

Appendix 1

The CUPM Curriculum Initiative

Begun at Mathfest 1999 under the leadership of CUPM chair Tom Berger, the CUPM curriculum initiative focused on what students should know and experience as they complete their coursework in mathematics. The working assumptions of the initiative were (1) One curriculum is not appropriate for all majors; students' needs and goals have expanded, and the mathematical preparation of a diverse audience calls for a broader, more flexible major. (2) The mathematics program must serve a wide variety of mathematics-intensive majors and be responsive to the needs of other disciplines. (3) It must serve the quantitative literacy needs of a very large population often enrolled in, but not optimally served by, college algebra courses.

Gathering information

CUPM began its work on the *CUPM Guide 2004* by examining past CUPM recommendations. The 1981 recommendations recognized that many students wish to combine the study of mathematics with that of other disciplines in order to broaden their knowledge base and enhance their future career opportunities. The 1981 recommendations were reissued in 1988 in *Reshaping College Mathematics* (MAA Notes 13). The 1991 CUPM recommendations appear at the end of *Heeding the Call for Change* (MAA Notes 22). By this time, a list of courses for the major was no longer easy to state and the mathematics community was engaged in substantial discussions about calculus. Therefore, the 1991 curriculum document is brief. Since 1991, *Models That Work* (MAA Notes 38) described exemplary programs in mathematics, and *Confronting the Core Curriculum* (MAA Notes 45) addressed preparation in the first two years. The recent book on assessment, *Assessment Practices in Undergraduate Mathematics* (MAA Notes 49), examined a broad range of assessment issues and included models illustrating productive approaches. The recommendations of the Joint committee on Undergraduate Statistics of the American Statistical Association and the MAA also appear in *Heeding the Call for Change*. The recommendations of the American Mathematical Association of Two-Year Colleges appear in *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus*, AMATYC, 1995. All of these documents remain useful for departments planning their programs. They also serve as a base for the current report.

Since the summer of 1999, CUPM has gathered information directly from the profession in the following ways.

Meeting sessions and panels

- At Mathfest 1999, CUPM sponsored a panel/audience reaction session in front of an involved and packed audience.
- At the January 2000 Joint Meetings CUPM sponsored a panel/audience reaction session again to a packed room, plus two very well-attended contributed paper sessions.
- At Mathfest 2000 CUPM sponsored a panel of industry members commenting on the curriculum for a full and active audience.

- More panels were held at the Joint Meetings in January 2001, January 2002, and January 2003 and at August 2001.

Focus groups

- In January 2000, CUPM invited mathematicians to participate in a number of focus groups discussing curricular issues, clustering participants by institution type.
- More focus groups met in January 2001 (including groups addressing the CRAFTY Curriculum Foundations project reports—see Appendix 2), January 2002, August 2002 (including groups representing AMS and AMATYC), and January 2003.

Interdisciplinary Conferences

- See Appendix 2 on the CRAFTY Curriculum Foundations project.

Invited papers followed by a workshop discussion—September 2000

- With funding from the Calculus Consortium for Higher Education, CUPM solicited papers from a number of mathematicians asking them to address issues central to developing a planning document for departments. Writers, members of CUPM and a few others met in September 2000 for a workshop informed by and taking off from the issues raised in the papers. The invited papers appear in the January 2001 MAA Report *CUPM Discussion Papers about Mathematics and the Mathematical Sciences in 2010: What Should Students Know?* along with a summary of the deliberations at the workshop and the first tentative formulation of these recommendations. This MAA report is still available on MAA Online.

Surveys spring 2001

- Random sample of 300 departments—see Appendix 4.
- Selected sample of departments (the source of some of the examples in Illustrative Resources).

Details on the selected sample:

Questionnaires were sent to 300 selected mathematics departments. Thirty-seven were returned. Of the 37 responses, 10 indicated no data to share, 4 had some descriptive data to share. Of the remaining 23, nearly all were contacted for further information.

Forming the selected sample of 300 mathematics departments:

Thirty departments were from 2-year colleges: 9 from a list of NSF grantees, 12 from a list of FIPSE grantees, and 9 suggested by CUPM members.

The list of 270 departments at 4-year institutions was put together as follows. Working from a list of all mathematics departments provided by Jim Maxwell of the AMS, three different indices were constructed, each based on the ratio of the number of mathematics majors to the total undergraduate enrollment in 1996. The 1996 total was available for almost every department, but the numbers of majors (in 1996, 1998 and 1999) were available in different years for different institutions. The 1996 numbers were from a different survey than the 1998 and 1999 numbers and were available for more than twice as many institutions. A list was constructed of all departments ranking in the top ten percent on any one of the indices, which gave a list of 199 departments. (There were only 51 duplicates on the three lists, of 134, 56 and 60 departments calculated for numbers of majors in 1996, 1998 and 1999 respectively.)

The ratios of mathematics enrollment to total enrollment (since this is a measure of service) and of major enrollment to mathematics enrollment (a measure of recruiting success) were also exam-

ined, but these ratios could only be formed for 1998 and 1999 (and the latter measure gave a list very similar to the major to total ratio), so the ratio of majors to total undergraduate enrollment was used.

Additional departments suggested by CUPM members were added; many were already on the list. More departments were added from lists of NSF grantees and FIPSE grantees; all departments on the NSF list and all the university departments on the FIPSE were added, since relatively few universities were among those having a high major ratio. That brought the total to 300.

Involvement of partner disciplines and other professional societies

- Representatives of professional societies in engineering, physics, and economics attended the September 2000 Workshop. The presidents of MAA, SIAM, and AMATYC, and the chair of the AMS Committee on Education contributed papers and participated in the workshop.
- The AMS has representation on CUPM both through two designated AMS members on CUPM (currently Ramesh Gangolli and Diane Herrmann; previously Amy Cohen-Corwin and Naomi Fisher) and also through a liaison to the CUPM curriculum initiative (David Bressoud) appointed by the President of the AMS.
- AMATYC has a liaison to the CUPM curriculum initiative.
- The immediate past chair of the ASA/MAA Joint Committee on Undergraduate Statistics (Allan Rossman) serves on CUPM.
- The Curriculum Foundations project (Appendix 2) involved scores of representatives of mathematics-intensive disciplines.
- The CUPM curriculum initiative has an advisory committee that includes representatives of biology (Lou Gross), computer science (Peter Henderson) and engineering (David Bigio).
- Professional organizations were invited to form Association Review Groups in spring 2003 to provide comments on and reactions to draft 4.2 of the *Guide*. Reports and suggestions for improvement were received from the American Mathematical Association of Two Year Colleges, the American Mathematical Society, the American Statistical Association, the Institute of Electrical and Electronic Engineers Computer Society, the National Council of Teachers of Mathematics, and the Society for Industrial and Applied Mathematics.

