

Please read this booklet completely upon receipt Exams must be administered over a continuous 75 -minute period to all students at the same time

## 2009 Contest Dates

# The MATHEMATICAL ASSOCIATION OF AMERICA American Mathematics Competitions 

Steven Dunbar AMC Director

To all Contest Managers:

I am very pleased you will be providing the opportunity for your students to participate in the $60^{\text {th }}$ annual American Mathematics Contest 12 (AMC 12) or its sister contest, the $10^{\text {th }}$ annual American Mathematics Contest 10 (AMC 10). I believe that you and your students will find these contests to be both interesting and challenging. Again this year the Committee on the American Mathematics Competitions and panelists under the leadership of Committee Chairs LeRoy Wenstrom for the ADC 10 and Bernardo Abrego for the AMC 12 developed a total of 4 contests (two AMC 10 and two AMC 12). I extend special thanks to both of them for the effort involved in developing these excellent contests.

This year, we continue to include in this manual several handouts, included in Section XIV:

* Worksheets for contest preparation (page 14)
* A handout for parents on why mathematics is important (page 22)
* A sample Press Release (page 23)
* A Certificate of Participation (pages $24 \& 25$ )

You may reproduce these pages for your students.

Very sincerely,


Steven Dunbar

## Director

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# Important Procedures <br> Format 

There are two official dates for the AMC $10 \&$ AMC 12. Give the AMC 10 and AMC 12 at the same time within each participating school on TUESDAY, February 10, 2009 (AMC 10-A \& AMC 12-A), or WEDNESDAY, February 25, 2009 (AMC 10-B \& AMC 12-B) in a convenient 75 -minute interval, preferably in the morning. All four contests consist of 25 questions.
N O T E : Each correct answer scores 6 points, a blank scores 1.5 points and an incorrect answer scores 0 points.
The AMC 10 and AMC 12 have several questions in common. The students in grades 10 and below should choose between the AMC 10 and AMC 12. Students in grades 11 and 12 may only take the AMC 12. All rules and awards apply to both contests for all schools and students. Any student who missed the exam may take it unofficially, and we will be happy to grade it. Students may take the contest booklets home with them the day of the contest.

## Answer Forms

The AMC 10 and AMC 12 each have their own color coded answer form (AMC 10-orange and AMC 12-red). Contest B answer forms have instructions in black ink. Please be careful to match the correct answer form with the appropriate contest when passing out the papers.

## AIME Qualification

Students who score 100 or above or finish in the top $5 \%$ on this AMC 12 or students who score 120 or above or finish in the top $1 \%$ on this AMC 10 will be invited to take the 27th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 17 or Wednesday, April 1.

## Results

We will send your school's results by email (if available) and first class mail as soon as the answer forms are scored. If you have not received your results from our office within 30 days after the contest, please contact us to verify that your answer forms were in fact received. All student Answer Forms are held for 90 days after the exam date, then they are recycled.

## School Flier

We are again providing a flier you can use to promote the contests within your school. It is included on page 17 of this Manual. It has space left for you to add school pertinent information, such as the location of the contests, and who to contact locally for more information. If you need both the A and B versions, or would like a color version, visit our web page to download a pdf version of the flier you need: www.unl.edu/amc/

## I. Preliminary Instructions for

## Administering the AMC 10/AMC 12

1. Inform students far in advance about the date for the AMC 10/AMC 12 and obtain a supply of No. 2 lead pencils.
2. Hand out the student Answer Forms and have the students complete the non-answer sections on the front and back. Have students use their full legal name, no nicknames or abbreviations. Have them pay special attention to marking their name and address accurately. Remind them that student names listed in the National Summary come from this form. The AMC Office will not do any editing of the information.
3. The name of the school, city, state and postal code must be stamped or written on each student answer form. This is very important because there is no other school identification on the answer forms. Please do not use stick-on labels to provide this information.
4. Announce that the students may use scratch paper, graph paper, ruler, compass, and/ or a protractor. CALCULATORS ARE NOT PERMITTED. No problems on the contest will require the use of a calculator.
5. Review past tests and solutions or inform your students how to order copies for themselves. The Publications Order Form can be printed from the AMC Web page at www.unl.edu/
amc, (or call, fax or write to the AMC Office for a copy).
6. Encourage participation by students who have not taken the contest before, especially younger students, but make sure students know what to expect. Let them know about typical scores at your school last year at their grade level. Show students the national statistics in our National Summary of Results and Awards from last year. Tell them to set appropriate goals for themselves.
7. Remind students the day before the contest about the time and place of the AMC 10/AMC 12. Also tell them your plan if the school should suddenly close. All students must take the contest at the same time, either in one group or in separate classrooms and be under the supervision of a proctor.
8. Make sure you have arranged to follow all the rules and procedures in this manual. Early administration of the contests is never permitted, and will lead to disqualification. To assure the validity of the results we report, we take our rules very seriously.

## II. Instructions For The Day of The AMC 10/AMC 12

If today is not yet Tuesday, February 10, 2009 STOP EVERYTHING. Under no circumstances is the contest to be given to anyone before the official day, nor should the contest package(s)
be opened before that date.

1. Sign the Certification Form to certify that the package was opened within an hour before the contest.
2. When the students arrive, seat them so they are separated by an empty space, if possible.
3. Hand out the Answer Forms which have been partially completed by the students (AMC 10-orange/AMC 12red).
4. As you hand out the papers, tell the students not to open the contest booklet. They should then read the entire front cover. Give them 5 minutes to do this.
5. Inform the students to, "Carefully read instructions 3 and 4 on the contest cover." (see pages 26 and 27 of this manual). The AMC 10/AMC 12 has a scoring system which has important consequences for guessing. Unless you are fairly sure of the answer, it is better to leave a question unanswered than to guess. Six points are given for a correct answer, 1.5 points for a blank answer and 0 points for an incorrect answer. If a student can reduce the problem to three possible answers, it is advantageous to guess one of the three possible answers. If a student can only reduce to 4 possible answers by eliminating 1 of the possibilities, then it is not advantageous to guess.
6. Inform the participants they may not talk or ask any questions during the contest, and they must do their own work.
7. Remind students they have 75 minutes, then tell them to BEGIN. (See page 6 for student disabilities policy)
8. Students who finish the contest early may be dismissed provided they will be under the supervision of a teacher during the remainder of the contest period.
9. You (and other teachers, if there are many participants) should proctor continually as you would for any important contest. Students whose eyes wander should be warned; students caught copying answers or collaborating must be disqualified. Try to provide as quiet an environment as possible.
10. Announce when there are 30 minutes remaining and when there are 5 minutes remaining.
11. When time is up, tell the students to STOP and have them sign their name in the space provided on the Student Answer Form. Collect the Answer Forms as quickly as possible.
12. Please do not grade the answer forms. They are to be sent to the AMC office for grading. Students may circle their answers on the contest booklet. However, the official answers will be the ones blackened on the answer form.
13. Fan the forms, making sure none are stuck together, place the School ID Form on top. There is only one ID FORM to be used with all the AMC 10/AMC 12 answer forms.
14. YOUR SCHOOL'S CEEB NUMBER IS THE NUMBER WRITTEN ON THE FRONT OF THE AMC 10/AMC 12 REPORT ENVELOPE.
15. Complete the Certification Form (only one form is needed) and place it on top of the School ID Form and answer forms and place all in the Report Envelope. Seal and send it by First Class (trackable recommended) within 24 hours or as soon as possible. Please affix the proper postage before mailing.
16. Please note: After the Answer Forms have been delivered to the school office to be mailed, you may discuss the contest with your students under the following conditions which take into consideration the fact that there will be schools
taking the contest in other locations at different times.
a. Inform the students that the contest may not be discussed with anyone outside of your school either orally, via email, www, copier or media of any type until after the contest period.
b. Students may keep the contest booklets and take them home.

