Curriculum Inspirations
Inspiring students with rich content from the MAA American Mathematics Competitions
MAA

James Tanton, MAA Mathematician-at-Large

TRY THE FIRST TEN PROBLEMS

AMC 10

Are you ever-so-slightly curious about the American Mathematics Competitions?

What does one look like?
Are they hard?
What does it mean to be "good" at one?
Do I have to be good at it?

The point of the AMC is to show students, like you, that there is lot of curious, interesting, and sometimes quirky mathematics to think about and play with. Sure, some people like the sense of "competition," but there are many others (and I am one of those "others") who aren't drawn to that element at all. That's okay!

So, here's an invitation: Try these first 10 problems from the 2018 AMC 10B competition. Have fun with them. See how they affect your brain and what new ideas they lead you to think about and wonder about. Just try them! And perhaps try the next upcoming AMC 10 competition too with the idea of just dwelling on the first 10 problems there and see which ones make you think and wonder.

Most people don't realize that making progress on the first 10 problems is actually a significant achievement!

Just go for the first ten!

2018 AMC 10B
The First Ten

Problem 1
Kate bakes a 20-inch by 18-inch pan of cornbread. The cornbread is cut into pieces that measure 2 inches by 2 inches. How many pieces of cornbread does the pan contain?

$\textbf{(A)}\ 90 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 180 \qquad \textbf{(D)}\ 200 \qquad \textbf{(E)}\ 360$

Problem 2
Sam drove 96 miles in 90 minutes. His average speed during the first 30 minutes was 60 mph (miles per hour), and his average speed during the second 30 minutes was 65 mph. What was his average speed, in mph, during the last 30 minutes?

$\textbf{(A)}\ 64 \qquad \textbf{(B)}\ 65 \qquad \textbf{(C)}\ 66 \qquad \textbf{(D)}\ 67 \qquad \textbf{(E)}\ 68$

Problem 3
In the expression $(\_\_\_\_ \times \_\_\_\_) + (\_\_\_\_ \times \_\_\_\_)$ each blank is to be filled in with one of the digits 1, 2, 3, or 4, with each digit being used once. How many different values can be obtained?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 24$

Problem 4
A three-dimensional rectangular box with dimensions X, Y, and Z has faces whose surface areas are 24, 24, 48, 48, 72, and 72 square units. What is $X + Y + Z$?

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 22 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 36$

Problem 5
How many subsets of $\{2,3,4,5,6,7,8,9\}$ contain at least one prime number?

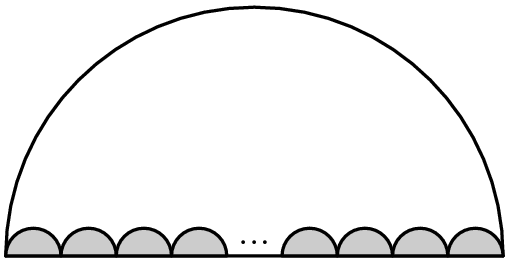
$\textbf{(A)}\ 128 \qquad \textbf{(B)}\ 192 \qquad \textbf{(C)}\ 224 \qquad \textbf{(D)}\ 240 \qquad \textbf{(E)}\ 256$

Problem 6
A box contains 5 chips, numbered 1, 2, 3, 4, and 5. Chips are drawn randomly one at a time without replacement until the sum of the values drawn exceeds 4. What is the probability that 3 draws are required?

$\textbf{(A)}\ \frac{1}{15} \qquad \textbf{(B)}\ \frac{1}{10} \qquad \textbf{(C)}\ \frac{1}{6} \qquad \textbf{(D)}\ \frac{1}{5} \qquad \textbf{(E)}\ \frac{1}{4}$

Problem 7

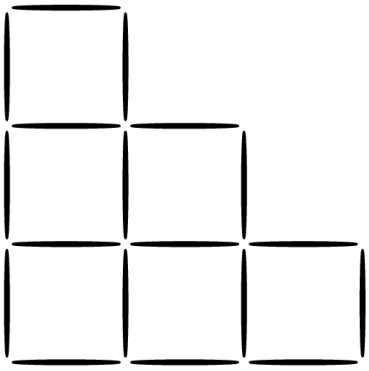
In the figure below, N congruent semicircles are drawn along a diameter of a large semicircle, with their diameters covering the diameter of the large semicircle with no overlap. Let A be the combined area of the small semicircles and B be the area of the region inside the large semicircle but outside the small semicircles. The ratio $A : B$ is $1 : 18$. What is N?



(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 36

Problem 8

Sara makes a staircase out of toothpicks as shown:



This is a 3-step staircase and uses 18 toothpicks. How many steps would be in a staircase that used 180 toothpicks?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 24 **(E)** 30

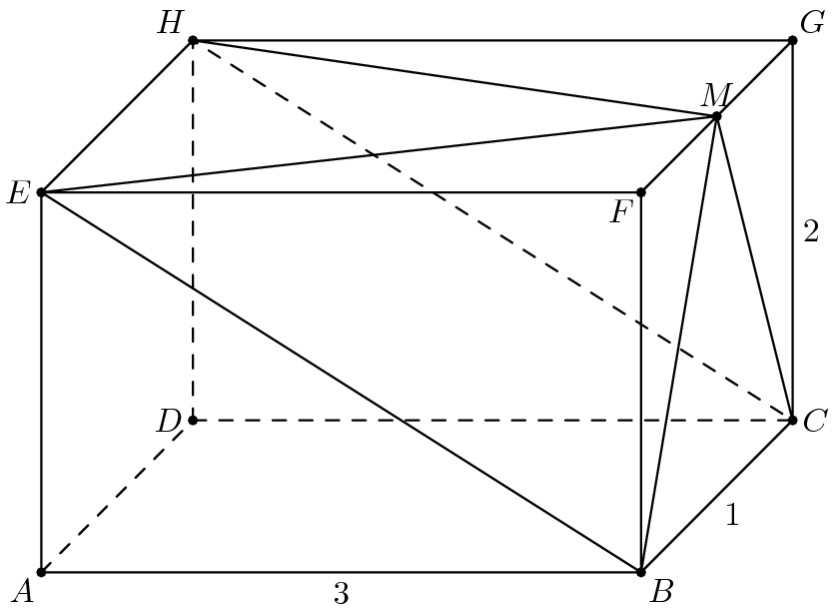
Problem 9

The faces of each of 7 standard dice are labeled with the integers from 1 to 6. Let p be the probability that when all 7 dice are rolled, the sum of the numbers on the top faces is 10. What other sum occurs with the same probability p?

(A) 13 **(B)** 26 **(C)** 32 **(D)** 39 **(E)** 42

Problem 10

In the rectangular parallelepiped shown, $AB = 3$, $BC = 1$, and $CG = 2$. Point M is the midpoint of $\overline{FG}$. What is the volume of the rectangular pyramid with base $BCHE$ and apex M?



(A) 1 **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{5}{3}$ **(E)** 2

EXTRA QUESTION

Which of these 10 questions was intriguing or interesting to you?
Which one "sticks" in your brain the most?

Discuss this question with your classmates and your teacher.

Are there different ways to answer it? (How do your colleagues think about it?)
Can you, yourself, find more than one way to approach it?

Could you design a similar question for your classmates to try?

Could you change some of the numbers or ideas in the question and make up a brand new idea to explore? Where can this question lead you?

And so you have them

Answers: 1.(A); 2.(D); 3.(B); 4.(B); 5.(D); 6.(D); 7.(D); 8.(C); 9.(D); 10.(E).

Curriculum Inspirations is brought to you by the Mathematical Association of America and the MAA American Mathematics Competitions.