Reports of other MAA committees and taskforces

In addition to CRAFTY's Curriculums Foundations project, this *Guide* has been informed by the work of the Committee on the Mathematical Education of Teachers and the report of the Conference Board of the Mathematical Sciences, *Mathematical Education of Teachers*, and reports from and information gathered by the Committee on the Teaching of Undergraduate Mathematics, the Committee on Computers in Mathematics Education, the CUPM subcommittee on Quantitative Literacy Requirements, the Committee on Articulation and Placement, and the informal working group on the first college course.

The steering committee and advisory committee

The project has been guided by a steering committee consisting of William Barker, Thomas Berger, David Bressoud (AMS representative), Susanna Epp, William Haver, Herbert Kasube and Harriet Pollatsek (chair). A larger advisory committee also includes James Lewis (chair of the MAA Coordinating Council on Education), and the members of other disciplines listed earlier. The associate directors of the MAA, first Tom Rishel and now Michael Pearson, have provided staff support.

The writing process

A writing group chaired by Harriet Pollatsek and consisting of William Barker, David Bressoud, Susanna Epp, Susan Ganter, and Bill Haver, was formed in January 2001. It developed a document for discussion at the August 2001 and January 2002 MAA meetings. In the spring and summer of 2002, with assistance from Barry Cipra, the group prepared a preliminary draft report for discussion at the August 2002 MAA meeting. Using feedback from the focus groups at that meeting, the members of the writing group began intensive work on the *CUPM Guide 2004*, which they wrote between September 2002 and August 2003. Help with assembling and editing the Illustrative Resources for CUPM Guide 2004 was provided by Kathleen Snook in the winter and spring of 2003. The writing and distribution of the *CUPM Guide 2004* was supported by grants from the National Science Foundation (DUE-0218773) and the Calculus Consortium for Higher Education.

Appendix 2

The Curriculum Foundations Project

The CUPM subcommittee *Curriculum Renewal Across the First Two Years* (CRAFTY⁸⁷) has gathered input from partner disciplines through a series of eleven workshops held across the country from November 1999 to February 2001, followed by a final summary conference in November 2001. Each Curriculum Foundations workshop consisted of 20–35 participants, the majority chosen from the discipline under consideration, the remainder chosen from mathematics. The workshops were not intended to be discussions between mathematicians and colleagues in the partner disciplines, although this certainly happened informally. Instead, each workshop was a dialogue among the representatives from the partner discipline, with mathematicians present only to listen and serve as resources when questions arose about the mathematics curriculum.

Each workshop produced a report summarizing its recommendations and conclusions. The reports were written by representatives of the partner disciplines and directed to the mathematics community. This insured accurate reporting of the workshop discussions while also adding credibility to the recommendations. Uniformity was achieved across the reports through a common set of questions that was used to guide the discussions at each workshop (see the end of this appendix). Having these common questions also made it easier to compare the recommendations from different disciplines.

Funding for most workshops was provided by the host institutions.⁸⁸ Such financial support—obtained with little advance notice—indicates the high level of support from university administrations for such interdisciplinary discussions about the mathematics curriculum. Workshop participants from the partner disciplines were extremely grateful—and surprised—to be invited by mathematicians to state their views about the mathematics curriculum. That their opinions were considered important and would be taken seriously in the development of the *CUPM Curriculum Guide* only added to their enthusiasm for the project as well as their interest in continuing conversations with the mathematics community.

In November 2001, invited representatives from each disciplinary workshop gathered at the U.S. Military Academy in West Point, NY for a final Curriculum Foundations Conference. The discussions resulted in *A Collective Vision*, a set of commonly shared recommendations for the first two years of undergraduate mathematics instruction.

All of the Curriculum Foundations reports, along with the *Collective Vision* recommendations, have been published as an MAA Report.⁸⁹ These disciplinary reports and *Collective Vision* were heavily drawn

⁸⁷Formerly Calculus Renewal And The First Two Years, but renamed when its charge widened.

⁸⁸The only exceptions were the two workshops on technical mathematics, which were hosted by two-year institutions and funded by the National Science Foundation, and the workshop on statistics, which was mostly funded by the American Statistical Association.

⁸⁹*The Curriculum Foundations Project: Voices of the Partner Disciplines*. Electronic versions of these materials are also available for downloading from www.maa.org/cupm/crafty.

upon during the construction of the *CUPM Curriculum Guide*. However, the workshop reports and *Collective Vision* have value independent of the *Guide*: they can and should serve as resources for starting multi-disciplinary discussions at individual institutions. Promoting and supporting informed interdepartmental discussions about the undergraduate curriculum might ultimately be the most important outcome of the Curriculum Foundations project.

The Curriculum Foundations Workshops

Physics and Computer Science

Bowdoin College, Maine, October, 1999
William Barker,

Interdisciplinary (Math, Physics, Engineering)

USMA, West Point, November, 1999
Don Small,

Engineering

Clemson University, May, 2000
Susan Ganter,

Health-related Life Sciences

Virginia Commonwealth University, May, 2000
William Haver,

Technical Mathematics (at two sites)

Los Angeles Pierce College, California, October, 2000
Bruce Yoshiwara,
J. Sargeant Reynolds CC, Virginia, October, 2000
Susan Wood,
Mary Ann Hovis,

Statistics

Grinnell College, October, 2000
Thomas Moore,

Business, Finance and Economics

University of Arizona, October, 2000
Deborah Hughes Hallett,
William McCallum,

Mathematics Education

Michigan State University, November, 2000
Sharon Senk,

Biology and Chemistry

Macalester College, November, 2000
David Bressoud,

Mathematics Preparation for the Major

Mathematical Sciences Research Institute, February, 2001
William McCallum,

Questions Provided at Disciplinary Workshops

Understanding and Content

- What conceptual mathematical principles must students master in the first two years?
- What mathematical problem solving skills must students master in the first two years?
- What broad mathematical topics must students master in the first two years? What priorities exist between these topics?
- What is the desired balance between theoretical understanding and computational skill? How is this balance achieved?
- What are the mathematical needs of different student populations and how can they be fulfilled?

Instructional Techniques

- What are the effects of different instructional methods in mathematics on students in your discipline?
- What instructional methods best develop the mathematical comprehension needed for your discipline?
- What guidance does educational research provide concerning mathematical training in your discipline?

Technology

- How does technology affect what mathematics should be learned in the first two years?
- What mathematical technology skills should students master in the first two years?
- What different mathematical technology skills are required of different student populations?

Instructional Interconnections

- What impact does mathematics education reform have on instruction in your discipline?
- How should education reform in your discipline affect mathematics instruction?
- How can dialogue on educational issues between your discipline and mathematics best be maintained?