## III. Eligibility

AMC 12 Eligibility - A student in a program leading to a high school diploma, and under 19.5 years of age on the day of the contest.
AMC 10 Eligibility - A student in a program leading to a high school diploma, and under 17.5 years of age on the day of the contest, and not enrolled in grades 11 or 12 or equivalent.
Please note: students in grades 11 \& 12 can not take the AMC 10. However, students in grades $9 \& 10$ may choose which contest they take.

## International Students \& Non-Citizens in USA Schools

US and Canadian Citizens and International Students residing in the United States (with qualifying scores) are eligible to take the USAMO.
Students learning "English as a Second Language" (ESL) may use a book nontechnical dictionary between their native language and English. A student may use the dictionary only the first time that he/she takes the AMC 10/AMC 12. The dictionary must be given to the school contest manager to examine and retain for the 24 -hour period preceding the contest. The proctor must announce to other students that the student(s) has/have been given special permission to use the dictionary during the contest.

## IV. Team Score Identification

TO RECEIVE OFFICIAL TEAM STATUS AND AWARDS, A SCHOOL MUST HAVE AT LEAST THREE PARTICIPANTS ON A CONTEST DATE. The team score for a school is the sum of its three highest student scores and will be determined by the AMC Office. The score of USA and Canadian teams is used to determine National School awards. In addition, the team score is used to select the top 60 schools to identify teachers who are eligible for the Edyth May Sliffe Award for Distinguished High School Teaching.

## V. School Results

The AMC office will send results by email (if available) and first class mail as soon as the answer forms are scored. If you have not received your results from our office within 30 days after the AMC 10/AMC 12 please contact us to verify that your answer forms were in fact received.
If you would like to receive your results by e-Mail, and have not previously sent us your email address, send a message, including your name, school name, address, and CEEB \# to:

## hstran@amc.unl.edu

Results are not official until you receive the postal mailed paper copy of your report, and that should be no longer than 30 days after AMC 10/AMC 12.
Please retain this report for future reference, either in your files
or with the student counselor's office. Many students cite these scores on college applications.

## VI. Policy Statements

## Early Administration

Administration on an earlier date is NEVER permitted and will lead to disqualification. Such an administration would jeopardize the validity of all scores from other participating schools.

## Official Administration

The AMC 10-A/AMC 12-A will be given officially on Tuesday February 10, 2009. The AMC 10-B/AMC 12 -B will be given officially on Wednesday, February 25, 2009. Only official participants, their school and their teacher are eligible for National Awards. In addition, official participants are eligible for all intramural awards and for participation in the AIME.

## Unofficial Administration

If you are unable to give Contest A on Tuesday, February 10, 2009 because:
a. your school is closed,
b. your school has an academic conflict,
c. the class periods have been shortened due to an assembly or other reason,
then you may give the second version of the contests (AMC 10-B/AMC 12-B) on the second official day, Wednesday, February 25, 2009 (See Contest B Registration Form on page 13). You may still take either exam unofficially on later dates, but those contests will not be eligible for state and national awards and will not be eligible for participation in the AIME. Unofficial participants are still eligible for intramural awards.
It is important to note that the only days eligible for official participation are the two official Contest days: February 10, and February 25, 2009.

## Contest Proctoring

The proctoring of the AMC 10 / AMC 12 contest should be by: $1^{\text {st }}$ preference, a math teacher at the school $2^{\text {nd }}$ preference, a teacher or administrator at the school
$3^{\text {rd }}$ preference, a college or university teacher of mathematics or a responsible adult who is a math club or team coach, and not associated with or related to any of the participants
$4^{\text {th }}$ preference, a responsible adult not associated with or related to any of the participants such as librarian, clergy, etc.

- The proctoring of the contest must take place in a public building, (E.g. school library, college or university, church, etc.).
- All students must take the contest at the same time, either in one group or in separate classrooms and be under the supervision of a proctor.
- The proctor should not be related to any of the participants
- If a parent wishes to help, they can arrange room reservations and set-up, help with student sign-up, provide treats, etc.


## One Contest per Date

A student may take only one exam on a given day but can participate on both contest dates if the school registers for both contests. The higher score will be used for individual awards.

## Students with Visual or Learning Disabilities

The AMC 10/AMC 12 time limit set by the CAMC for students
who are visually impaired or learning disabled is 120 minutes. A teacher or a school administrator may read the questions to the student and mark the answers as directed by the student. The cost of a Braille or Large Print exam is $\$ 7.00$ shipping and handling plus $\$ 1.60$ per exam for the AMC 12 and $\$ 1.50$ per exam for the AMC 10 . They are mailed separately and must be ordered no later than three weeks before the test.

## Sickness and Other Special Situations

A student who is sick or on a field trip on the first contest day may register and take the alternate Contest B on Wednesday, February 25. YOU MUST REGISTER FOR CONTEST B if you have not already done so. (see page 13 for a Registration Form).

## Questionable Scores

If it is clear to the Contest Manager from personal observation that a student has cheated, then the Manager must disqualify the student. If the Contest Manager receives an accusation of cheating, or obtains other indirect evidence of cheating, then the Manager must hold back the student's paper and immediately report all the facts of the situation to the AMC Director, who in conjunction with the Chair of the CAMC, will determine what to do. UNDER NO CIRCUMSTANCES may the school decide on its own to accept a questionable score, nor should a school retest the student in question before receiving instructions from the AMC Office.

## Follow-up Inquiries and Reexamination

The results of this contest helps to identify students with unusual mathematical ability. To assure that this purpose is served, the CAMC reserves the right to retest students before deciding whether to grant official status to individual or team scores. Reexamination will be requested when, after an inquiry, there is a reasonable basis to disbelieve a score. Official status will not be granted if a student or school does not agree to a requested retesting.

## Policy for Changes

The CAMC may, from time to time, change the program rules, regulations, awards and conditions of participation in whole or in part. Whenever possible you will be notified of these changes ahead of time.

## Refund/Credit Policy

If your school is unable to take the contests, please use the materials as practice sets for the next year. Do not return them. WE CAN NOT GIVE REFUNDS OR CREDITS AFTER THE CONTEST MATERIALS ARE SHIPPED.

## Request for Student Names Policy

The following statement appears on the student answer forms for the AMC 10 and AMC 12:

> The American Mathematics Competitions (AMC) receives requests from educational institutions and organizations for the names, addresses and grade levels of high scoring AMC 12 (or AMC 10) students. The optional personal data on ethnic origin and gender is used for recruiting and academic purposes..
> Blacken this circle ifyou give the AMC permission to release this information to these organizations. (Your score will not be affected ifyou do not blacken the circle.)

Receiving information is an "opt-in" decision for each individual

The AMC handles requests from institutions and organizations on a case-by-case basis and evaluates each individually for appropriateness. We provide legitimate educational institutions of all levels, both secondary and collegiate/university level, with one-time use of selected names and addresses for postal mailings. We also provide professional and scholarly organizations such as those listed as contributors to the AMC with one-time use of names and addresses for postal mailings, generally for professional or career information.
The only information we provide is the name, address, city, state, and zip code necessary for a postal mailing. We do not list individual scores or awards.

