Curriculum Inspirations
Inspiring students with rich content from the MAA American Mathematics Competitions
MAA

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TRY THE FIRST TEN PROBLEMS

AMC 12

Are you ever-so-slightly curious about the American Mathematics Competitions?

What does one look like?
Are they hard?
What does it mean to be "good" at one?
Do I have to be good at it?

The point of the AMC is to show students, like you, that there is lot of curious, interesting, and sometimes quirky mathematics to think about and play with. Sure, some people like the sense of "competition," but there are many others (and I am one of those "others") who aren't drawn to that element at all. That's okay!

So, here's an invitation: Try these first 10 problems from the 2020 AMC 12A competition. Have fun with them. See how they affect your brain and what new ideas they lead you to think about and wonder about. Just try them! And perhaps try the next upcoming AMC 12 competition too with the idea of just dwelling on the first 10 problems there and see which ones make you think and wonder.

Most people don't realize that making progress on the first 10 problems is actually a significant achievement!

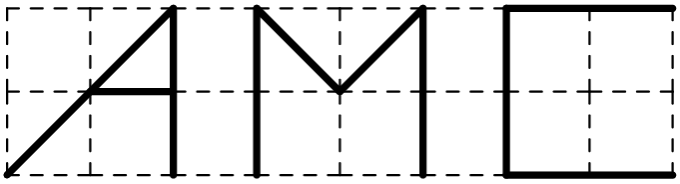
Just go for the first ten!

2020 AMC 12A
The First Ten

Problem 1
Carlos took 70% of a whole pie. Maria took one third of the remainder. What portion of the whole pie was left?

$\textbf{(A)}\ 10\%\qquad\textbf{(B)}\ 15\%\qquad\textbf{(C)}\ 20\%\qquad\textbf{(D)}\ 30\%\qquad\textbf{(E)}\ 35\%$

Problem 2
The acronym AMC is shown in the rectangular grid below with grid lines spaced 1 unit apart. In units, what is the sum of the lengths of the line segments that form the acronym AMC?



$\textbf{(A)}\ 17\qquad\textbf{(B)}\ 15+2\sqrt{2}\qquad\textbf{(C)}\ 13+4\sqrt{2}\qquad\textbf{(D)}\ 11+6\sqrt{2}\qquad\textbf{(E)}\ 21$

Problem 3
A driver travels for 2 hours at 60 miles per hour, during which her car gets 30 miles per gallon of gasoline. She is paid $\$0.50$ per mile, and her only expense is gasoline at $\$2.00$ per gallon. What is her net rate of pay, in dollars per hour, after this expense?

$\textbf{(A)}\ 20\qquad\textbf{(B)}\ 22\qquad\textbf{(C)}\ 24\qquad\textbf{(D)}\ 25\qquad\textbf{(E)}\ 26$

Problem 4
How many 4-digit positive integers (that is, integers between 1000 and 9999, inclusive) having only even digits are divisible by 5?

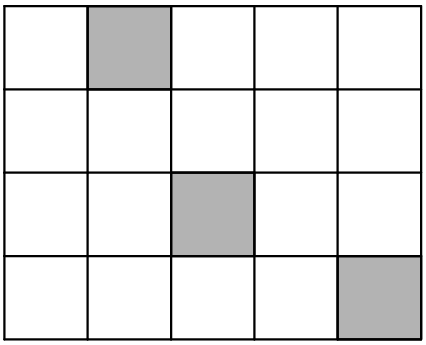
$\textbf{(A)}\ 80\qquad\textbf{(B)}\ 100\qquad\textbf{(C)}\ 125\qquad\textbf{(D)}\ 200\qquad\textbf{(E)}\ 500$

Problem 5
The 25 integers from -10 to 14, inclusive, can be arranged to form a 5-by-5 square in which the sum of the numbers in each row, the sum of the numbers in each column, and the sum of the numbers along each of the main diagonals are all the same. What is the value of this common sum?

$\textbf{(A)}\ 2\qquad\textbf{(B)}\ 5\qquad\textbf{(C)}\ 10\qquad\textbf{(D)}\ 25\qquad\textbf{(E)}\ 50$

Problem 6

In the plane figure shown below, 3 of the unit squares have been shaded. What is the least number of additional unit squares that must be shaded so that the resulting figure has two lines of symmetry?



$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Problem 7

Seven cubes, whose volumes are $1, 8, 27, 64, 125, 216,$ and 343 cubic units, are stacked vertically to form a tower in which the volumes of the cubes decrease from bottom to top. Except for the bottom cube, the bottom face of each cube lies completely on top of the cube below it. What is the total surface area of the tower (including the bottom) in square units?

$\textbf{(A)}\ 644 \qquad \textbf{(B)}\ 658 \qquad \textbf{(C)}\ 664 \qquad \textbf{(D)}\ 720 \qquad \textbf{(E)}\ 749$

Problem 8

What is the median of the following list of 4040 numbers?

$$1, 2, 3, \ldots, 2020, 1^2, 2^2, 3^2, \ldots, 2020^2$$

$\textbf{(A)}\ 1974.5 \qquad \textbf{(B)}\ 1975.5 \qquad \textbf{(C)}\ 1976.5 \qquad \textbf{(D)}\ 1977.5 \qquad \textbf{(E)}\ 1978.5$

Problem 9

How many solutions does the equation $\tan(2x) = \cos\left(\frac{x}{2}\right)$ have on the interval $[0, 2\pi]$?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 10

There is a unique positive integer n such that $\log_2\left(\log_{16} n\right) = \log_4\left(\log_4 n\right)$. What is the sum of the digits of n?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 13$

EXTRA QUESTION

Which of these 10 questions was intriguing or interesting to you?
Which one "sticks" in your brain the most?

Discuss this question with your classmates and your teacher.

Are there different ways to answer it? (How do your colleagues think about it?)
Can you, yourself, find more than one way to approach it?

Could you design a similar question for your classmates to try?

Could you change some of the numbers or ideas in the question and make up a brand new idea to explore? Where can this question lead you?

And so you have them

Answers: 1.(C); 2.(C); 3.(E); 4.(B); 5.(C); 6.(D); 7.(B); 8.(C); 9.(E); 10.(E).

Curriculum Inspirations is brought to you by the Mathematical Association of America and the MAA American Mathematics Competitions.