A Collective Vision

Voices of the Partner Disciplines

Susan L. Ganter
Clemson University

William Barker
Bowdoin College

Introduction

Mathematics can and should play an important role in the education of undergraduate students. In fact, few educators would dispute that students who can think mathematically and reason through problems are better able to face the challenges of careers in other disciplines—including those in non-scientific areas. Add to these skills the appropriate use of technology, the ability to model complex situations, and an understanding and appreciation of the specific mathematics appropriate to their chosen fields, and students are then equipped with powerful tools for the future.

Unfortunately, many mathematics courses are not successful in achieving these goals. Students do not see the connections between mathematics and their chosen disciplines; instead, they leave mathematics courses with a set of skills that they are unable to apply in non-routine settings and whose importance to their future careers is not appreciated. Indeed, the mathematics many students are taught often is not the most relevant to their chosen fields. For these reasons, faculty members outside mathematics often perceive the mathematics community as uninterested in the needs of non-mathematics majors, especially those in introductory courses.

The mathematics community ignores this situation at its own peril since approximately 95% of the students in first-year mathematics courses go on to major in other disciplines. The challenge, therefore, is to provide mathematical experiences that are true to the spirit of mathematics yet also relevant to students' futures in other fields. The question then is not *whether* they need mathematics, but *what* mathematics is needed and in what context.

The Mathematical Association of America (MAA) is currently studying the undergraduate program in mathematics, informed by the views of a broad segment of the mathematics community and its partner disciplines. The goal is a set of recommendations from the MAA Committee on the Undergraduate Program in Mathematics (CUPM)—the *CUPM Curriculum Guide 2004*—that will assist mathematics departments as they plan their programs through the first decade of the 21st century. The current efforts by CUPM are being informed by an unprecedented amount of information on the mathematical needs of partner disciplines, obtained through a series of disciplinary-based workshops known as the *Curriculum Foundations Project*.

The Curriculum Foundations Project

The CUPM subcommittee *Curriculum Renewal Across the First Two Years* (CRAFTY) has gathered input from partner disciplines through a series of eleven workshops held across the country from November 1999 to February 2001, followed by a final summary conference in November 2001 (see Appendix A). Each Curriculum Foundations workshop consisted of $20-35$ participants, the majority chosen from the discipline under consideration, the remainder chosen from mathematics. The workshops were not intended to be discussions between mathematicians and colleagues in the partner disciplines, although this certainly happened informally. Instead, each workshop was a dialogue among the representatives from the partner discipline, with mathematicians present only to listen and serve as a resource when questions about the mathematics curriculum arose.

Each workshop produced a report directed to the mathematics community summarizing the workshop's recommendations and conclusions. Each report was written by representatives of the partner discipline, insuring accurate reporting of the workshop discussions while also adding credibility to the recommendations. The Curriculum Foundations Steering Committee supplied a common set of questions to guide the discussions at each workshop (see Appendix B). This helped achieve uniformity in structure for the reports and made it easier to compare the recommendations from different disciplines.

The host institutions funded most of the workshops.[1] Such financial support—obtained with little advance notice—indicates the high level of support from university administrations for such interdisciplinary discussions about the mathematics curriculum. Workshop participants from the partner disciplines were extremely grateful—and surprised—to be invited by mathematicians to state their views about the mathematics curriculum. That the opinions of the partner disciplines were considered important and would be taken seriously in the development of the *CUPM Curriculum Guide 2004* only added to their enthusiasm for the project as well as their interest in continuing conversations with the mathematics community.

In addition to the workshop reports, the Curriculum Foundations Project has resulted in a number of publications that describe the workshops, their outcomes, and related work (see Publications). These publications include articles in journals of the disciplinary societies as well as the general press. Conversations have also continued via panels and invited colloquia at professional meetings, both in mathematics and the partner disciplines. For example, the Joint Mathematics Meetings (January 2001–2004) and MathFests (August 2001–2003) featured a number of events about the Curriculum Foundations Project. In addition, the partner disciplines have enthusiastically continued the conversations from the workshops at their own disciplinary meetings and have included mathematicians in these discussions.