## VII. AIME Instructions

The $27^{\text {th }}$ annual American Invitational Mathematics Examination (AIME) will be held on TUESDAY, March 17, 2009 with a second alternate exam given on the alternate date of WEDNESDAY, April 1, 2009. These are the only days the exam may be taken officially. You may give the exam for practice (unofficially), after the official dates. We will be pleased to grade it for you but your students will not be eligible to take the USAMO. The contest is provided free of charge to all those taking the exam on the first date, however those taking the exam on the second alternate date will be charged a processing fee to cover expedited delivery.

## AIME Rules for AMC 10/AMC 12

Students who score 100 or above or finish in the top $5 \%$ on this AMC 12 or students who score 120 or above or finish in the top $1 \%$ on this AMC 10 will be invited to take the $27^{\text {th }}$ annual American Invitational Mathematics Examination (AIME) on Tuesday, March 17, 2009 or Wednesday, April 1, 2009. PLEASE read the following participation rules to your students as soon as you receive the AMC 10/AMC 12 package so potential AIME students will be able to plan accordingly.

## AIME School Manager

1. The AMC office will include all materials relating to the examination (including instructions for the exam) with your AMC 10/AMC 12 results.
2. All questions or problems concerning the AIME should be directed to the AMC office (800-527-3690).
3. The AIME is a three-hour examination. Each of its 15 questions requires a three digit integer answer and each correct answer will receive one (1) point.
4. Calculators are not allowed.
5. The AIME Answer Forms are sent directly to the AMC office for grading and processing.
6. Each participating school will receive a report of their results, an AIME solution pamphlet, and a list of students who qualify for the USAMO.
7. All AMC $10 / \mathrm{AMC} 12$ procedures for disqualification, follow-up inquiries and reexamination apply to the AIME as appropriate.
8. If you have students who you feel may qualify for the AIME please order prior year AIME exams and solutions for practice now. This way you will have these practice materials on hand when you receive your AMC 10/AMC 12 results.

## Second AIME Testing Date

Situations in which a student may take a second version of the AIME to be held on WEDNESDAY, April 1, 2009, keeping their USAMO eligibility open are:

1. School is closed on March 17 (i.e. spring break, weather).
2. Student is out of school the entire day due to attendance at an academic/school related event.
3. Student is ill and can not attend school on March 17.

There will be a processing fee for the second AIME as follows: $1-10$ students $=\$ 25,11+$ students $=\$ 50$. We will need your payment before the answer forms can be graded. A special envelope and payment form will be included with your AIME material, if you have AIME qualifiers. All AIME answer forms must arrive in the AMC office by April 3, 2009.
Email requests for the second AIME may be sent to:
AIMEQUAL@AMC.UNL.EDU
or, you can call the AMC office at 1-800/527-3690. Please have your school identification number (CEEB) and charge card information available before calling. E-Mail requests should include the school's CEEB number, and complete mailing address.
Under no circumstances can a student take both AIME's.

## VIII. USAMO Participant Selection

The USA Mathematical Olympiad (USAMO) is a two day, nine-hour, six-question, essay-proof examination. Selection for the USAMO will be explained in the AIME Teacher Manual. The goal is to select about 500 of the top scorers from the prior AIME and AMC 12A, AMC 12B, AMC 10A and AMC 10B contests to participate in the USAMO.
The USAMO is scheduled for Tuesday and Wednesday, April $28 \& 29,2009$ at your school. If you feel you may have a qualifier, please arrange for a space and proctor for these dates.
The top 12 scoring students on the USAMO will be invited to attend an award ceremony held in Washington, D.C., on June 7-8, 2009.

## IX. The MOSP Program

The Mathematical Olympiad Summer Program (MOSP) is a 3week, academic challenge designed to broaden participants' view of mathematics while fostering excitement toward further math study. It is held each year at the University of Nebraska-Lincoln in June-July. Invited students include the top 12 USAMO winners, 12-18 high-scoring USAMO participants, who are current juniors and below, and an additional 30 ninth-grade USAMO participants, grant funding permitting.
Watch for further details to be announced in the 2009 AIME/ USAMO Teachers' Manual and on the AMC website at www. unl.edu/amc.

## X. Regions of the AMC 10/AMC 12

The USA and Canada are partitioned into the following regions. National Awards are given to a minimum of 10 high scoring students and 5 schools (based on the team score) in each of these regions.
Region
0 Connecticut, Maine, Massachusetts, New Hampshire, Pennsylvania, Rhode Island, Vermont

1 New Jersey, New York
2 Delaware, District of Columbia, Maryland, North Carolina, South Carolina, Virginia, West Virginia
3 Alabama, American Embassy and APO/FPO Schools, Florida, Georgia, Puerto Rico, Virgin Islands
4 Indiana, Michigan, Ohio
5 Arkansas, Iowa, Kansas, Minnesota, Nebraska, North Dakota, Oklahoma, South Dakota, Wisconsin
6 Illinois, Kentucky, Missouri, Tennessee
7 Louisiana, Mississippi, Texas
8 Alaska, Arizona, Colorado, Guam, Hawaii, Idaho, Montana, Nevada, New Mexico, Oregon, Utah, Washington, Wyoming
9 California
10 Canada

## XI. Intramural and National Awards

## Intramural Awards for Students

The AMC Office will send you Intramural awards, along with your school results. Your registration fee entitles you to pins, medals, certificates, and a copy of the National Summary of Results and Awards. An order form for additional intramural awards will be included with your results. Committee and Donor Awards will be sent later.

1. AMC 10 School Winner Pin - For each school making a report on three or more students the Mathematics Award Pin is awarded to the student with the highest score. No listing is made for this award.
2. AMC 12 School Winner Pin - For each school making a report on three or more students the Mathematics Award Pin is awarded to the student with the highest score. No listing is made for this award
3. AMC 10 Honor Roll of Distinction Pin - The Honor Roll of Distinction Pin is awarded to the top $1 \%$ of all participating students.
4. AMC 12 Honor Roll of Distinction Pin - The Honor Roll of Distinction Pin is awarded to the top $1 \%$ of all participating students.
5. Gold Medal - The Gold Medal is awarded to participants who place first in their respective school for four consecutive years.
6. Silver and Bronze Medals - These medals are awarded to participants who place first in their school for three and two consecutive years, respectively. The names of these medal winners are listed in the Summary of Results and Awards published by the State Director.
7. AMC 10/12 Certificate of Distinction - A Certificate of Distinction is awarded to those students who qualify to take the American Invitational Mathematics Examination.
8. AMC $10 / 12$ Certificate of Achievement - Awarded to students who score 100 or above in grade $10 \&$ below on the AMC 12 or grade $8 \&$ below on the AMC 10.

## Committee and Donor Awards for Students

1. Large Plaque - A large plaque, donated by the Committee on the American Mathematics Competitions, is awarded to each student with a perfect score.
2. Small Plaque - A small plaque, donated by the Committee on the American Mathematics Competitions, is awarded to
the student with the highest score in each region in which there was no perfect score.
3. Mathematics Magazine* - A one-year subscription to the Mathematics Magazine, donated by the Committee of the American Mathematics Competitions, is awarded to a maximum of four participants with high score in each of the eleven regions.
4. NCTM Book* - A book, donated by the National Council of Teachers of Mathematics, is awarded to three high-scoring participants in each of the eleven regions.
5. Mu Alpha Theta Book* - A book, donated by Mu Alpha Theta, is awarded to one participant with a high score in each of the eleven regions.
6. AMATYC Review* - A one-year subscription to the American Mathematical Association of Two-Year Colleges Review is awarded to one student in each of the eleven regions.
7. CAMC Problem Book - One Problem Book donated by The American Mathematics Competitions, is awarded to the top-scoring student in grade 9 or less in each of the eleven regions.