Appendix 3

Data on Numbers of Majors

The declining annual number of bachelor's degrees in mathematics

As noted in the introduction, the annual number of degrees in the mathematical sciences has remained overall flat and the number of degrees in mathematics has fallen during a time when the numbers of students earning degrees in science, engineering and technology is growing. Table 3-1 shows data on the number of degrees in the mathematical sciences from the Fall 2000 CBMS Survey.⁹⁰

Table 3-1. *Number of bachelor's degrees awarded by departments of mathematics and statistics in academic years 1979–80, 1984–85, 1989–90, 1994–95, 1999–2000.*

| Academic year | 79–80 | 84–85 | 89–90 | 94–95 | 99–00 |
|----------------------|--------|--------|--------|--------|--------|
| Math | 11,541 | 13,171 | 13,303 | 12,456 | 10,759 |
| Math Ed | 1,752 | 2,567 | 3,116 | 4,829 | 4,991 |
| Subtotal | 13,293 | 15,738 | 16,419 | 17,285 | 15,750 |
| Stat-related* | 613 | 659 | 987 | 1,839 | 1,123 |
| Total** | 13,90 | 19,237 | 19,380 | 20,154 | 19,299 |

*Stat-related majors include statistics, actuarial mathematics and joint mathematics/statistics majors. Of 664 statistics majors in 99–00, 394 (59%) were awarded by statistics departments; all 425 actuarial degrees in 99–00 were awarded by mathematics departments.

** The additional majors contributing to the total awarded by mathematics and statistics departments are operations research, computer science, joint mathematics/computer science, and “other” (a mixed category growing from 0 in 1979–80 to 1,507 in 1999–2000).

The data in Table 3-1 show strong growth in statistics-related degrees from 1980 until 1995, but then a 39% decline in the most recent five year period. Much of the growth in degrees awarded annually by mathematics and statistics departments was in mathematics education, which rose almost 55% from 1990 to 1995 but then remained essentially unchanged. The subtotal of mathematics and mathematics education degrees grew 30% from 1980 to 1995, but then it fell, dropping 9% in the most recent period. The annual number of degrees in mathematics grew 15 % from 1980 to 1990, but then it fell 19% in the next decade.

The serious shortage of teachers of mathematics in secondary schools makes the data on majors especially troubling. In 2000 the number of new graduates who were interested in secondary teaching fell far short of the need for mathematics teachers. The American Association for Employment in Education studied teacher supply and demand in 2000. They surveyed all institutions of higher education listed in the Higher

⁹⁰*Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States: Fall 2000 CBMS Survey*, D.J. Lutzer, J.W. Maxwell, S.R. Rodi, AMS, 2002, Table SE 4, p. 14. Data from other sources vary somewhat. For a discussion of this variation, see “Mathematics Majors 2002,” D.J. Lutzer, *Notices of the AMS*, **50** (February 2003).

Education Directory as preparing teachers. Respondents (deans and career service representatives) rated the job market for their graduates in different fields from 1 (many more applicants than jobs) to 5 (vice versa). For mathematics education, the national average was 4.44 (up from 4.18 in 1999) and varied from 3.5 in Alaska to 4.85 in the Rocky Mountain states.⁹¹ An American Federation of Teachers survey of certified applicants to school districts in 1998–99 obtained similar results. Using the same 1–5 scale as the AAEE study, the AFT average for mathematics was 4.51.⁹²

The authors of *Models that Work* point out that in the 1960s, 5% of freshmen entering colleges and universities were interested in mathematics, and 2% majored in mathematics, and this led some to the conclusion that departments were “filtering” prospective mathematics majors. Both percentages have fallen. The percentage of mathematics degrees among all bachelor’s degrees was 1.54% in 1985, 1.33% in 1991, 1.17% in 1995 and 1.05% in 1998, based on NSF data.⁹³ For the past twenty years, the percentage of entering freshmen intending to major in mathematics has been smaller than the percentages graduating with majors in the discipline; currently the percentage is about 0.6%. Therefore, some students have been making their decision to major in mathematics during the first two years of college-level study. An optimistic interpretation of the data suggests that introductory mathematics courses have been awakening interest and encouraging students to consider majors. But departments are up against a stiff challenge, because the numbers of students entering college interested in mathematics is low, especially among groups traditionally under-represented in mathematics. Table 3-2 shows NSF figures for mathematics and, for comparison, for computer science.⁹⁴

Table 3-2. *Percent of freshmen intending to major in mathematics/statistics (MA) or in computer science (CS), by race and gender*

| Year | 1975 | 1980 | 1985 | 1990 | 1995 | 2000 |
|-----------------|------|------|------|------|------|------|
| Whites | | | | | | |
| All (MA) | 1.5 | 0.9 | 1.1 | 0.9 | 0.7 | 0.7 |
| All (CS) | 0.8 | 2.4 | 2.0 | 1.3 | 1.8 | 3.0 |
| Females (MA) | 1.5 | 0.9 | 1.1 | 0.8 | 0.7 | 0.6 |
| Females (CS) | 0.6 | 2.0 | 1.2 | 0.7 | 0.6 | 0.8 |
| Blacks | | | | | | |
| All (MA) | 0.8 | 0.8 | 0.7 | 0.5 | 0.7 | 0.5 |
| All (CS) | 0.7 | 4.0 | 6.4 | 3.9 | 4.6 | 6.0 |
| Females (MA) | 0.7 | 0.7 | 0.6 | 0.5 | 0.7 | 0.5 |
| Females (CS) | 0.8 | 3.9 | 5.9 | 3.5 | 3.7 | 4.2 |
| Hispanic | | | | | | |
| All (MA) | 1.8 | 0.8 | 0.7 | 0.8 | 0.5 | 0.6 |
| All (CS) | 1.0 | 2.4 | 2.6 | 1.5 | 2.2 | 2.7 |
| Females (MA) | 1.3 | 1.0 | 0.5 | 0.8 | 0.5 | 0.5 |
| Females (CS) | 0.6 | 2.8 | 1.7 | 1.2 | 1.1 | 1.1 |

While the percentage of White students entering college intending to major in mathematics held steady from 1995 to 2000, the percentage of females and of Blacks continued to fall. The percentages intending to major in computer science have steadily increased (except for White and Hispanic females).

⁹¹ See www.ub-careers.buffalo.edu/aaee/S_DReport2000.pdf.

⁹² See www.aft.org/research/survey/figures/, Figure IV-1

⁹³ From NSF *Science and Engineering Indicators 2002*, quoted in “Mathematics Majors 2002,” D.J. Lutzer, *Notices of the AMS*.

⁹⁴ www.nsf.gov/sbe/srs Science and Engineering Indicators 2002, Appendix Table 2-11. “Hispanic” refers to Mexican American, Chicano, and Puerto Rican American.

Some of these racial and gender patterns are also visible in the number of degrees in mathematics and computer science. Tables 3-3a and 3-3b⁹⁵ show that both disciplines have improved the representation of these groups, and that women make up a much higher proportion of bachelor's degrees in mathematics than in computer science, while Blacks make up a slightly higher proportion of degrees in computer science than in mathematics. A cautionary note on these tables: some computer science degrees (about 3300 per year according to CBMS2000) are granted by mathematical sciences departments.