In January 2001, a series of small focus groups was organized by CUPM to discuss the disciplinary workshop reports. Each invited focus group discussed and analyzed the implications of the report(s) for a specific discipline. Input generated from these discussions was used to supplement the workshop reports in the development of the *CUPM Curriculum Guide 2004*.[2]

In November 2001, invited representatives from each disciplinary workshop gathered at the United States Military Academy in West Point, NY for a final Curriculum Foundations Conference. The discussions resulted in *A Collective Vision*, a set of commonly shared recommendations for the first two years of undergraduate mathematics instruction.

This volume includes CRAFTY's official report on the *Collective Vision* recommendations, as well as the individual disciplinary reports.[3] The disciplinary reports and *Collective Vision* were used by CUPM

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[1] The only exceptions were the two workshops on technical mathematics, which were hosted by two-year institutions and funded by the National Science Foundation.

[2] Information about these and other resources is available on the MAA website (`www.maa.org`) or through the authors.

[3] Electronic versions of these materials are also available for downloading from `www.maa.org/cupm/crafty`.

to inform the preparation of the *CUPM Curriculum Guide 2004*, a set of recommendations for the mathematics community on the undergraduate curriculum that will focus on desired student outcomes. However, the workshop reports and *Collective Vision* have value independent of the *CUPM Curriculum Guide 2004*, since they can serve as resources for multi-disciplinary discussions at individual institutions. Promoting and supporting informed interdepartmental discussions about the undergraduate curriculum might ultimately be the most important outcome of the Curriculum Foundations Project.

Summary Recommendations: A Collective Vision

Understanding, Skills, and Problem Solving

Emphasize conceptual understanding.

- *Focus on understanding broad concepts and ideas in all mathematics courses during the first two years.*
- *Emphasize development of precise, logical thinking. Require students to reason deductively from a set of assumptions to a valid conclusion.*
- *Present formal proofs only when they enhance understanding. Use informal arguments and well-chosen examples to illustrate mathematical structure.*

There is a common belief among mathematicians that the users of mathematics (engineers, economists, etc.) care primarily about computational and manipulative skills, forcing mathematicians to cram courses full of algorithms and calculations to keep “them” happy. Perhaps the most encouraging discovery from the Curriculum Foundations Project is that this stereotype is largely false. Though there are certainly individuals from the partner disciplines who hold the more strict algorithmic view of mathematics, the disciplinary representatives at the Curriculum Foundations workshops were unanimous in their emphasis on the overriding need to develop in students a conceptual understanding of the basic mathematical tools.

The partner disciplines also value the precise, logical thinking that is an integral part of mathematics. They would like to see this emphasized in early collegiate mathematics instruction in a way that enhances understanding of the underlying concepts. However, the partner disciplines vary widely in the level of rigor and formal reasoning their students need to master. For instance, business students do not need much formal mathematical proof, while students in computer science or software engineering must develop the skill to apply formal logic and construct simple but rigorous proofs. It is therefore important that each institution carefully consider the nature of its students and their intended major disciplines when planning how to incorporate mathematical reasoning into the curriculum. Precise thinking and formal proof must be present, but in appropriate courses, at the appropriate time, and delivered in the appropriate manner. When carelessly introduced, formal proof can confuse many students and negatively affect their ability to understand and apply mathematics.

Emphasize problem solving skills.

- *Develop the fundamental computational skills the partner disciplines require, but emphasize **integrative skills**: the ability to apply a variety of approaches to single problems, to apply familiar techniques in novel settings, and to devise multi-stage approaches in complex situations.*

Fundamental computational skills are important and must be developed in students. However, colleagues in the partner disciplines confirm that applying mathematics to unfamiliar problems requires far more than computational skill. Mathematics courses must include more sophisticated problem-solving experiences than simple situations in which students look in the book for an example of the same type and change the numbers.

Students must learn to recognize how and when to use mathematics outside of familiar contexts. In the partner disciplines they will have to make judgments about what mathematical techniques (and what technologies) are appropriate for specific problems. Therefore, students must learn more than how to copy the behavior of their instructor; they must develop their own processes for solving problems. Support must be given as students make this transition.

Emphasize mathematical modeling.