## National School Awards

The AMC divides the United States and Canada into eleven regions. In each of these regions the five schools with the highest team scores (sum of the highest three scores by participants) are recognized by Donor or Committee Awards.

1. Charles T. Salkind Memorial Silver Cup - This silver cup, donated by the L.G. Balfour Company, Attleboro, MA, is awarded to the school with the highest team score (in any region).
2. William H. Fagerstrom Memorial Silver Cup - This silver cup is awarded to the school with the second highest team score (in any region) and it is provided by the L.G. Balfour Company, Attleboro, MA.
3. Committee Bronze Cups - In each of the regions in which a silver cup is not awarded the Committee on the American Mathematics Competitions provides a Bronze Cup to the school with the highest team score.

## Awards for Schools not Receiving a Cup

4. CAMC Mathematics Books* - In each of the eleven regions, five books are donated by the Committee on the American Mathematics Competitions to one school having a high team score.
5. W. H. Freeman Books* - In each of the eleven regions, a set of books, donated by W.H. Freeman and Company, San Francisco is awarded to one school having a high team score.
6. Mathematics Magazine* - In each of the eleven regions, a one-year subscription is donated by the Committee on the American Mathematics Competitions to one or more schools having a high team score.

## School Intramural Awards

7. Certificate of Honor - This is awarded to schools with a team score of 400 or greater.
8. Certificate of Merit - This is awarded to schools with a team score of 300-399.

## XII. Contest A Certification

The AMC 10 and AMC 12 must be administered by a teacher or an adult not associated with or related to any of the participants. The administration of the contest must take place in a public building (e.g. school, library, church). Please send all Answer Forms from your school or group at one time.
The Contest Manager and the Principal, Vice Principal, or Headmaster must sign this form which is to be returned with your student Answer Forms.

## Certification by the Contest Manager:

I certify that the following statements are true or that, if there are any exceptions, I have checked the box at the bottom of this page and have listed them on a separate page. I understand that the absence of either signature from this form, and a consideration of the exceptions may result in DISQUALIFICATION of all scores from our school.

1. I certify that the exam package(s) were retained in their sealed condition within an hour of the start of the contest.
2. I accept for our school the rules and procedures described on this page and pages 4-7, and accept that failure to follow these rules and procedures may result in DISQUALIFICATION from official standing of all scores from our school.
3. The contest was held on TUESDAY, February 10, 2009.
4. The AMC 10A / AMC 12A were given at the same time.
5. The participants were continually monitored during the contest, and they were separated by an empty space, if possible.
6. No aids were permitted other than scratch paper, graph paper, ruler, compass, and protractor. NO CALCULATORS WERE USED - (see Section I. Item 4).
7. Participants had exactly 75 minutes working time. (See page 6 for Student Disabilities Policy)
8. No students were permitted to proctor or grade the contest.
9. The instructions relating to the opening of the "Complimentary Solutions Envelope" and/or Solution Packets were followed.
10. After the contest, the answer forms were kept secure and no changes were made in the answers.
11. No parent assisted in the administration of the contest.
12. I have followed all the rules as stated in this Teachers' Manual.

Signature $\qquad$ E-mail (please print) $\qquad$
Day \& Date Test was given
Name of School $\qquad$
City $\qquad$ State/Province
Telephone ( $\qquad$ ) $\qquad$ School CEEB Number $\qquad$
Certification by the Principal, official or person of comparable title:
I agree that the Contest Manager followed all the rules and procedures listed above.

Signature $\qquad$ Date $\qquad$

Title $\qquad$

PLEASE INDICATE THE EXACT NUMBER
OF AMC 10A /AMC 12A ANSWER FORMS RETURNED FOR GRADING. $\qquad$

## Service Questionnaire

This year's questions have to do with the math curriculum at your school. Answer to the best of your ability, but if you don't know give us a "best estimate".

1. What is the average number of students in your school's math classes?
(0) 1-10
(1) 11-20
(2) 21-25
(3) 26-30
(4) $31-35$
(5) 36+
2. What grade levels are covered in your school? Check all that apply.
(0) pre-k
(1) K-4
(2) 5-6
(3) 7-8
(4) $9-10$
© 11-13
3. How many students attend your school?
(0) 1-99
(1) 100-399
(2) 400-799
(3) 800-1,199
(4) 1,200-1,999
© $2,000+$
4. How many "High School" programs (schools) are there in your school district?
(0) 1
(1) 2-5
(2) 6-10
(3) 11-15
(4) 16-19
© $20+$
5. Who makes the decision in your school on what math curriculum to follow? Check all that apply.
(0) Individual Teacher makes the choice
(3) School District Math Supervisor
(1) School Committee of teachers who teach the classes
(4) State Mandated Curricula/Guidelines
(2) School District Committee--teachers from the various schools
(5) Other $\qquad$
6. Which of the following curricula are used at your school? Check only the item most used.

0 University of Chicago School Mathematics Project (UCSMP)
(1) Contemporary Mathematics in Context (CORE-Plus)
(2) Interactive Mathematics Program (IMP)
(3) Saxon Math - Secondary, for grades 9-12
(4) Mathematics: Modeling Our World (ARISE)
© Other $\qquad$
7. Most comprehensive math programs provide more than just the text book. Which of the supplemental materials do you use? Check all that apply.
(0) Supplemental Worksheets
(3) Pre-designed unit tests
(1) Teacher study guides
(4) On-line help for students
(2) Pre-designed lesson plans
© Other
$\qquad$
8. On the whole, are you happy with the formal materials you are using? Check only one.
(0) Definitely
(3) Unhappy
(1) Somewhat happy
(4) A Disaster
(2) Indifferent
9. Do you feel the need to supplement the established curricula with additional material? Check only one.
(0) No - No need
(4) Yes - 2-3 times a week
(1) No - do not have time
(6) Yes - daily
(2) No - District/supervisor frowns on deviating from established curricula
(3) Yes - limited, 1x a week
10. From where do you draw your supplemental material? Check all that apply.
(0) Teacher workshops
(3) Other companies formal curricula
(1) Supplemental math problem books
(4) Networking with other teachers
(2) On-line materials
© Other

## XIII. Additional Forms used <br> - (if the form you need is not here, please see our web site) <br> Additional Bundles Form

Please fill in the information below and FAX your order. The administrator or authorized person of the school agrees to pay the American Mathematics Competitions for the following materials:
School Name $\qquad$ CEEB \# $\qquad$
Address $\qquad$
City $\qquad$ State $\qquad$ Zip $\qquad$
Teacher placing the order


To order either the 2008-2009 AMC 10/12 Math Club Package or the $21^{\text {st }}$ Century CD with pdf's of all contests 2001-2008, download the Publications Order Form from the AMC website at www.unl.edu/amc/

Postage/handling Fee (see chart below) .. $\$$ $\qquad$
Total.............................................................................................................................................. \$ $\qquad$
P.O. Number $\qquad$
VISA/MC\# : Address:
Name (Please Print): $\qquad$
Exp. Date: $\qquad$

AMC ORDERING -- TERMS

1. VISA \& MasterCard accepted.
2. Make checks payable to:

AMERICAN MATHEMATICS COMPETITIONS
3. PAYMENT IN U.S. FUNDS ONLY.
4. U.S.A.: Order TOTAL Shipping Charge*

| $\$ 10.00--\$ 40.00$ | $\$ 7.00$ |
| :--- | ---: |
| $\$ 40.01--\$ 50.00$ | $\$ 9.00$ |
| $\$ 50.01--\$ 75.00$ | $\$ 12.00$ |
| $\$ 75.01-$ - UP | $\$ 15.00$ |

5. OUTSIDE U.S.A.: Add additional $\$ 10$ to U.S.A. shipping costs.
*Orders after February $1^{\text {st }}$ add $\$ 10.00$ additional for 1-day Service.