Table 3-3a. *Annual number (percent of total) of bachelor's degrees in mathematics*

| Year | 1977 | 1998 |
|---------------------------|---------------|---------------|
| Total | 14,303 | 12,363 |
| Women | 5,949 (41.6%) | 5,659 (45.8%) |
| Blacks | 712 (5.0%) | 1,030 (8.3%) |
| Hispanic Americans | 321 (2.2%) | 642 (5.2%) |

Table 3-3b. *Annual number (percent) of bachelor's degrees in computer science*

| Year | 1977 | 1998 |
|---------------------------|-------------|---------------|
| All | 6,426 | 27,674 |
| Women | 956 (14.9%) | 7,439 (26.9%) |
| Blacks | 361 (5.6%) | 2,580 (9.3%) |
| Hispanic Americans | 114 (1.8%) | 1,410 (5.1%) |

The annual number of women earning bachelor's degrees in mathematics fell from 1977 to 1998, but the annual number of White men earning these degrees fell even more, so the percentage of female degree recipients rose. For the same period, the annual number of computer science degrees grew more than seven-fold for women and for Blacks and more than twelve-fold for Hispanics; percentages grew as well. For Blacks and Hispanics, the number and percentage of mathematics degrees also grew, but much less dramatically. It should be added that more recent data show the annual number of majors in computer science is leveling off and that interest in computer science among entering students is declining.⁹⁶

⁹⁵www.nsf.gov/sbe/srs Science and Engineering Indicators 2002, Table 2-17.

⁹⁶“Tech’s Major Decline: College Students Turning Away from Bits and Bytes, E. McCarthy, *Washington Post*, August 27, 2002, p. E01; www.washingtonpost.com.

Appendix 4

Data on Student Goals, Department Practices, and Advanced Courses

The increasing diversity of student interests and goals

Mathematics majors—like all post-secondary students—are more diverse than they were even 30 years ago. The 1997 National Survey of Recent College Graduates⁹⁷ found that 34% of bachelor’s degree recipients in the mathematical and related sciences also attended a two-year college. The annual number of degrees awarded by two-year colleges increased by 24% from 1985 to 2000.⁹⁸ While enrollments in remedial courses at four-year colleges and universities fell 13% during the same period,⁹⁹ they rose 58% at two-year colleges. The two-year institutions thus play a critical role in preparing students for mathematics-intensive majors, including mathematical sciences majors.

At many institutions a large fraction of the undergraduates work at part-time or sometimes even full-time jobs while they are going to school. Even with jobs, many students graduate with a substantial debt, and that fact may influence their choices of academic programs and career paths. English is a second language for many students.

Some students arrive in college and university classrooms well-prepared, highly interested in mathematics and intending doctoral study in mathematics. Other students are less intrinsically interested in mathematics and less confident of their mathematical abilities; they may choose to major in mathematics because of its applicability in other disciplines or because it offers the promise of employment opportunities. The 1997 National Survey of Recent College Graduates found that 92% of students earning bachelor’s degrees in the mathematical sciences during the academic years 1994–95 and 1995–96 went directly into the workforce after graduation.¹⁰⁰ Data from 2002 show that approximately 3.8% of US citizen mathematics majors go on to receive doctoral degrees in the mathematical sciences within 6 years.¹⁰¹

⁹⁷Every year the NSF publishes a National Survey of Recent College Graduates, sampling those who completed their degrees during the preceding two academic years. The 1997 survey was based on those who received degrees between July 1, 1994 and June 30, 1996. The survey separately tracks degrees in mathematical and related sciences and in computer and information sciences. The 1997 study was based on 26,800 bachelor’s degrees in mathematical and related sciences. See www.nsf.gov/sbe/srs/nsf01337, Table B3. Table D1 on p. 92, Table D5 on p. 97.

⁹⁸www.nces.ed.gov website, Table 247.

⁹⁹CBMS 2000, Table SE 3.

¹⁰⁰See Table D1 on p. 92 and Table D5 on p. 97.

¹⁰¹“Mathematics Majors 2002,” D.J. Lutzer, *Notices of the AMS*.

The CUPM recommendations in 1981 and 1991 especially cautioned departments to design their programs to be responsive to the needs of the overwhelming majority of majors headed for the workforce rather than graduate study in mathematics.

Department practices

In 1990, a task force of the MAA and the Association of American Colleges examined the practices of mathematical sciences departments at many institutions.¹⁰² They found a number of common features, including “a multiple track system that addressed diverse student objectives, emphasis on breadth of study in the major, and requirements for depth that help students achieve critical sophistication.” The 1991 CUPM report in fact recommended exactly these practices.

In the spring of 2001, CUPM collected information from a random sample of 300 mathematics departments offering a bachelor’s degree. The CUPM sample of mathematical sciences departments was stratified according to the Carnegie classification of institutions: Pub xy for public and Pri xy for private, where $x = 1-3$ denotes the highest degree offered (1 = doctorate, 2 = master’s, 3 = bachelor’s) and $y = 1-4$ denotes the Carnegie rank of the institution (1 is most highly rated). Thirty percent of the questionnaires sent to the random sample of departments were returned. Table 4-1 below shows the response rates for different classes of institutions.

In the responding departments, 30% of the majors earning bachelor’s degrees in 2001 were either *double* majors (completing two full majors) or *joint* majors (completing a coordinated program that is less than two full majors), so linkages with other disciplines were highly significant. In fact, about two thirds of the interdisciplinary majors reported were double majors. Obviously, such cross-disciplinary choices are more difficult if the mathematics major requires a large number of courses and only double majors are possible.

Table 4-1. *Questionnaires returned from the CUPM random sample of departments, spring 2001.*

| | Carnegie class. | received/sent | percent |
|--------------|-----------------|---------------|---------|
| Pri | 11 | 2/7 | 29% |
| | 12 | 0/2 | 0% |
| | 13 | 0/10 | 0% |
| | 14 | 4/5 | 80% |
| | 21 | 16/43 | 37% |
| | 22 | 5/15 | 33% |
| | 31 | 17/35 | 49% |
| | 32 | 17/78 | 22% |
| Pub | 11 | 6/14 | 43% |
| | 12 | 2/6 | 33% |
| | 13 | 3/6 | 50% |
| | 14 | 0/8 | 0% |
| | 21 | 16/54 | 30% |
| | 22 | 1/5 | 20% |
| | 31 | 0/1 | 0% |
| | 32 | 2/16 | 13% |
| Total | | 91/305 | 30% |

¹⁰²From “Challenges for College Mathematics: An Agenda for the Next Decade, the report of the MAA-AAC Taskforce on Study in Depth, 1990,” reprinted in *Heeding the Call for Change: Suggestions for Curricular Action*, MAA Notes 22, edited by Lynn Steen, p. 188.