- *Expect students to create, solve, and interpret mathematical models.*
- *Provide opportunities for students to describe their results in several ways: analytically, graphically, numerically, and verbally.*
- *Use models from the partner disciplines: students need to see mathematics in context.*

The importance ascribed to mathematical modeling by *every* disciplinary group in *every* workshop was quite striking. After the first two years, students should be able to create, solve, and interpret basic mathematical models from informal problem statements; to provide logical arguments (at an appropriate level) that the models constructed are valid; and to use the models to solve problems.

Emphasize modeling through student projects that are engaging, meaningful, and relevant to student learning and interests. Modeling is a powerful problem solving process that helps students use their skills, knowledge, and creativity to produce results and products that can benefit society. Therefore, modeling can build student confidence, introduce them to useful and powerful elements of mathematics, and provide a mechanism for communication, expression, and reasoning that is cross-cultural and cross-disciplinary.

The need to increase the emphasis on mathematical modeling in the first two years of the undergraduate program is a strong message from the Curriculum Foundations Project.

Emphasize communication skills.

- *Incorporate development of reading, writing, speaking, and listening skills into courses.*
- *Require students to explain mathematical concepts and logical arguments in words. Require them to explain the meaning — the hows and whys — of their results.*

A theme that ran through nearly every disciplinary workshop was the importance of being able to communicate mathematical and quantitative ideas. Though there are successful examples of instructors who teach writing and speaking in the mathematics classroom, there is still a need for more universal implementation of these activities. Many mathematicians view instruction in writing and oral presentation as hard, time-consuming, and foreign to their own training. However, these skills are critical to students, and faculty members in all disciplines have a responsibility to incorporate them into class instruction. Such activities can take the form of written lab assignments, technical reports, group projects, professional presentations in class, short essays on exams, and the like.

Communication skills are related to logical reasoning: if you can't explain it, you don't understand it. Therefore, these skills are essential to the development of students' mathematical knowledge in the first two collegiate years.

Emphasize balance between perspectives.

- *Continuous and discrete*
- *Linear and nonlinear*
- *Deterministic and stochastic*
- *Deductive and inductive*
- *Exact and approximate*

- *Pure and applied*
- *Local and global*
- *Quantitative and qualitative*

A broader view of mathematics needs to be communicated to students. The perspectives on the left in the above list are those that are usually covered in the undergraduate mathematics curriculum. Those on the right are, for the most part, not covered. The consensus of the Summary Curriculum Foundations Conference participants was that students should be exposed to both views, although the depth, breadth, sequence, and methods of how this should be done would depend on the nature of the local institution.

Priorities for Content, Topics, and Courses

Strive for depth over breadth. Explore locally what topics can be omitted and teach the remaining topics in more depth.

Topics can and should be eliminated to achieve a depth of conceptual understanding on a limited number of mathematical tools. Specifically, topics currently taught in some introductory courses can be omitted, and replaced with a more detailed treatment of the remaining material or with new material that better supports the desired conceptual objectives. For example, most workshops placed emphasis on derivatives and integrals as conceptual ideas and tools while downplaying the importance of intricate computational techniques. Time can be gained for conceptual underpinnings by focusing only on calculations using the most basic functions and eliminating more tedious calculations that students will not likely remember past the final exam.

Offer non-calculus-based descriptive statistics and data analysis in the first two years (either as a separate course or integrated into other courses).

The importance of data analysis for so many of the partner disciplines argues strongly for an increased presence of basic statistical training in the first two years of undergraduate mathematics. The experience provided should be primarily concerned with descriptive statistics, needing only a brief introduction to probability. The material should be motivated by a variety of examples and real data sets, including data collected by students.

Offer discrete mathematics and mathematical reasoning in the first two years. Do not have calculus as a prerequisite.

Discrete mathematics, including a serious introduction to proof techniques, is absolutely essential for students majoring in computer science. In many institutions, the instruction of discrete mathematics has been taken over by computer science departments because mathematicians do not consider it important, believe a high level of mathematical maturity is required, or do not teach it in a way that is useful to computer scientists. This is unfortunate, especially since proof techniques are an integral part of discrete mathematics.

