5:00 p.m., Eastern Daylight Time
 11:30–4:00 p.m., Central Daylight Time
 10:30–3:00 p.m., Mountain Daylight Time
 9:30–2:00 p.m., Pacific Daylight Time

All other time zones, contact the AMC office at amcinfo@maa.org

FORM C – AIME Order Form for Prior Years Exams

If you wish to order any of these publications complete the order form below and mail to:
 American Mathematics Competitions, ATTN: Publications, PO Box 81606, Lincoln, NE 68501-1606.
 To expedite delivery you may fax your order (402-472-6087). VISA & MASTERCARD accepted. (Call 1-800-527-3690).

For each publication below write the quantity of each you wish to order in the blank beside the year or volume

AMERICAN INVITATIONAL MATHEMATICS EXAMINATION

1989_____1990__NA__1991_____1992__NA__1993_____

1994__NA__1995_____1996_____1997_____1998_____1999_____

From 2000 on there were 2 versions of the AIME given:

	2000	2001	2002	2003	2004	2005	2006	2007
AIME 1	_____	_____	__ <u>NA</u> __	_____	_____	_____	__ <u>NA</u> __	_____
AIME 2	_____	_____	__ <u>NA</u> __	_____	_____	_____	_____	_____

Total quantity of AIMEs_____ @ \$ 2.00 = \$ _____

CD - 21st Century Contests \$ 20 each

A CD with all the 21st Century Contests – AMC 8 (1999–2007)
 AMC 10A&B, AMC 12A&B (2000–2007)
 AIME, and USAMO (2001–2007).

All the questions and solutions included are available in PDF form.
 Finished AMC 8/10/12 worksheets are also included.

21st Century Contests CD_____ @ \$ 20.00 = \$ _____

No Shipping

Subtotal \$ _____

CANADIAN & INTERNATIONAL POSTAGE:

Under 2 pounds	\$30.00	
Over 2 pounds	\$50.00	\$ _____

PAYABLE ONLY IN U.S. FUNDS

\$\$\$ GRAND TOTAL \$ _____

ORDER RECORD (Please Print)

CEEB # _____

Name _____

School _____

City _____

State _____

Zip _____

VISA or MASTERCARD Number:

Exp. Date _____

Cardholder Signature _____

Email _____

AIME 2008 Teacher Manual
FORM D – Alternate AIME Payment Form
2008 Alternate AIME
Wednesday, April 2, 2008

The Alternate AIME on Wednesday, April 2, 2008 is a new set of 15 questions, completely different from, but comparable to the AIME on March 18, 2008. Please send payment before April 2, 2008.

PLEASE PRINT

School CEEB # -- -- -- -- -- --

Contest Manager: -----

School Name: -----

School Address: -----

City: ----- State: ----- Zip: -----

School Phone #: -----

E-mail: -----

There will be a processing fee for the second AIME as follows: 1-10 students = \$25, 11+ students = \$50. We will need your payment before the answer forms can be graded. A special envelope and payment form has been included with your AIME material. All AIME answer forms must be returned by "express mail" so that they arrive in the AMC office by April 4, 2008.

Number of AIME qualifiers who will be taking the Alternate AIME on Wednesday, April 2, 2008:

Number of Alternate Exams Requested: # -----

1-10 student qualifiers @ \$25.00 = \$ -----
OR

11+ student qualifiers @ \$50.00 = \$ -----

Method of Payment:

Check enclosed (US funds only) made payable and mailed with this form in the BUSINESS REPLY/postpaid envelope provided:

AMERICAN MATHEMATICS COMPETITIONS
University of Nebraska-Lincoln
PO Box 880658
Lincoln, NE 68588-0658

Charge to Visa/Mastercard #: -----

Name on card (print): -----

Signed: -----

Expiration Date: ----- Telephone: -----

1-800-527-3690

E-mail: aimequal@amc.unl.edu

Fax: 402-472-6087

American Invitational Mathematics Examination (AIME)

On-site Official

Section 1 – Student

This disclaimer statement is to be completed by all students taking the AIME in their school on the scheduled date of the contest, but at a later time from the other students, due to the requirement of attendance at an official academic function, which conflicted with the regularly scheduled time of the administration of the AIME. Reexamination may be requested if, after an inquiry, there is a reasonable basis to believe that a high score is well beyond a student's ability due to extremely lucky guessing, dishonesty or some other circumstance.

I certify that prior to my taking of the 2008 AIME on Tuesday, March 18, 2008, I had absolutely no contact or communication with any student who participated in the contest, nor had I seen any of this year's contest questions.