CUPM also collected data on major requirements at 40% of the departments in the random sample.¹⁰³ Information on major requirements was obtained from web pages (plus some follow-up phone calls) for 120 departments in the random sample.¹⁰⁴ Because the data collection does not cover all of the random sample, the findings must be treated with caution. Note that if a department required “algebra or analysis” it was counted as specifically requiring neither. Overall 79% of the departments require abstract algebra of all of their majors and 78% require analysis. But it is striking that 100% of the highly ranked doctoral departments require algebra, and 95% require analysis. The percent requiring analysis varied more from one kind of institution to another, ranging from 100% at the highest rated public doctoral departments to 40% at the highest rated public master’s level departments. See Table 4-2 below.

Table 4-2. *Percent of departments of the specified type requiring the course of all of their majors, based on 120 departments in the CUPM 2001 random sample.*

| Carnegie Class. (no.) | Alg 1 | Alg 2 | Ana 1 | Ana 2 | Geo | Pro | Mod | Sem |
|------------------------------|--------------|--------------|--------------|--------------|------------|------------|------------|------------|
| Pub 11 (14) | 100% | 64% | 100% | 100% | 21% | 0% | 7% | 0% |
| Pri 11 (7) | 100% | 71% | 86% | 86% | 29% | 14% | 0% | 0% |
| All 11 (21) | 100% | 67% | 95% | 95% | 24% | 5% | 5% | 0% |
| All 12 (8) | 75% | 75% | 88% | 88% | 38% | 50% | 13% | 25% |
| Pub 21 (40/54) | 70% | 5% | 40% | 8% | 8% | 13% | 8% | 30% |
| Pub 32 (16) | 69% | 63% | 69% | 69% | 19% | 38% | 13% | 13% |
| Pri 31 (35) | 83% | 49% | 89% | 89% | 6% | 14% | 3% | 20% |
| Total (120) | 79% | 41% | 78% | 60% | 13% | 18% | 7% | 19% |

Alg = abstract algebra; Ana = analysis or advanced calculus; Geo = geometry; Prob = probability (or, in some cases a single probability/statistics course); Mod = mathematical modeling; Res = one or more of project, research, internship or senior seminar. If a department required “algebra or analysis” it was counted as requiring neither. No distinction was made between semester and quarter courses.

Advanced courses

The variation among institutions of different kinds found in the 2001 CUPM sample data mirrors the results of the 2000 CBMS survey on the percentage of departments offering certain advanced courses. See Table 4-3 below. The CBMS 2000 survey found that, with the single exception of number theory, every advanced course tracked was *less* available in 2000–2001 than in 1995–96. (CBMS did not collect data on the availability of statistics courses in 1995–96.) Their survey also found that a wider array of advanced courses was available in doctoral level departments than in master’s or, especially, bachelor’s degree level departments.

Percentages below 50% in the CBMS data—which occur for at least one kind of institution for every course except algebra—suggest that there may be departments not even offering these courses in alternate years, so the courses are completely unavailable to their students. Also, between 1995–96 and 2000–01, the percentage of all departments offering advanced courses in these six topics declined significantly, with

¹⁰³ Because of limitations of time, it was not possible to examine all 300 departments. The higher ranking departments were included in every case, which may bias the results. The omitted groups are the public and private doctoral institutions of ranks 3 and 4, the public and private master’s institutions of rank 2, the private bachelor’s degree institutions of rank 2, and the single rank 1 public bachelor’s degree institution in the sample. Also just 40 of 54 departments were included in classification Pub 21; other categories included were complete.

¹⁰⁴ CUPM thanks Anna Zwahlen-Tronick and Muluwork Geremew, student workers at Colby and Mount Holyoke Colleges respectively, for their contributions to this data collection and analysis.

an 8% decline in the availability of modern algebra, and declines from 19% to 46% in the availability of analysis, geometry, modeling, operations research and combinatorics. Note that this is not a description of what is required of majors but rather of what is *available* to students.

Table 4-3. *Percent of departments offering certain advanced courses in 1995–96 and 2000–2001, by kind of department.*¹⁰⁵

| | in 95–96 | in 00–01 | in 00–01 | | |
|-----------------------------|----------|----------|----------|-----|------|
| | all | all | PhD | MA | BA |
| # depts | 1369 | 1430 | 187 | 233 | 1010 |
| algebra | 77% | 71% | 87% | 88% | 63% |
| analysis^a | 70% | 56% | 90% | 77% | 45% |
| geometry | 69% | 56% | 75% | 88% | 46% |
| models^m | 35% | 24% | 51% | 51% | 13% |
| OR^o | 24% | 13% | 14% | 26% | 10% |
| combin.^c | 24% | 18% | 48% | 24% | 11% |

a: advanced calculus or real analysis
o: operations research

m: mathematical modeling or applied mathematics
c: combinatorics

One might assume that diminished availability of algebra or analysis reflects a shift to various applied tracks within the major, but note the decline in availability of modeling and operations research courses. Also both algebra and analysis are recommended for prospective secondary teachers, and their availability has fallen despite the increase in the number of bachelor's degrees in mathematics education.

Not only has the availability of advanced courses declined, the enrollment in advanced courses has also declined, as Table 4-4 shows.

Table 4-4. *Fall enrollment in advanced courses in mathematics and statistics*¹⁰⁶

| Enrollments in 1000s | 1980 | 1985 | 1990 | 1995 | 2000 |
|-----------------------------|------|------|------|------|------|
| Advanced mathematics | 91 | 138 | 120 | 96 | 102 |
| Upper statistics | | | | | |
| math depts | | | 38 | 28 | 35 |
| math+stat depts | 43 | 63 | 52 | 44 | 45 |

Enrollments in both mathematics and statistics courses recovered somewhat in 2000 from the sharp declines in 1995, although not to their 1990 levels. A study in 1989 by Garfunkel and Young examined enrollments in courses in departments other than mathematics that consisted entirely or mainly of advanced mathematics.¹⁰⁷ (By advanced they meant having a prerequisite at or above the level of calculus.) These courses were offered not only in departments of physics, engineering, computer science, but also in economics, political science, biology, and management. Based on a sample of 425 schools they determined that each year there are over 170,000 enrollments in advanced mathematics courses being taught outside mathematics and statistics departments. It is hard to compare to the CBMS data, partly because it's full year versus fall only and partly because the definitions of advanced are not the same. As a rough estimate, this would mean that the number of student enrollments in advanced mathematical and

¹⁰⁵CBMS 2000, Table SE.5, p. 16.

¹⁰⁶CBMS 2000, Tables A1 and A2, pp. 178, 180.