Mathematics departments should offer at least one introductory course in discrete mathematics, preferably at the freshman level, which serves the needs of computer science and provides a bridge from introductory mathematics to more theoretical and abstract upper-division courses. Ideally, such discrete mathematics courses would require no calculus prerequisite. Computer science faculty should be heavily involved in the design and implementation of these courses.

Continue to offer calculus and linear algebra in the first two years, but make the curriculum more appropriate for the needs of the partner disciplines.

Continuous mathematics, most commonly seen in calculus courses and linear algebra, has not lost its importance for many of the partner disciplines. However, these courses have often drifted in a theoretical direction that has made them obscure, formidable, and seemingly irrelevant to other disciplines. This is unhealthy for both the partner disciplines and mathematics. Making these courses more appropriate for the partner disciplines will mean confronting the issues raised by the calculus reform movement and the less widely known, but still vital, reform efforts in linear algebra instruction. Specifically, workshop participants stressed that only the most fundamental and applicable results from calculus are needed, and that linear algebra should stress matrix algebra through eigenvalues and eigenvectors. In most workshops, representatives of the partner disciplines advocated attitudes and actions that were *exactly* those put forward during the calculus reform movement. Yet in almost every instance these colleagues had not even heard of the calculus reform movement.

Replace traditional college algebra courses with courses stressing problem solving, mathematical modeling, descriptive statistics, and applications in the appropriate technical areas. Deemphasize intricate algebraic manipulation.

College algebra courses serve two distinct student populations: the overwhelming majority for whom this is a terminal course in mathematics, and the relatively small minority for whom it is a gateway to further mathematics. Neither group is well-served by the traditional version of the college algebra course. Many of the disciplinary workshop reports recommend the reorganization of college algebra and precalculus courses to better meet the needs of various student populations. In particular, the obvious mismatch between a curriculum designed to prepare students for calculus and the reality that very few of these students subsequently enroll in calculus caused the Summary Curriculum Foundations Conference participants to recommend changes stressing problem solving, modeling, statistics, and applications.

Emphasize two- and three-dimensional topics.

The most urgent plea for an earlier introduction of multidimensional topics came from the chemists: introductory chemistry courses utilize many topics in multivariable calculus. However, colleagues from many disciplines expressed dissatisfaction with the level of student understanding of concepts—geometric and otherwise—in three dimensions. Specifically, they stressed the need for vectors in two and three dimensions, geometric and graphical reasoning, linear systems, and three-dimensional visualization skills.

Pay attention to units, scaling, and dimensional analysis.

Using units and dimensional analysis, as well as realistic scaling, is important in helping students make the transition between the concrete physical world of observations and data and the abstract mathematical world of formulas and models. Specifically, appropriate units and scale can help students to apply meaning to algebraic formulas and equations.

Instructional Techniques and Technology

Use a variety of teaching methods since different students have different learning styles. In particular, encourage the use of active learning, including

- *in-class problem solving opportunities*
- *class and group discussions*
- *collaborative group work, and*
- *out-of-class projects.*

Colleagues in partner disciplines support the need for alternatives to traditional lecture courses. Mathematics classes that are based primarily on lectures may discourage student interest and minimize opportunities for students to develop mathematical understanding. Some students learn best by demonstration, some by visual recognition, some by hands-on experience, and others by conceptualization. Therefore, it is important to create diverse learning environments within each class. In addition, active learning by students is far more likely to increase the "residue of knowledge" that students retain after a course is over.

Improve interdisciplinary cooperation.

Mathematicians should seek out projects from partner disciplines to be used in mathematics courses and increase team teaching opportunities. Disciplinary workshop participants were so excited by the possibility of increasing the use of real models in mathematics courses that many volunteered to help with the development of such models. The support from colleagues in the partner disciplines for interdisciplinary cooperation with mathematics departments was so strong that mathematicians at individual institutions should be encouraged to seek out such support locally.

Interdisciplinary cooperation can help students overcome the transfer problem from mathematics courses to partner discipline courses. Specifically, students often have difficulty seeing the relationships between problems in non-mathematics disciplines and material studied in mathematics courses. Colleagues in partner disciplines believe that exposing students in mathematics courses to discipline-specific contexts for various mathematical topics will have a positive effect on their ability to transfer knowledge between courses.