Signed: _____ Date: _____

Name of School: _____

Section 2 – Contest Administrator

This section is to be completed by the teacher who administered the AIME on the day of the contest (Tuesday, March 18, 2008) but at a time different from the regularly scheduled time.

This student (these students) participated in an officially sanctioned academic function. The student(s) had no contact or communication with any student who participated in the contest, nor had they seen any of this year's AIME. All contest papers were collected after any previous administrations, and students were instructed not to discuss the questions.

I administered the AIME to the student(s) indicated in Section 1 on (DATE): _____

Contest time period: From _____ to _____

Name (please print): _____

Title (please print): _____

Signature: _____

School name and address: _____

Date this form completed: _____

PLEASE INCLUDE THIS FORM WITH THE ANSWER FORMS OF THE STUDENTS.

IX. EXAM AUXILIARY MATERIAL

Practice Worksheets

A 100 foot long moving walkway moves at a constant rate of 6 feet per second. Al steps onto the start of the walkway and stands. Bob steps onto the start of the walkway two seconds later and strolls forward along the walkway at a constant rate of 4 feet per second. Two seconds after that, Cy reaches the start of the walkway and walks briskly forward beside the walkway at a constant rate of 8 feet per second. At a certain time, one of these three persons is exactly halfway between the other two. At that time, find the distance in feet between the start of the walkway and the middle person.

2007 AIME 1, Problem #2—

“List equations with variable t for Al, Bob, or Cy be the middle person, and solve for t .”

Solution(052)

Let t be Al's travel time. Then $t - 2$ is Bob's time, and $t - 4$ is Cy's time, and $t \geq 4$. If Cy is in the middle, then $10(t - 2) - 8(t - 4) = 8(t - 4) - 6t$, which has no solution. If Bob is in the middle, then $10(t - 2) - 8(t - 4) = 6t - 10(t - 2)$, which has solution $t = 4/3$. But $t \geq 4$, so this is impossible. If Al is in the middle, then $6t - 8(t - 4) = 10(t - 2) - 6t$, which has solution $t = 26/3$. In this case, Al is 52 feet from the start and is $44/3$ feet from both Bob and Cy. Thus the required distance is 52.

Difficulty: Easy

NCTM Standard: Algebra Standard: Represent and analyze mathematical situations and structures using algebraic symbols

Mathworld.com Classification: Number Theory > Arithmetic > General Arithmetic

The formula for converting a Fahrenheit temperature F to the corresponding Celsius temperature C is $C = \frac{5}{9}(F - 32)$. An integer Fahrenheit temperature is converted to Celsius and rounded to the nearest integer; the resulting integer Celsius temperature is converted back to Fahrenheit and rounded to the nearest integer. For how many integer Fahrenheit temperatures T with $32 \leq T \leq 1000$ does the original temperature equal the final temperature?

2007 AIME 1, Problem #5—

“First note that a temperature T converts back to T if and only if $T + 9$ converts back to $T + 9$. Explore the behavior of the first nine consecutive numbers, and note that this behavior is going to repeat itself for every nine consecutive number cycle.”

Solution(539)

Note that a temperature T converts back to T if and only if $T + 9$ converts back to $T + 9$. Thus it is only necessary to examine nine consecutive temperatures. It is easy to show that 32 converts back to 32, 33 and 34 both convert back to 34, 35 and 36 both convert back to 36, 37 and 38 both convert back to 37, and 39 and 40 both convert back to 39. Hence out of every nine consecutive temperatures starting with 32, five are converted correctly and four are not. For $32 \leq T < 32 + 9 \cdot 107 = 995$. There are $107 \cdot 5 = 535$ temperatures that are converted correctly. The remaining six temperatures 995, 996, \dots , 1000 behave like 32, 33, \dots , 37, so four of the remaining six temperatures are converted correctly. Thus there is a total of $535 + 4 = 539$ temperatures.

OR

Because one Fahrenheit degree is $5/9$ of a Celsius degree, every integer Celsius temperature is the conversion of either one or two Fahrenheit temperatures (nine Fahrenheit temperatures are being converted to only five Celsius temperatures) and converts back to one of those temperatures. The Fahrenheit temperatures 32 and 1000 convert to 0 and 538, respectively, which convert back to 32 and 1000. Therefore there are 539 Fahrenheit temperatures with the required property, corresponding to the integer Celsius temperatures from 0 to 538.

Difficulty: Medium

NCTM Standard: Number and Operations Standard: Compute fluently and make reasonable estimates.