## Proof of Intent to Pay

This document is intended to be used in lieu of pre-payment when calling or faxing in an order. Please indicate if you wish to be billed or will be sending a "check in the mail" (to be received within 2 weeks of order or you will be billed). Mail orders not wishing to be billed should include a check when returning this form. The person who signs this form must be authorized to pay the order that is placed by the teacher.

BILLED
Name of Person Authorized to Pay (please print): $\qquad$
Signature: $\qquad$
Title: $\qquad$ Date: $\qquad$

## Rescoring Request Form

I would like to have the following student's answer form rescored. I understand that there is a $\$ 5.00$ charge for each student answer form rescored
\$ 5.00/each
Student Name $\qquad$ \$ $\qquad$
Contest taken: AMC 10-A AMC 10-B AMC12-A AMC12-B
Student Name $\qquad$ \$ $\qquad$
Contest taken: AMC 10-A AMC 10-B AMC12-A AMC12-B
Grand Total $\qquad$ \$ $\qquad$

Teacher's Name $\qquad$ CEEB \# $\qquad$
School Name $\qquad$
Address
City $\qquad$ State $\qquad$ Zip $\qquad$

## Method of Payment:

Check (US funds only) made payable and mailed with this form to the:
AMERICAN MATHEMATICS COMPETITIONS
University of Nebraska-Lincoln
P.O. Box 81606

Lincoln, NE 68501-1606
Charge to Visa/Mastercard\#: $\qquad$
Name on card (print): $\qquad$
Signed
Expiration Date: $\qquad$ Telephone: $\qquad$
FAX to: 402/472-6087

## PLEASE PRINT <br> High School CEEB\# ( 6 digits):

## Contest Manager

School Name

School Street Address

| City | State | Zip |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |


| City | State | Zip |
| :--- | :--- | :--- |
| $\left(\begin{array}{ll}\text { Phone \# } & \end{array}\right.$ |  |  |
|  |  |  |

E-mail (for sending results) Please Print Clearly
(Circle appropriate responses, below)
Learning Center, Type of Public Private Home Outside School Group: School School School Class/Group
Grades: PreK K 12345678910111213 School Size: 0-200 201-400 401-1000 1001+

## PAYMENT OPTIONS

Do not send payment alone. The Registration Form must be included with your payment option. Checks sent without appropriate registration information cannot be processed and will be returned to sender.
Check P.O. \# Visa Master Card Terms - Payment in U.S. Funds only.
Make checks payable to: MAA American Math Competitions

VISA/MC \#
Exp. Date:

Name (Please Print)
Give an address for mailing the charge receipt in "Billing Address" above
All orders Non-Refundable once shipped. Mail along with your payment or Purchase Order to

MAA American Mathematics Competitions
ATTN: AMC 10/12 Registration
P.O. Box 81606

Lincoln, NE 68501-1606

Step 1 - Pick registration by current date, if you are Canadian, US nonadjacent, or Int'I please include additional shipping, below.

## REGISTRATION - REQUIRED

- One fee covers 10B/12B Registration and US contiguous 48 states shipping (choose 1)
$\square$ Registration/Expedited Shipping . $\$ 50.00$
- REQUIRED for Short Shipping window
- 2-day shipping

OR
Registration/Overnight Shipping .. $\$ 60.00$

- REQUIRED after January 26 (Jan. 27--Feb 20).
- Overnight shipping
U. S. REGISTRATION SUBTOTAL
. \$


## Step 2 - Add extra shipping if not in US contiguous 48 states

ADDITIONAL SHipping

- Canada, Alaska, Guam, Hawail, \& Puerto RICO ADDITIONAL SHIPPING............... $\$ 10.00 \rightarrow$.. $\$$ $\qquad$
or
- InTERNATIONAL SCHOOLS ADDITIONAL SHIPPING (\$70 MAXIMUM) \# EXAM BUNDLES ___ X $\$ 7.00 /$ BUNDLE $\qquad$ \$


## Step 3 - Indicate your selection of contests (and solutions)

## CONTEST BUNDLES OF TEN

- AMC 10B -- Ten contests per bundle

| English 10B contests \# x \$ |  |
| :---: | :---: |
| $\square$ Spanish 10B contests \#___ $x$ \$15/bundle.. |  |
| - French 10B contests \# __ $x \$ 15 / \mathrm{bundle}$.. | \$ |
| [optional) English 10B Solutions \#___@ \$ 6/set. |  |
| - AMC 12B -- Ten contests per bundle |  |
| English 12B contests \#___ x \$16/bundle. |  |
| $\square$ Spanish $12 B$ contests \#___ x \$16/bundle.. |  |
| $\square$ French 12B contests \#__ $x$ \$ $16 / \mathrm{bundle.}$. |  |
| $]_{\text {(optional) }}$ English 12B Solutions \#__ $x$ \$ 6/set.... |  |
|  |  |

\$

## Braille \& Large Print Contests

Priced individually, not available for International Schools:


## Step 4 - Decide if you want to include a Math Club Package

AMC 10/12 MATH Club Package:
Study Guide, CD, Web materials, see brochure.
Sent separately; available Fall 2008
With Shipping for Contiguous US 48 states $\ldots .$. . $\$ 25.00=\$$ $\qquad$
Additional postage required for International/Overseas Addresses,
please email AMC office: amcinfo@maa.org
Step 5 - Add sub-totals for steps 1,2, 3, and 4
TOTAL ORDER

# The Mathematical Association of America <br> American Mathematics Competitions 

NEW this year CD's with all the Contests - \$20 each

1. AJHSME \& AMC 8 (1985-2007) + worksheets (1999-2007)
2. AHSME 1 (1950-1974)
3. AHSME 2 (1975-1999)

OR . . . Buy the Math Club Package! - \$25 each
This includes a book with teaching ideas and club activities, and a CD which contains contests and solutions from the last 10 years of the AMC 8 (98-07), AMC 10 A\&B (00-08), and AMC 12 A\&B (99-08), plus AIMEs (01-08), and USAMOs (01-08). All the
questions and solutions from these years are available in PDF form. We have also included the 275 AMC 8 worksheets and 395 AMC 10 and AMC 12 worksheets we have developed thus far.


CD --AJHSME \& AMC 8 (1985-2007) ......\#
CD --AHSME 1 (1950-1974)........................ \#
CD --AHSME 2 (1974-1999)..............\#——@ \$20/ea = \$

- @ 25/ea=
@
DVD -- Hard Problems, Deluxe Edition ...\# ___ @ \$25/ea = \$
MAA AMC T-Shirt -- 60th Anniversary ..\#___ @ \$15/ea = \$ Current Sizes:Small__ Medium___ Large__ X-Large__
Postage \& Handling (See TERMS) ...................................... \$
GRAND TOTAL........................................................ \$
PAYABLE in U.S. Funds ONLY


## ALLOW 2-3 WEEKS FOR DELIVERY

## MAA AMC Ordering -. TERMS

- VISA and MasterCard accepted.
- Payment by check will also be accepted. Make checks payable to: AMERICAN MATHEMATICS COMPETITIONS
- PAYMENT IN U.S. FUNDS ONLY.
- All 50 U.S states, only: No additional fee for shipping.
- All other including Canadian \& International Orders:

Please includee-mail orfaxnumber. AMC willsend youatotal orderconfirmation with the shipping chargeaddedtoyour order. When paying byVisa orMasterCard we need your charge card number, expiration date, and name of card holder. Unpaid Purchase Orders will be accepted. Payment in U.S. FUNDS ONLY.