Interdisciplinary class sessions—both in mathematics courses and courses in the partner disciplines—were enthusiastically endorsed by workshop participants as a means for increasing interaction between mathematics faculty and partner discipline faculty. For example, an engineering professor presenting specific applications to a mathematics class will increase the sense of relevancy for students. Conversely, mathematics professors coming into engineering courses when specific mathematics topics are being reintroduced would reinforce earlier mathematics instruction.

Cooperative teaching arrangements between mathematics and other disciplines can be expensive. In addition to funding issues, there are often administrative barriers that need to be addressed, including questions about tenure and promotion. MAA should take a visible leadership role in helping institutions confront these issues.

Emphasize the use of appropriate technology.

Technology should be used in introductory mathematics courses to provide students with tools for solving problems. Current technologies make possible the discussion of important problems that were previously inaccessible, such as problems without analytic solutions. Technology is thus a powerful tool that should be utilized fully in the mathematics classroom. However, mathematics faculty must stress to students the importance of choosing the appropriate method of calculation (mental, paper-and-pencil, or technological) for the desired task.

Colleagues in other disciplines do not need to be convinced of the importance of technology in mathematics instruction since they know how critical technology is to their own fields. And they are well aware of the importance of choosing appropriate tools for each problem solving activity, as well as properly interpreting the results so obtained. Therefore, mathematics courses should stress intelligent and careful interpretation of results obtained from technology. Blind, unquestioning belief in the results obtained from a calculator or computer can be disastrous.

A more surprising statement from workshop participants was that spreadsheets are the technology of choice for a large number of partner disciplines. Although individual workshop reports stopped short of recommending spreadsheets as the primary technology in mathematics instruction, their widespread use is relevant to the technology choices made in mathematics courses that primarily serve other disciplines.

A related observation was the unimportance of graphing calculators; very few workshop participants reported their use in disciplinary courses. Therefore, if calculators are chosen as the technology for a mathematics course, it must be understood that this is done for pedagogical reasons, not to support uses in other disciplines.

The bottom line: mathematics faculty members need to be aware of the preferred tools of the partner disciplines—both globally and locally.

Emphasize the use of appropriate assessment.

The important relationship between assessment and student learning was discussed extensively at the workshops; i.e., how and what you assess directly affects how and what students learn. Because assessment can be difficult, time-consuming, and tedious, instructors often put less thought and effort into this aspect of course design. However, since effective assessment is critical to learning, instructors must invest in the development of a variety of assessment strategies to measure achievement of course objectives.

WYTIWYG ("What you test is what you get") was adopted at the Summary Curriculum Foundations Conference as a central message about assessment. For example, discussions focused on the need for conceptual questions on examinations as opposed to only algorithmic computations and problems that can be solved mechanically with a calculator. It underscores the importance assigned by colleagues in partner disciplines to the development of conceptual understanding.

Recommendations for Departments and the Profession

Promote professional development.

Changes in the mathematics curriculum that are advocated in this volume will not occur easily or naturally. They will require a great deal of effort, insight, and hard work, and will not happen until the mathematics community believes in the need for these changes. It is also important that MAA actively support these recommendations from the partner disciplines. However, meaningful implementation needs to be developed at the local level. To assist in this curriculum development, MAA can promote the urgency of the recommendations and widely disseminate a variety of curricular models as they become available.

Establish mechanisms for the development, review, and dissemination of effective instructional materials and techniques, including collaborative efforts between mathematicians and partner disciplines that result in innovative instructional materials.

MAA should take the lead in establishing mechanisms by which colleagues from partner disciplines can collaborate with mathematics faculty, and the products of these collaborations can be made readily available to a wider audience. Courses taught by faculty teams that include both mathematicians and partner discipline representatives should be encouraged by institutions. Several colleges and universities have implemented programs that encourage the development of interdisciplinary courses. Such programs can be studied and adapted for local implementation.

Encourage institutional assessment of programmatic changes.