Mathworld.com Classification: Number Theory > Arithmetic > General Arithmetic

Let a sequence be defined as follows: $a_1 = 3$, $a_2 = 3$, and for $n \geq 2$,
 $a_{n+1}a_{n-1} = a_n^2 + 2007$. Find the largest integer less than or equal
 to $\frac{a_{2007}^2 + a_{2006}^2}{a_{2007}a_{2006}}$.

2007 AIME 1, Problem #14—

“The fact that the equation $a_{n+1}a_{n-1} = a_n^2 + 2007$ holds for $n \geq 2$ implies that $a_n a_{n-2} = a_{n-1}^2 + 2007$ for $n \geq 3$.”

Solution(539)

The fact that the equation $a_{n+1}a_{n-1} = a_n^2 + 2007$ holds for $n \geq 2$ implies that $a_n a_{n-2} = a_{n-1}^2 + 2007$ for $n \geq 3$. Subtracting the second equation from the first one yields $a_{n+1}a_{n-1} - a_n a_{n-2} = a_n^2 - a_{n-1}^2$, or $a_{n+1}a_{n-1} + a_{n-1}^2 = a_n a_{n-2} + a_n^2$. Dividing the last equation by $a_{n-1}a_n$ and simplifying produces $\frac{a_{n+1}+a_{n-1}}{a_n} = \frac{a_n+a_{n-2}}{a_{n-1}}$. This equation shows that $\frac{a_{n+1}+a_{n-1}}{a_n}$ is constant for $n \geq 2$. Because $a_3 a_1 = a_2^2 + 2007$, $a_3 = 2016/3 = 672$. Thus $\frac{a_{n+1}+a_{n-1}}{a_n} = \frac{672+3}{3} = 225$, and $a_{n+1} = 225a_n - a_{n-1}$ for $n \geq 2$. Note that $a_3 = 672 > 3 = a_2$. Furthermore, if $a_n > a_{n-1}$, then $a_{n+1}a_{n-1} = a_n^2 + 2007$ implies that

$$a_{n+1} = \frac{a_n^2}{a_{n-1}} + \frac{2007}{a_{n-1}} = a_n \left(\frac{a_n}{a_{n-1}} \right) + \frac{2007}{a_{n-1}} > a_n + \frac{2007}{a_{n-1}} > a_n.$$

Thus by mathematical induction, $a_n > a_{n-1}$ for all $n \geq 3$. Therefore the recurrence $a_{n+1} = 225a_n - a_{n-1}$ implies that $a_{n+1} > 225a_n - a_n = 224a_n$ and therefore $a_n \geq 2007$ for $n \geq 4$. Finding a_{n+1} from $a_{n+1}a_{n-1} = a_n^2 + 2007$ and substituting into $225 = \frac{a_{n+1}+a_{n-1}}{a_n}$ shows that $\frac{a_n^2+a_{n-1}^2}{a_n a_{n-1}} = 225 - \frac{2007}{a_n a_{n-1}}$. Thus the largest integer less than or equal to the original fraction is 224.

Difficulty: Hard

NCTM Standard: Algebra Standard: Understand patterns, relations, and functions.

Mathworld.com Classification: Number Theory > Sequences

Publicity

The sample Contest Announcement news release below, should be prepared and distributed to the newspapers, radio and television stations in your region. To make preparation of the news release easier, visit our web site, and download the text from the on-line copy of the AIME Teachers' Manual. See the AMC website, or the 2007 Summary of Results for statistics and figures from the 2007 contest. Statistics and figures for the 2008 Contest will be available on our website in late March and early April 2008. Printing this on a sheet of school stationery gives an official look to the page.

(School or School District)
FOR IMMEDIATE RELEASE

(School) STUDENTS PARTICIPATE IN NATIONAL MATHEMATICS COMPETITION

(#) students at (School) participated in the 26th annual American Invitational Mathematics Examination (AIME). (Student names) qualified for the AIME by scoring in the top 1% of the American Mathematics Contest 10 (AMC 10, 10th grade and below). (Student names) qualified for the AIME by scoring 100 or more out of a possible 150 (approximately the top 5%) on the American Mathematics Contest 12 (AMC 12, 12th grade and below).

The AIME was held on Tuesday, March 18, 2008, (or Wednesday, April 2, 2008). This contest was established in 1973 as an intermediate step between the high school (AMC 10 and AMC 12) contests and the U. S. A. Mathematical Olympiad (USAMO). The AIME is a 15 question, 3 hour examination in which each answer is an integer number from 0 to 999, students are very unlikely to obtain the correct answer by guessing. The questions on the AIME cover high school mathematics, and are much more difficult than those on the AMC 10 and AMC 12. All problems on the AIME can be solved by pre-calculus methods. The use of calculators is not allowed. This contest leads to the USAMO, MOSP (Mathematical Olympiad Summer Program) and the selection of the USA team sent to the International Mathematical Olympiad (IMO), the premier international high school level problem solving contest. This year the IMO will be held in Madrid, Spain, July 10-22, 2008.