- Prices good until September 1, 2009.
- Please allow 2 weeks for delivery.

FAX 402-472-6087 or call 1-800-527-3690 Please Send Your Order To:
MAA American Mathematics Competitions
ATTN: AMC Publications
P.O. Box 81606

Lincoln, NE 68501-1606

## Hard Problems DVD, Deluxe Edition - \$25 each

A DVD documentary about the extraordinarily gifted students who represented the United States in 2006 at the world's toughest math competition - the International Mathematical Olympiad (IMO). It is the story of six American high school students who competed with 500 others from 90 countries in Ljubljana, Slovenia. The film shows the dedication and perseverance of these remarkably talented students, the rigorous preparation they undertake, and the joy they get out of solving challenging problems. It captures the spirit that infuses the mathematical quest at the highest level. Includes Feature version: 82 minutes; Classroom version: 45 minutes; and several bonus features.


Current sizes: Small, Sports the 2008-2009 theme of "6 Decades of the American Mathematics Competitions." Black, 100\% cotton, full-color design on back, blue and white logo on front left chest.

Medium, Large and X-Large

## Shipping Address <br> PLEASE FILL IN ALL THE INFORMATION

If other than a school, please strike out school and any other irrelevant headers and fill in the name, and ship-to address.
(Please Print)
Name:
School Name: $\qquad$
Street Address: $\qquad$

City: $\qquad$ State: $\qquad$ Zip: $\qquad$
Phone \#: $\square$ 1

E-mail address: $\qquad$
CHARGE INFORMATION:
Card Name (Print): $\qquad$
VISA/MC \#: $\qquad$
Exp. Date: $\qquad$
E-mail address: $\qquad$
Address: $\qquad$

For additional publications please visit the MAA online store (hhtps://enterprise.ma.org/ecomtpro/Timssnet/Common/tnt_frontpage.cfm), or the NCTM online store (http://www.nctm.org/publications/index.htm).

# XIV. Classroom Accessories 

## Publicity

The sample Contest Announcement news release below, should be prepared and distributed to the newspapers, radio and television stations in your region. To make preparation of the news release easier, visit our web site, and download the text from the on-line copy of the AMC 10/12 Teachers' Manual. See the AMC website, or the 2008 Summary of Results for statistics and figures from the 2008 contest. Statistics and figures for the 2009 Contest will be available on our website in March 2009.

## (School or School District) <br> FOR IMMEDIATE RELEASE

## (School) STUDENTS PARTICIPATE IN NATIONAL MATHEMATICS COMPETITION

$(\#)$ students at (School) participated in the $60^{\text {th }}$ annual American Mathematics Contest 12, and/or the $10^{\text {th }}$ annual American Mathematics Contest 10. The contests were held on Tuesday, February 10, 2009 (or Wednesday, February 25, 2009). The students competed for local, regional and national student and school awards. The contest, which covers high school mathematics, is given in participating schools. Its purpose is to spur interest in mathematics and develop talent through the excitement of friendly competition at problem solving in a timed format. In 2008 over 250,000 students from 4,200 schools participated in the AMC 10 \& AMC 12 contest including (\#) students from (\#) schools in (State). Top scorers at (school) were ( $\qquad$ _,
$\qquad$ , etc.).
According to Prof. Steven Dunbar, who serves as Director of the American Mathematics Competitions, the AMC 12 (first offered in 1950), and the AMC 10 (first offered in 2000), are part of a series of contests sponsored each year by The Mathematical Association of America, through their program, the American Mathematics Competitions. The AMC offers the only math competition series in the country leading to the United States of America Mathematical Olympiad (USAMO) and the Mathematical Olympiad Summer Program (MOSP). From this group of students, the AMC sends the highly competitive USA Team to the prestigious annual International Mathematical Olympiad. The AMC program includes:

| American Mathematics Contest 8 (AMC8) | Grades 6-8 | November |
| :--- | :--- | :--- |
| American Mathematics Contest 10 (AMC 10) | Grades 10 \& below | 2 dates in February |
| American Mathematics Contest 12 (AMC 12) | Grades 12 \& below | 2 dates in February |
| American Invitational Mathematics Examination (AIME) | All who qualify | 2 dates in March-April |
| USA Mathematical Olympiad (USAMO) | All who qualify | late-April |
| Mathematical Olympiad Summer Program (MOSP) | Qualify thru USAMO | June |
| International Mathematical Olympiad (IMO) | Top six from USAMO, MOSP | July |

The AMC is located at the University of Nebraska - Lincoln. and receives support from the Akamai Foundation, Academy of Applied Sciences, American Mathematical Association of Two Year Colleges, American Mathematical Society, American Society of Pension Actuaries, American Statistical Association, Art of Problem Solving, Awesome Math, Canada/USA Mathcamp, Casualty Actuarial Society, Clay Mathematics Institute, IDEA Math, Institute for Operations Research and the Management Sciences, L.G.Balfour, Math Zoom Academy, Mu Alpha Theta, National Assessment and Testing, National Council of Teachers of Mathematics, Pi Mu Epsilon, the Society of Actuaries, U.S.A Math Talent Search, W.H. Freeman and Wolfram Research, Inc.. The Contests are given across the U.S.A, Canada, and in many schools abroad.

Details concerning the 2009 AMC 10/12 contests for High School, as well as the rest of AMC's programs are available on the AMC web site: www.unl.edu/amc/.

For further information contact the AMC -- telephone: 800/527-3690, email: amcinfo@maa.org.

# The MATHEMATICAL ASSOCIATION OF AMERICA American Mathematics Competitions 

Steven Dunbar Director

February, 2009
Dear Parent or Guardian:

On February 10, 2009 and/or February 25, 2009 your son or daughter will participate in the $60^{\text {th }}$ annual American Mathematics Competitions contest. This contest has grown from a single city-wide competition in New York City in 1950, organized by the local section of The Mathematical Association of America to a sequence of contests. In 2008, over 250,000 students from over 4,200 US and international schools competed for school, regional, and national awards in this contest and found it fun and rewarding. Top 10, well-known U.S. universities and colleges, including internationally recognized U.S. technical institutions, ask for AMC scores on their application forms. Your students deserve the chance to list these scores on their applications!

Each year the AMC 10 and AMC 12 are on the National Association of Secondary School Principals
 Advisory List of Contests and Activities. The AMC Contests are sponsored by the Mathematical Associatimon of America, and are considered to be such a valuable stimulus to student interest in mathematics that 23 professional societies and organizations, including the National Council of Teachers of Mathematics and those represented below, support the contests with financial contributions.


With these contests, there are awards in each school for the student with the highest score, certificates for high-scoring students in each school, state-wide awards, regional awards, and even national awards. These contests lead to other more selective math contests, even all the way to the USA team sent to the International Mathematical Olympiad, the premier international high school level problem solving contest.
But the real rewards come from challenging each student with mathematics that is new, different, and "outside of the box." The problems on the contest are hard, but designed to be within reach. Just by participating in the contest your student should still feel accomplishment, because these problems are meant to be more challenging than routinely encountered in mathematics courses.
Mathematics is increasingly important in our technological and scientific age. Taking enough mathematics in high school is the gateway to jobs and careers of all kinds, even those that are not explicitly mathematical, scientific, or technological. We hope that by offering these contests, we can challenge and inspire students to learn more mathematics. We hope that your son or daughter enjoyed the contests, and will continue to take mathematics courses and competitions in high school and beyond.