The recommendations made in this volume are fundamental and far-reaching. It is therefore important that MAA, in cooperation with the mathematics community and individual institutions, engage in a sustained effort to collect and analyze information regarding the influence of these recommendations on undergraduate mathematics education. In particular,

- How do mathematics departments implement the recommendations?
- How do colleagues in other disciplines respond to the recommendations?
- How do students adjust their modes of interaction with the mathematics curriculum as a result of the recommended curricular changes?
- How do textbook writers, publishers, and educational website developers react to the recommendations?
- How is the "residue of student learning" affected at institutions that implement these recommendations?

Utilizing the Reports in this Volume

Although some differences between disciplines have been observed in the Curriculum Foundations workshops, the guiding messages are strong. These messages can be clearly understood when organized into the single Collective Vision outlined here. However, this common vision is not a substitute for the unique contributions of colleagues in the many disciplines participating in this effort. And, depending on the situation and circumstances at individual institutions, there may be a need for a greater focus on the needs of certain disciplines.

Therefore, the full reports as submitted by the participants in each of the Curriculum Foundations workshops are included here. These reports provide the details of individual discussions at these workshops as interpreted by leaders from the partner disciplines. The words are their words, the recommendations are their beliefs about their students. Individual reports can serve as a resource to guide one-on-one discussions between mathematicians and colleagues in partner disciplines. And because the reports were written by the partner disciplines, they are more likely to be received by colleagues in those disciplines as credible ideas from like-minded colleagues—not a mandate handed down by mathematicians. In this way, it is hoped that the reports will foster open, collegial, and constructive conversations between academic departments.

Finally, the reports to follow give renewed impetus for continuing efforts of the mathematics community in renewing the undergraduate curriculum. Ultimately, it is the responsibility of mathematicians to create courses and curricula that embrace the spirit of these recommendations while retaining the intellectual integrity that is so integral to the discipline of mathematics. MAA continues to provide resources for this effort including the publication of the *CUPM Curriculum Guide 2004*, addressing the undergraduate program in mathematics. However, all mathematics departments—and all mathematicians in those departments—must make the development of high quality, relevant mathematics courses at the introductory collegiate level a priority of great importance. The students who take these courses will be the leaders of tomorrow in government, business, and the arts. Valuable and meaningful mathematical experiences will contribute positively to the welfare of society and insure future support for mathematics.

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PUBLICATIONS RESULTING FROM THE CURRICULUM FOUNDATIONS PROJECT

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APPENDIX A: The Curriculum Foundations Workshops

Physics and Computer Science

Bowdoin College, October 1999

Interdisciplinary (Mathematics, Physics, Engineering)

United States Military Academy, West Point, November 1999

Engineering

Clemson University, May 2000

Health-Related Life Sciences

Virginia Commonwealth University, May 2000

Technical Mathematics (at two sites)

Los Angeles Pierce College, October 2000

J. Sargeant Reynolds Community College, October 2000

Statistics

Grinnell College, October 1999 and 2000

Business and Management

University of Arizona, October 2000

Teacher Preparation and Mathematics Education

Michigan State University, November 2000

Biology and Chemistry

Macalester College, November 2000

Mathematics Preparation for the Major

Mathematical Sciences Research Institute, February 2001

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The Summary Curriculum Foundations Conference

United States Military Academy, West Point, November 2001

APPENDIX B: Curriculum Foundations Workshop Questions

Understanding and Content

- *What conceptual mathematical principles must students master in the first two years?*
- *What mathematical problem solving skills must students master in the first two years?*
- *What broad mathematical topics must students master in the first two years? What priorities exist between these topics?*
- *What is the desired balance between theoretical understanding and computational skill? How is this balance achieved?*
- *What are the mathematical needs of different student populations and how can they be fulfilled?*

Technology

- *How does technology affect what mathematics should be learned in the first two years?*
- *What mathematical technology skills should students master in the first two years?*
- *What different mathematical technology skills are required of different student populations?*

Instructional Interconnections

- *What impact does mathematics education reform have on instruction in your discipline?*
- *How should education reform in your discipline affect mathematics instruction?*
- *How can dialogue on educational issues between your discipline and mathematics best be maintained?*

Instructional Techniques

- *What are the effects of different instructional methods in mathematics on students in your discipline?*
- *What instructional methods best develop the mathematical comprehension needed for your discipline?*
- *What guidance does educational research provide concerning mathematical training in your discipline?*