¹⁰⁷See "Mathematics Outside Mathematics Departments," SA Garfunkel and GS Young, *Notices of the AMS*, **37** (April 1990) for a summary of their COMAP study, and a follow-up article "The Sky is Falling," in the *Notices* **45** (February 1998).

statistical courses taught outside of mathematics and statistics departments was about 70% of student enrollments in advanced courses taught inside of mathematics and statistics departments. No follow-up of the Garfunkel and Young study has been published, but anecdotal information suggests the proportion of advanced mathematics taught outside mathematics and statistics departments in 2000 would have been at least as large.

The decline in undergraduate geometry courses

Geometric/visual thinking is an important strand in many areas of mathematics, including contemporary applied mathematics—problems rooted in geometric questions are central to the development of new technologies such as medical imaging and robotics. However, there has been a dramatic decline in undergraduate offerings in geometry; see Table 4-5 below.¹⁰⁸ Further, solid geometry has long been gone from the high school curriculum.

Table 4-5. *Fall enrollments in selected undergraduate courses, 1970–2000*

| Enrollments in 1000's | 1970 | 1980 | 1985 | 1990 | 1995 | 2000 |
|-------------------------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Calculus level | 413 | 590 | 637 | 647 | 539 | 570 |
| Discrete math (intro) | na | na | 14 | 17 | 16 | 20 |
| Modern algebra I, II | 23 | 10 | 13 | 12 | 13 | 11 |
| Discrete structures | na | na | 7 | 3 | 3 | 5 |
| Geometry | 13 | 4 | 7 | 8 | 6 | 6 |
| Total adv. mathematics | | 91 | 138 | 120 | 96 | 102 |
| Total mathematics | | 1525 | 1619 | 1621 | 1471 | 1614 |

Table 4-3 on the availability of advanced courses supports the point about the decline in geometry offerings: the percentage of departments offering geometry courses has fallen from 69% to 56% from 1995–96 to 2000–2001. Also, Table 4-5, on enrollments in advanced courses, shows geometry enrollments falling along with the decline in total enrollments in advanced mathematics courses from 1985 to 1995 and then remaining flat (at almost half the 1970 enrollment) from 1995 to 2000. Notice also that both total enrollments and enrollments in advanced courses rose from 1995 to 2000, but the percentage of mathematics enrollments at the advanced level fell slightly, from 6.5% to 6.3%.

¹⁰⁸Data on undergraduate enrollment data from Table A-1 on pp. 127 ff. of *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States, Fall 1995 CBMS Survey* and Table A-1 on pp. 176 ff of *2000 CBMS Survey*.

Appendix 5

Summary of Recommendations

Part I. Recommendations for departments, programs and all courses in the mathematical sciences

1. Understand the student population and evaluate courses and programs

Mathematical sciences departments should

- Understand the strengths, weaknesses, career plans, fields of study, and aspirations of the students enrolled in their courses;
- Determine the extent to which the goals of courses and programs offered are aligned with the needs of students, as well as the extent to which these goals are achieved;
- Continually strengthen courses and programs to better align with student needs, and assess the effectiveness of such efforts.

2. Develop mathematical thinking and communication skills

Every course should incorporate activities that will help all students progress in developing analytical, critical reasoning, problem-solving, and communication skills and acquiring mathematical habits of mind. More specifically, these activities should be designed to advance and measure students' progress in learning to

- State problems carefully, modify problems when necessary to make them tractable, articulate assumptions, appreciate the value of precise definition, reason logically to conclusions, and interpret results intelligently;
- Approach problem solving with a willingness to try multiple approaches, persist in the face of difficulties, assess the correctness of solutions, explore examples, pose questions, and devise and test conjectures;
- Read mathematics with understanding and communicate mathematical ideas with clarity and coherence through writing and speaking.

3. Communicate the breadth and interconnections of the mathematical sciences

Every course should strive to

- Present key ideas and concepts from a variety of perspectives;
- Employ a broad range of examples and applications to motivate and illustrate the material;
- Promote awareness of connections to other subjects (both in and out of the mathematical sciences) and strengthen each student's ability to apply the course material to these subjects;
- Introduce contemporary topics from the mathematical sciences and their applications, and enhance student perceptions of the vitality and importance of mathematics in the modern world.

4. Promote interdisciplinary cooperation

Mathematical sciences departments should encourage and support faculty collaboration with colleagues from other departments to modify and develop mathematics courses, create joint or cooperative majors, devise undergraduate research projects, and possibly team-teach courses or units within courses.

5. Use computer technology to support problem solving and to promote understanding

At every level of the curriculum, some courses should incorporate activities that will help all students progress in learning to use technology

- Appropriately and effectively as a tool for solving problems;
- As an aid to understanding mathematical ideas.

6. Provide faculty support for curricular and instructional improvement

Mathematical sciences department and institutional administrators should encourage, support and reward faculty efforts to improve the efficacy of teaching and strengthen curricula.

Part II. Additional Recommendations Concerning Specific Student Audiences

A. Students taking general education or introductory collegiate courses in the mathematical sciences

General education and introductory courses enroll almost twice as many students as all other mathematics courses combined.¹⁰⁹ They are especially challenging to teach because they serve students with varying preparation and abilities who often come to the courses with a history of negative experiences with mathematics. Perhaps most critical is the fact that these courses affect life-long perceptions of and attitudes toward mathematics for many students—and hence many future workers and citizens. For all these reasons these courses should be viewed as an important part of the instructional program in the mathematical sciences.

This section concerns the student audience for these entry-level courses that carry college credit.

A.1. Offer suitable courses

All students meeting general education or introductory requirements in the mathematical sciences should be enrolled in courses designed to

- Engage students in a meaningful and positive intellectual experience;
- Increase quantitative and logical reasoning abilities needed for informed citizenship and in the workplace;
- Strengthen quantitative and mathematical abilities that will be useful to students in other disciplines;
- Improve every student's ability to communicate quantitative ideas orally and in writing;
- Encourage students to take at least one additional course in the mathematical sciences.

¹⁰⁹ According to the CBMS study in the Fall of 2000, a total of 1,979,000 students were enrolled in courses it classified as "remedial" or "introductory" with course titles such as elementary algebra, college algebra, Pre-calculus, algebra and trigonometry, finite mathematics, contemporary mathematics, quantitative reasoning. The number of students enrolled in these courses is much greater than the 676,000 enrolled in calculus I, II or III, the 264,000 enrolled in elementary statistics, or the 287,000 enrolled in all other undergraduate courses in mathematics or statistics. At some institutions, calculus courses satisfy general education requirements. Although calculus courses can and should meet the goals of Recommendation A.1, such courses are not the focus of this section.

A.2. Examine the effectiveness of college algebra

Mathematical sciences departments at institutions with a college algebra requirement should

- Clarify the rationale for the requirement and consult with colleagues in disciplines requiring college algebra to determine whether this course—as currently taught—meets the needs of their students;
- Determine the aspirations and subsequent course registration patterns of students who take college algebra;
- Ensure that the course the department offers to satisfy this requirement is aligned with these findings and meets the criteria described in A.1.