Sincerely,


Dr. Steven R. Dunbar
AMC Director

## American Mathematics Contest 10 American Mathematics Contest 12

AMC 10 A I AMC 12 A Tuesday, February 10, 2009 and / or
AMC 10 B / AMC 12 B Wednesday, February 25, 2009

For more information Contact:

Practice materials are available at:
http://www.unl.edu/amc/


The Mathematical Association of America American Mathematics Competitions

CERTIFICATE
Awarded to
for participating in the
Annerican Mathematics Contest 10
(AMC 10)

$$
2009
$$



CERTIFICATE
Ywartect to
for participating in the
Annericaun Mathematics Contest 10 (ADC iO)



The Mathematical Association of America
American Mathematics Competitions
CERTIEICATE
for participating in the
American Mathematics Contest 12 (AMC 12)


American $\mathcal{M a t h e m a t i c s ~ C o m p e t i t i o n s ~}$


AMC 12 Subcommittee


The Mathematical Association of America
American Mathematics Competitions
CERTIFICATE
Awarded to
for participating in the
Anneriicaun Mathemnatics Conntest 12


American Mathematics Competitions


AMC 12 Subcommittee

## Tuesday, FEBRUARY 10, 2009



1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR GIVES THE SIGNAL TO BEGIN.
2. This is a 25 -question, multiple choice test. Each question is followed by answers marked $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ and E . Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a \#2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, and erasers. No calculators are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have 75 MINUTES to complete the test.
8. When you finish the exam, sign your name in the space provided on the Answer Form.
Students who score in the top $1 \%$ on this AMC 10 will be invited to take the $27^{\text {th }}$ annual American Invitational Mathematics Examination (AIME) on Tuesday, March 17, 2009 or Wednesday, April 1, 2009. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.
The publication, reproduction or communication of the problems or solutions of the AMC 10 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules. After the contest period, permission to make copies of problems in paper or electronic form including posting on web-pages for educational use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear the copyright notice.

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## Facsimile of AMC 12 Front Cover

## Tuesday, FEBRUARY 10, 2009

$60^{\text {th }}$ Annual American Mathematics Contest 12


The MATHEMATICAL ASSOCIATION of AMERICA American Mathematics Competitions

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR GIVES THE SIGNAL TO BEGIN.
2. This is a 25 -question, multiple choice test. Each question is followed by answers marked $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ and E . Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a \#2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, and erasers. No calculators are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have 75 MINUTES to complete the test.
8. When you finish the exam, sign your name in the space provided on the Answer Form.
Students who score 100 or above or finish in the top $5 \%$ on this AMC 12 will be invited to take the $27^{\text {h }}$ annual American Invitational Mathematics Examination (AIME) on Tuesday, March 17, 2009 or Wednesday, April 1, 2009. More details about the AIME and other information are on the back page of this test booklet.
The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.
The publication, reproduction or communication of the problems or solutions of the AMC 12 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules. After the contest period, permission to make copies of problems in paper or electronic form including posting on web-pages for educational use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear the copyright notice.

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## AMC 10 Student Practice Questions

You will find these and additional problems for the AMC 10 and AMC 12 on AMC's web site: http://www.unl.edu/amc, available from the current and previous AMC 10/12 Teacher Manuals, (http://www.unl.edu/amc/e-exams/e6-amc12/archive12.shtml) or from our Problems page archives (http://www.unl.edu/amc/a-activities/a7-problems/problem81012archive.shtml).

Each of the sides of a square $S_{1}$ with area 16 is bisected, and a smaller square $S_{2}$ is constructed using the bisection points as vertices. The same process is carried out on $S_{2}$ to construct an even smaller square $S_{3}$. What is the area of $S_{3}$ ?
(A) $\frac{1}{2}$
(B) 1
(C) 2
(D) 3
(E) 4

## 2008 AMC 10 A, Problem \#10-

"The sides of $S_{2}$ have length $\sqrt{2^{2}+2^{2}}=2 \sqrt{2}$."

## Solution

Answer (E): The sides of $S_{1}$ have length 4, so by the Pythagorean Theorem the sides of $S_{2}$ have length $\sqrt{2^{2}+2^{2}}=2 \sqrt{2}$. By similar reasoning the sides of $S_{3}$ have length $\sqrt{(\sqrt{2})^{2}+(\sqrt{2})^{2}}=2$. Thus the area of $S_{3}$ is $2^{2}=4$.


OR
Connect the midpoints of the opposite sides of $S_{1}$. This cuts $S_{1}$ into 4 congruent squares as shown. Each side of $S_{2}$ cuts one of these squares into two congruent triangles, one inside $S_{2}$ and one outside.
Thus the area of $S_{2}$ is half that of $S_{1}$. By similar reasoning, the area of $S_{3}$ is half that of $S_{2}$, and one fourth that of $S_{1}$.

Yesterday Han drove 1 hour longer than lan at an average speed 5 miles per hour faster than lan. Jan drove 2 hours longer than lan at an average speed 10 miles per hour faster than lan. Han drove 70 miles more than Ian. How many more miles did Jan drive than lan?
(A) 120
(B) 130
(C) 140
(D) 150
(E) 160

2008 AMC 10 A, Problem \#15-
"Set up equation to represent the relations with lan's total time, $h$ hours, and average speed, $r$ miles."

## Solution

Answer (D): Suppose that lan drove for $t$ hours at an average speed of $r$ miles per hour. Then he covered a distance of rt miles. The number of miles Han covered by driving 5 miles per hour faster for 1 additional hour is

$$
(r+5)(t+1)=r t+5 t+r+5
$$

Since Han drove 70 miles more than lan,

$$
70=(r+5)(t+1)-r t=5 t+r+5, \quad \text { so } \quad 5 t+r=65
$$

The number of miles Jan drove more than lan is consequently

$$
(r+10)(t+2)-r t=10 t+2 r+20=2(5 t+r)+20=2 \cdot 65+20=150
$$

## OR

Represent the time traveled, average speed, and distance for lan as length, width, and area, respectively, of a rectangle as shown. A similarly formed rectangle for Han would include an additional 1 unit of length and 5 units of width as compared to lan's rectangle. Jan's rectangle would have an additional 2 units of length and 10 units of width as compared to lan's rectangle.


Ian


Han


Given that Han's distance exceeds that of lan by 70 miles, and Jan's $10 \times t$ and $2 \times r$ rectangles are twice the size of lan's $5 \times t$ and $1 \times r$ rectangles, respectively, it follows that Jan's distance exceeds that of lan by

$$
2(70-5)+20=150 \text { miles. }
$$

Difficulty: Medium-hard
NCTM Standard: Algebra Standard: use symbolic algebra to represent and explain mathematical relationships.
Mathworld.com Classification: Algebra > Algebraic Equations > Linear Equation

Assume that $x$ is a positive real number. Which is equivalent to $\sqrt[3]{x \sqrt{x}}$ ?
(A) $x^{1 / 6}$
(B) $x^{1 / 4}$
(C) $x^{3 / 8}$
(D) $x^{1 / 2}$
(E) $x$

## 2008 AMC 10 B, Problem \#3-

"Use the fact that $\sqrt[a]{x}=x^{\frac{1}{a}}$. "

## Solution

Answer (D): The properties of exponents imply that

$$
\sqrt[3]{x \sqrt{x}}=\left(x \cdot x^{\frac{1}{2}}\right)^{\frac{1}{3}}=\left(x^{\frac{3}{2}}\right)^{\frac{1}{3}}=x^{\frac{1}{2}}
$$

Suppose that $\left(u_{n}\right)$ is a sequence of real numbers satisfying $u_{n+2}=2 u_{n+1}+u_{n}$, and that $u_{3}=9$ and $u_{6}=128$. What is $u_{5}$ ?
(A) 40
(B) 53
(C) 68
(D) 88
(E) 104

## 2008 AMC 10 B, Problem \#11-

"Rewrite $u_{6}$ in term of $u_{4}$ and solve for $u_{4}$."