According to Prof. Steven Dunbar, who serves as Director of the American Mathematics Competitions, the AIME is one of a series of contests sponsored each year by The Mathematical Association of America, through their program, the American Mathematics Competitions (AMC). The AMC offers the only math competition series in the country leading to the USAMO and the Mathematical Olympiad Summer Program (MOSP). From this group of students, the AMC sends the highly competitive USA Team to the prestigious annual International Mathematical Olympiad. The AMC program includes:

American Mathematics Contest 8 (AMC8)	Grades 6- 8	November
American Mathematics Contest 10 (AMC 10)	Grades 10 & below	2 dates in January/February
American Mathematics Contest 12 (AMC 12)	Grades 12 & below	2 dates in January/February
American Invitational Mathematics Examination (AIME)	All who qualify	2 dates in March
USA Mathematical Olympiad (USAMO)	All who qualify	mid- to late-April
Mathematical Olympiad Summer Program (MOSP)	Qualify thru USAMO	June
International Mathematical Olympiad (IMO)	Top six from USAMO, MOSP	July

The AMC is located at the University of Nebraska – Lincoln. and receives direct financial contributions from the Akamai Foundation, American Mathematical Association of Two Year Colleges, American Mathematical Society, American Society of Pension Actuaries, American Statistical Association, Art of Problem Solving, Canada/USA Mathpath, Canada/USA Mathcamp, Casualty Actuarial Society, Clay Mathematics Institute, Idea Math, Institute for Operations Research and the Management Sciences, Mu Alpha Theta, National Association of Mathematicians, National Council of Teachers of Mathematics, Pedagogy, Inc., Pi Mu Epsilon, Society of Actuaries and the USA Math Talent Search. The Contests are given across the U.S.A, Canada, and in many schools abroad.

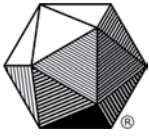
Details concerning the 2008 AIME contests for High School, as well as the rest of AMC's programs are available on the AMC web site: <http://www.unl.edu/amc/>.

For further information contact the AMC --

amcinfo@maa.org

800/527-3690

<http://www.unl.edu/amc/>



The MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

Steven Dunbar
Director

March, 2008

Dear Parent or Guardian:

On March 18, 2008 or April 2, 2008 your son or daughter participated in the 26th annual American Invitational Mathematics Examination (AIME). This contest was established in 1973 as an intermediate step between the high school (AHSME) contest and the U. S. A. Mathematical Olympiad (USAMO). The AIME is taken by students who achieved a score of 100 or more out of a possible 150 (approximately the top 5%) on the AMC 12, and students who achieve a score of 120 or more (approximately the top 1%) on the AMC 10 .

The AIME is a 15 question, 3 hour examination in which each answer is an integer number from 0 to 999. The questions on the AIME are much more difficult than those on the AMC 10 and AMC 12, and students are very unlikely to obtain the correct answer by guessing. All problems on the AIME can be solved by pre-calculus methods. The use of calculators is not allowed.

This contest leads to the USAMO, MOSP (Mathematical Olympiad Summer Program) and the selection of the USA team sent to the International Mathematical Olympiad (IMO), the premier international high school level problem solving contest. This year the IMO will be held in Madrid, Spain, July 10-22, 2008.

The AIME provides the exceptional students who are invited to take it with yet another opportunity to challenge their mathematical abilities. Like all examinations, it is but a means towards furthering mathematical development and interest. The real value of the examination is in the learning that can come from the preparation beforehand and from further thought and discussion of the solutions.

Our organization regularly receives requests from well-known colleges and universities for the names of high scoring students. A few colleges offer scholarships to students in their region with high scores on the contests. But the real rewards come from challenging each student with mathematics that is new, different, and "outside of the box." The problems on the contest are hard, but designed to be within reach. Even so, if your son or daughter managed to solve only one or two problems, one should still feel that they accomplished something, because these problems are meant to be more challenging than they routinely encounter in their mathematics courses.

Mathematics is increasingly important in our technological and scientific age. Taking enough mathematics in high school is the gateway to jobs and careers of all kinds, even those that are not explicitly mathematical, scientific, or technological. We hope that by offering these contests, we can challenge and inspire students to want to learn more mathematics. We hope that your son or daughter enjoyed the contests, and will continue to take mathematics courses and competitions in high school and beyond.

Sincerely,

Dr. Steven R. Dunbar
AMC Director