A.3. Ensure the effectiveness of introductory courses

General education and introductory courses in the mathematical sciences should be designed to provide appropriate preparation for students taking subsequent courses, such as calculus, statistics, discrete mathematics, or mathematics for elementary school teachers. In particular, departments should

- Determine whether students that enroll in subsequent mathematics courses succeed in those courses and, if success rates are low, revise introductory courses to articulate more effectively with subsequent courses;
- Use advising, placement tests, or changes in general education requirements to encourage students to choose a course appropriate to their academic and career goals.

B. Students majoring in partner disciplines

Partner disciplines vary by institution but usually include the physical sciences, the life sciences, computer science, engineering, economics, business, education, and often several social sciences.¹¹⁰ It is especially important that departments offer appropriate programs of study for students preparing to teach elementary and middle school mathematics. Recommendation B.4 is specifically for these prospective teachers.

B.1. Promote interdisciplinary collaboration

Mathematical sciences departments should establish ongoing collaborations with disciplines that require their majors to take one or more courses in the mathematical sciences. These collaborations should be used to

- Ensure that mathematical sciences faculty cooperate actively with faculty in partner disciplines to strengthen courses that primarily serve the needs of those disciplines;
- Determine which computational techniques should be included in courses for students in partner disciplines;
- Develop new courses to support student understanding of recent developments in partner disciplines;
- Determine appropriate uses of technology in courses for students in partner disciplines;
- Develop applications for mathematics classes and undergraduate research projects to help students transfer to their own disciplines the skills learned in mathematics courses;
- Explore the creation of joint and interdisciplinary majors.

B.2. Develop mathematical thinking and communication

Courses that primarily serve students in partner disciplines should incorporate activities designed to advance students' progress in

¹¹⁰See Appendix 2 for a list of the disciplines represented at the Curriculum Foundations workshops.

- Creating, solving, and interpreting basic mathematical models;
- Making sound arguments based on mathematical reasoning and/or careful analysis of data;
- Effectively communicating the substance and meaning of mathematical problems and solutions.

B.3. Critically examine course prerequisites

Mathematical topics and courses should be offered with as few prerequisites as feasible so that they are accessible to students majoring in other disciplines or who have not yet chosen majors. This may require modifying existing courses or creating new ones. In particular,

- Some courses in statistics and discrete mathematics should be offered without a calculus prerequisite;
- Three-dimensional topics should be included in first-year courses;
- Prerequisites other than calculus should be considered for intermediate and advanced non-calculus-based mathematics courses.

B.4. Pre-service elementary (K–4) and middle school (5–8) teachers

Mathematical sciences departments should create programs of study for pre-service elementary and middle school teachers that help students develop

- A solid knowledge—at a level above the highest grade certified—of the following mathematical topics: number and operations, algebra and functions, geometry and measurement, data analysis and statistics and probability;
- Mathematical thinking and communication skills, including knowledge of a broad range of explanations and examples, good logical and quantitative reasoning skills, and facility in separating and reconnecting the component parts of concepts and methods;
- An understanding of and experience with the uses of mathematics in a variety of areas;
- The knowledge, confidence, and motivation to pursue career-long professional mathematical growth.

C. Students majoring in the mathematical sciences

The recommendations in this section refer to all major programs in the mathematical sciences, including programs in mathematics, applied mathematics, and various tracks within the mathematical sciences such as operations research or statistics. Also included are programs designed for prospective mathematics teachers, whether they are “mathematics” or “mathematics education” programs, although requirements in education are not specified in this section.

Although these recommendations do not specifically address minors in the mathematical sciences, departments should be alert to opportunities to meet student needs by creating minor programs—for example, for students preparing to teach mathematics in the middle grades.

These recommendations also provide a basis for discussion with colleagues in other departments about possible joint majors with any of the physical, life, social or applied sciences.

C.1. Develop mathematical thinking and communication skills

Courses designed for mathematical sciences majors should ensure that students

- Progress from a procedural/computational understanding of mathematics to a broad understanding encompassing logical reasoning, generalization, abstraction and formal proof;
- Gain experience in careful analysis of data;
- Become skilled at conveying their mathematical knowledge in a variety of settings, both orally and in writing.

C.2. Develop skill with a variety of technological tools

All majors should have experiences with a variety of technological tools, such as computer algebra systems, visualization software, statistical packages, and computer programming languages.

C.3. Provide a broad view of the mathematical sciences

All majors should have significant experience working with ideas representing the breadth of the mathematical sciences. In particular, students should see a number of contrasting but complementary points of view:

- Continuous and discrete,
- Algebraic and geometric,
- Deterministic and stochastic,
- Theoretical and applied.

Majors should understand that mathematics is an engaging field, rich in beauty, with powerful applications to other subjects, and contemporary open questions.

C.4. Require study in depth

All majors should be required to

- Study a single area in depth, drawing on ideas and tools from previous coursework and making connections, by completing two related courses or a year-long sequence at the upper level;
- Work on a senior-level project that requires them to analyze and create mathematical arguments and leads to a written and an oral report.

C.5. Create interdisciplinary majors

Mathematicians should collaborate with colleagues in other disciplines to create tracks within the major or joint majors that cross disciplinary lines.

C.6. Encourage and nurture mathematical sciences majors

In order to recruit and retain majors and minors, mathematical sciences departments should

- Put a high priority on effective and engaging teaching in introductory courses;
- Seek out prospective majors and encourage them to consider majoring in the mathematical sciences;
- Inform students about the careers open to mathematical sciences majors;
- Set up mentoring programs for current and potential majors, and offer training and support for any undergraduates working as tutors or graders;
- Assign every major a faculty advisor and ensure that advisors take an active role in meeting regularly with their advisees;
- Create a welcoming atmosphere and offer a co-curricular program of activities to encourage and support student interest in mathematics, including providing an informal space for majors to gather.

D. Mathematical sciences majors with specific career goals

D.1. Majors preparing to be secondary school (9–12) teachers

In addition to acquiring the skills developed in programs for K–8 teachers, mathematical sciences majors preparing to teach secondary mathematics should

- Learn to make appropriate connections between the advanced mathematics they are learning and the secondary mathematics they will be teaching. They should be helped to reach this understanding in

courses throughout the curriculum and through a senior-level experience that makes these connections explicit.

- Fulfill the requirements for a mathematics major by including topics from abstract algebra and number theory, analysis (advanced calculus or real analysis), discrete mathematics, geometry, and statistics and probability with an emphasis on data analysis;
- Learn about the history of mathematics and its applications, including recent work;
- Experience many forms of mathematical modeling and a variety of technological tools, including graphing calculators and geometry software.

D.2. Majors preparing for the nonacademic workforce

In addition to the general recommendations for majors, programs for students preparing to enter the nonacademic workforce should include

- A programming course, at least one data-oriented statistics course past the introductory level, and coursework in an appropriate cognate area; and
- A project involving contemporary applications of mathematics or an internship in a related work area.