## Solution

Answer (B): Note that $u_{5}=2 u_{4}+9$ and $128=u_{6}=2 u_{5}+u_{4}=5 u_{4}+18$.
Thus $u_{4}=22$, and it follows that $u_{5}=2 \cdot 22+9=53$.

Three red beads, two white beads, and one blue bead are placed in a line in random order. What is the probability that no two neighboring beads are the same color?
(A) $\frac{1}{12}$
(B) $\frac{1}{10}$
(C) $\frac{1}{6}$
(D) $\frac{1}{3}$
(E) $\frac{1}{2}$

## 2008 AMC 10 B, Problem \#22- <br> "Note that there are $6!/(3!2!1!)=60$ distinguishable orders of the beads on the line."

## Solution

Answer (C): There are $6!/(3!2!1!)=60$ distinguishable orders of the beads on the line. To meet the required condition, the red beads must be placed in one of four configurations: positions 1,3 , and 5 , positions 2,4 , and 6 , positions 1,3 , and 6 , or positions 1,4 , and 6 . In the first two cases, the blue bead can be placed in any of the three remaining positions. In the last two cases, the blue bead can be placed in either of the two adjacent remaining positions. In each case, the placement of the white beads is then determined. Hence there are $2 \cdot 3+2 \cdot 2=10$ orders that meet the required condition, and the requested probability is $\frac{10}{60}=\frac{1}{6}$.

Three cubes are each formed from the pattern shown. They are then stacked on a table one on top of another so that the 13 visible numbers have the greatest possible sum. What is that sum?

(A) 154
(B) 159
(C) 164
(D) 167
(E) 189

## 2008 AMC 12 A, Problem \#11- <br> "The three pairs of opposite faces have numbers with sums $1+32=33,2+16=18$, and $4+8=12 . "$

## Solution

Answer (C): The sum of the six numbers on each cube is $1+2+4+8+$ $16+32=63$. The three pairs of opposite faces have numbers with sums $1+32=33,2+16=18$, and $4+8=12$. On the two lower cubes, the numbers on the four visible faces have the greatest sum when the 4 and the 8 are hidden. On the top cube, the numbers on the five visible faces have the greatest sum when the 1 is hidden. Thus the greatest possible sum is $3 \cdot 63-2 \cdot(4+8)-1=164$.

What is the area of the region defined by the inequality $|3 x-18|+|2 y+7| \leq 3 ?$
(A) 3
(B) $\frac{7}{2}$
(C) 4
(D) $\frac{9}{2}$
(E) 5

2008 AMC 12 A, Problem \#14-
"The boundaries of the region are the two pairs of parallel lines $(3 x-18)+(2 y+7)= \pm 3$ and $\quad(3 x-$ 18) $-(2 y+7)= \pm 3 . "$

## Solution

Answer (A): The boundaries of the region are the two pairs of parallel lines

$$
(3 x-18)+(2 y+7)= \pm 3 \quad \text { and } \quad(3 x-18)-(2 y+7)= \pm 3
$$

These lines intersect at $(6,-2),(6,-5),\left(5,-\frac{7}{2}\right)$, and $\left(7,-\frac{7}{2}\right)$. Thus the region is a rhombus whose diagonals have lengths 2 and 3 . The area of the rhombus is half the product of the diagonal lengths, which is 3 .

Vertex $E$ of equilateral $\triangle A B E$ is in the interior of unit square $A B C D$. Let $R$ be the region consisting of all points inside $A B C D$ and outside $\triangle A B E$ whose distance from $\overline{A D}$ is between $\frac{1}{3}$ and $\frac{2}{3}$. What is the area of $R$ ?
(A) $\frac{12-5 \sqrt{3}}{72}$
(B) $\frac{12-5 \sqrt{3}}{36}$
(C) $\frac{\sqrt{3}}{18}$
(D) $\frac{3-\sqrt{3}}{9}$
(E) $\frac{\sqrt{3}}{12}$

## 2008 AMC 12 B, Problem \#13-

"Sketch the figure, and identify region $R$ on the figure."

## Solution

Answer (B): Draw a line parallel to $\overline{A D}$ through point $E$, intersecting $\overline{A B}$ at $F$ and intersecting $\overline{C D}$ at $G$. Triangle $A E F$ is a $30-60-90^{\circ}$ triangle with hypotenuse $A E=1$, so $E F=\frac{\sqrt{3}}{2}$. Region $R$ consists of two congruent trapezoids of height $\frac{1}{6}$, shorter base $E G=1-\frac{\sqrt{3}}{2}$, and longer base the weighted average

$$
\frac{2}{3} E G+\frac{1}{3} A D=\frac{2}{3}\left(1-\frac{\sqrt{3}}{2}\right)+\frac{1}{3} \cdot 1=1-\frac{\sqrt{3}}{3} .
$$

Therefore the area of $R$ is

$$
2 \cdot \frac{1}{6} \cdot \frac{1}{2}\left(\left(1-\frac{\sqrt{3}}{2}\right)+\left(1-\frac{\sqrt{3}}{3}\right)\right)=\frac{1}{6}\left(2-\frac{5 \sqrt{3}}{6}\right)=\frac{12-5 \sqrt{3}}{36}
$$



Place $A B C D$ in a coordinate plane with $B=(0,0), A=(1,0)$, and $C=(0,1)$. Then the equation of the line $B E$ is $y=\sqrt{3} x$, so $E=\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, and the point of $R$ closest to $B$ is $\left(\frac{1}{3}, \frac{\sqrt{3}}{3}\right)$. Thus the region $R$ consists of two congruent trapezoids with height $\frac{1}{6}$ and bases $1-\frac{\sqrt{3}}{2}$ and $1-\frac{\sqrt{3}}{3}$. Then proceed as in the first solution.

A pyramid has a square base $A B C D$ and vertex $E$. The area of square $A B C D$ is 196 , and the areas of $\triangle A B E$ and $\triangle C D E$ are 105 and 91 , respectively. What is the volume of the pyramid?
(A) 392
(B) $196 \sqrt{6}$
(C) $392 \sqrt{2}$
(D) $392 \sqrt{3}$
(E) 784

## 2008 AMC 12 B, Problem \#18-

## "Construct a triangle whose altitude is the altitude of the pyramid. Apply Heron's Formula to find the altitude."

## Solution

Answer (E): Square $A B C D$ has side length 14. Let $F$ and $G$ be the feet of the altitudes from $E$ in $\triangle A B E$ and $\triangle C D E$, respectively. Then $F G=14, E F=2 \cdot \frac{105}{14}=15$ and $E G=2 \cdot \frac{91}{14}=13$. Because $\triangle E F G$ is perpendicular to the plane of $A B C D$, the altitude to $\overline{F G}$ is the altitude of the pyramid. By Heron's Formula, the area of $\triangle E F G$ is $\sqrt{(21)(6)(7)(8)}=84$, so the altitude to $\overline{F G}$ is $2 \cdot \frac{84}{14}=12$. Therefore the volume of the pyramid is $\left(\frac{1}{3}\right)(196)(12)=784$.

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