D.3. Majors preparing for post-baccalaureate study in the mathematical sciences and allied disciplines

Mathematical sciences departments should ensure that

- A core set of faculty members are familiar with the master's, doctoral and professional programs open to mathematical sciences majors, the employment opportunities to which they can lead, and the realities of preparing for them;
- Majors intending to pursue doctoral work in the mathematical sciences are aware of the advanced mathematics courses and the degree of mastery of this mathematics that will be required for admission to universities to which they might apply. Departments that cannot provide this coursework or prepare their students for this degree of mastery should direct students to programs that can supplement their own offerings.

Appendix 6

Sample Questions for Department Self-Study

These sample questions are meant to be suggestive. Every department should draft its own set of questions to gauge its progress in meeting the CUPM recommendations.

1. Do we have data on subsequent course taking in mathematics by students enrolled in our introductory courses? (For example, do we know how many of our Pre-calculus students successfully complete Calculus I? How many of our Calculus I students successfully complete a second course in the department?)
2. Do we know the intended majors of the students enrolled in our introductory courses?
3. Do we know how many of our majors enter the job market directly after graduation, and what kinds of jobs they take?
4. In the past five years, have we asked our majors who graduated recently what they think of the quality of their undergraduate preparation in mathematics?
5. In the past year, has at least one member of our department had a conversation with someone from a nearby business or industry to discuss the mathematics they need in their job and the skills they look for when hiring?
6. Does every course we offer have examinations or assignments *that affect a student's grade* requiring students to
 - a. explain their reasoning?
 - b. solve multi-step problems?
 - c. generalize from examples?
 - d. solve a problem two different ways?
 - e. read new material and use it in some way?
 - f. use a mathematical tool to solve a problem in another discipline?
7. Does the syllabus of every course we offer include
 - a. an application to at least one discipline outside mathematics?
 - b. at least one topic or application that is less than fifty years old?
8. Have we used *The Curriculum Foundations Project: Voices of the Partner Disciplines* to initiate and support conversations with faculty in other disciplines?
9. In the past year, has at least one member of our department had a conversation with a faculty member from another discipline* about
 - a. a course we offer that their students take?
 - b. a course we might offer that would be valuable for their students?
 - c. applications to their field that we might include in a course we teach?
 - d. possible undergraduate research projects?

- e. team-teaching (or guest lectures in) a course or a unit within a course?
(*specifically with someone from the biological sciences? business or economics? chemistry? computer science? engineering? physics?)
- 10. Have we had a conversation with another department about creation of a joint major?
- 11. Do we offer at least one introductory course in which an examination or assignment *that affects a student's grade* requires students
 - a. to use computer technology to solve a problem?
 - b. to use computer technology to investigate examples or visualize a concept?
- 12. Same as (11) for intermediate courses.
- 13. Same as (11) for advanced courses.
- 14. Are faculty in our department rewarded for extra teaching effort (such as learning substantial new material, extensive consultation with colleagues outside the department, or taking leadership of the curriculum and teaching of a multi-section introductory course) by one or more of the following?
 - * Released time.
 - * Credit toward merit pay, promotion or tenure.
 - * Travel money for professional development.
 - * Institutional recognition (teaching awards etc.).
- 15. Are faculty in our department offered support in using new technology or in learning new pedagogical strategies by one or more of the following?
 - * In-house workshops.
 - * Support to attend workshops/minicourses off campus.
 - * Released time.
 - * Extra student assistants.
- 16. Do we offer at least one introductory course that satisfies Recommendation A.1?
- 17. Do we make effective use of advising, placement tests and/or consultation with colleagues in other disciplines to ensure that students take appropriate introductory courses?
- 18. Do we offer a statistics course with an emphasis on data analysis and without a calculus prerequisite?
- 19. Do we offer a discrete mathematics course without a calculus prerequisite that meets the needs of computer science majors?
- 20. Do we incorporate geometric thinking and visualization in two and three dimensions — including vectors — in our first-year courses? In our second-year courses?
- 21. Have we examined the prerequisites for our intermediate and advanced courses with an eye to making them more accessible to students majoring in other disciplines or not yet decided on majors?
- 22. Have we consulted with colleagues in education about our programs for prospective teachers?
- 23. Do we offer a program for prospective elementary and middle school teachers that satisfies Recommendation B.4?
- 24. Are school systems that hire our graduates satisfied with their performance?
- 25. Can we see progress in our majors' abilities to reason, solve problems and think abstractly as they move through our program? How do we gauge their progress?
- 26. Can we see progress in our majors' abilities to read and write mathematics and present their ideas orally as they move through our program? How do we gauge their progress?
- 27. Do our majors have experience with a variety of technological tools? Which courses provide these experiences?

28. Does every major complete a set of courses that encompasses the breadth specified in Recommendation C.3? Which courses include which aspects of the contrasting but complementary elements listed in C.3?
29. By graduation, does every major know several substantial applications of mathematics and a number of contemporary open questions?
30. Do we assure that every major studies a single area in depth as specified in Recommendation C.4? What are the ways a student can satisfy this requirement?
31. Does every major complete a senior year project as specified in Recommendation C.4? What are the ways a student can satisfy this requirement?
32. Is our major flexible and adapted to connections to other disciplines? How do we know?
33. How do we recruit and retain majors and minors? Are we satisfied with our efforts? Are our students satisfied with our efforts?
34. How do we advise current and prospective majors? Are we satisfied with our efforts? Are our students satisfied with our efforts?
35. Do we effectively inform students about career opportunities for majors in the mathematical sciences? About what our alumni do after graduation?
36. Do courses and senior year projects for prospective teachers of secondary mathematics make explicit connections between new material and the material of the high school curriculum? What specific elements help students make these connections?
37. Does our program for prospective teachers of secondary mathematics include the topics listed in Recommendation D.1?
38. Do all prospective teachers of secondary mathematics experience many forms of mathematical modeling and a variety of technological tools? Which courses provide these experiences?
39. Do our majors preparing for the nonacademic workforce complete courses in programming, statistics and a cognate area as recommended in D.3? Which specific courses do we recommend to our students to meet these requirements?
40. Does every major preparing for the nonacademic workforce complete an internship or a project involving a contemporary application of mathematics or statistics?
41. Are employers who hire our graduates satisfied with their mathematical preparation?
42. How do we inform students about special opportunities like internships, summer research programs and visiting programs at other universities?
43. Which faculty have responsibility for advising students about post-baccalaureate study? Are students satisfied with the guidance they're getting? Are the programs that admit our students satisfied with their preparation?
44. Which faculty have responsibility for advising students intending doctoral study in the mathematical sciences? Have they contacted the graduate programs to which our students apply? Are the programs that admit our students satisfied with their preparation?