Summary

The conference participants reached agreement on the following aspects of a productive interaction between mathematics and the life sciences:

1. We affirm the value of the fundamentals of mathematics for the life sciences. A mastery of basic mathematical concepts is required, and the mathematical way of thinking is an essential part of the training of future life scientists.

2. We recognize a need for a core mathematics curriculum for students of the life sciences. Recommendations are made here for this mathematical core, although it is recognized that in practice there will be different local, institutional implementations of this core.

3. We recognize the necessity for flexibility in client discipline curricula. There is “not just one math student” coming to mathematics from the life sciences. Some students will want and need to take mathematical sciences courses beyond the recommended core, and the program for these students in client disciplines of the life sciences should allow for this diversity through customized individual programs.

4. We encourage integration in teaching efforts whenever possible. Progress will require communication and cooperation between mathematics and life sciences departments.

Our discussion of curricular issues should be an ongoing, dynamic process to improve our understanding of the details of these basic points of agreement. The interdisciplinary dialogue must continue in order to help drive this controlled evolution.

Narrative

Introduction and Background

We affirm that mathematics offers a language for precise communication, a way of thinking, and training in reading and writing technical material. As noted by a life sciences participant, students can “gain confidence in their ability to do things because of the mastery gained through their mathematical training.”

Throughout these recommendations, the definition of mastery of a mathematical concept recognizes the importance of both conceptual understanding at the level of definition and understanding in terms of use/implementation/computation:

\[ \text{Mastery} = (\text{Conceptual understanding}) + (\text{Implementation/Computation}) \]
Basic Concepts that should be mastered include:

- variables, parameters, functions, symbolic notation
- limits and their use especially in definitions of derivatives and integrals
- iterations
- probability, conditional probability
- approximation (as contrasted with exact value)
- logic and mathematical thinking, generalization, deductive reasoning

At the conceptual level, students should be able to explain these concepts in words.

Many participants put special emphasis on the use of models. Models are a way of organizing information for the purpose of gaining insight and providing intuition into systems that are too complex to understand any other way. There should be discussion about limitations of models (“what models cannot do”)—these aspects are not generally covered in standard mathematics curricula. Some participants felt this view of modeling is not given sufficient status in biology.

The Core Curriculum should include:

(a) Basic concepts course(s): The previous paragraphs detail some specific content for this coursework. We believe it is appropriate to emphasize the use of models and the “meanings” of the mathematical concepts involved. Participants noted that mathematical modeling requires “a solid mathematical background in techniques and structures.”

(b) Statistics—with an emphasis on the use of models.

(c) Use of computers and mathematics—we recognize a need for integrating mathematical concepts and relevant computer calculations for solving problems—true mastery should be the goal.

Although mathematics in biology uses discrete mathematics frequently, we affirm the importance of standard introductory “continuous” mathematics such as the standard content of introductory calculus courses.

Flexibility in Curriculum in the Life Sciences is needed. Conference participants from the life sciences repeatedly affirmed that the value of mathematics is not restricted to the specific content of any particular course or combination of courses.

The conference participants recommend to client disciplines that:

(a) There should be a long-term view of integration efforts that build productive interaction between mathematics and life sciences. Life sciences students should be permitted and indeed encouraged to take more mathematical sciences courses in lieu of courses within the client departments.

(b) Some students may want and need to take additional courses beyond the recommended core material. The “most likely optional list” as discussed by conference participants included coursework with coverage of:
- trigonometric functions (anything from basic through Fourier analysis)
- matrices and linear algebra, including least squares problems
- elementary differential equations or difference equations
- non-parametric statistics
- computer modeling and graphics
- discrete mathematics (combinatorics, set theory, sorting)
- advanced dynamics (nonlinear phenomena, time series analysis, equilibrium, stability, oscillations and limit cycles, bifurcations)
In the words of one participant:

*The flexibility needed for the benefit of life sciences students is that the content of specific courses is not as important as the package of courses a life sciences student is encouraged to take.*

Another scientist commented on the needs of students with academic ambitions beyond undergraduate level:

*The current areas of biological interest will multiply and branch into further areas, and mathematical issues will be crucial in forming links between these areas.*

**Integration in Teaching Efforts.** Integration can refer to specific courses (team teaching), or to an entire program structure. Some participants advocated “more mathematics concepts in biology courses, and more biology concepts in mathematics courses.” Some mathematics participants invited client disciplines to “say what they want to see included in core course content.”

At the undergraduate program level, *life sciences students might be allowed a program track that includes substantial mathematics course offerings. And mathematics departments might offer program tracks with substantial life sciences course offerings* for students with interests in those areas.

Teachers and researchers working across traditional academic boundaries need support and encouragement. Successful interdisciplinary work often involves overcoming the difficulties of language or academic culture differences.

**Understanding and Content**

The working definition of mastery is:

\[
\text{Mastery} = (\text{Conceptual understanding}) + (\text{Implementation/Computation})
\]

This definition is motivated by the participants’ acknowledgement that there has been a distancing between these aspects of fundamental understanding of mathematical concepts.

**What conceptual mathematical principles must students master in the first two years?**

We recommend that the entire Basic Concepts list be included here. We recognize that mathematics is cumulative in its development, and consequently, at the undergraduate level, it is essential that we work to maintain and build students’ confidence in the value of mathematics. Students in the life sciences must obtain the necessary prerequisite mathematical skills in order to succeed in their chosen fields.

**What mathematical problem solving skills must students master in the first two years?**

Students should have a mastery of all aspects of linear functions in one variable.

All students should master graphing and visualization skills, including the use of log scales. Some participants observed that “chemistry students tend to think more visually than physics students, who tend to think in terms of formulae.”

Students should master the use of computers to implement mathematical concepts for problem solving, and the use of statistics and models, through the use of a higher level interface as described below in the **Technology** section. Indeed, in the future many computations that are now typically done by hand or not done at all, will be performed analytically on computers.

**What broad mathematical topics must students master in the first two years?**

Here specific topics are distinguished from the broader list of basic concepts given previously.

Students should have a mastery of exponential functions (including base e exponentials), and logarithm functions, including properties of these functions required for their successful implementation in problem-solving.
What priorities exist between these topics?
The first priority is the basic core coverage as outlined above, with next priority to areas of specialization.

What is the desired balance between theoretical understanding and computational skill?
Our definition of mastery requires that both theoretical understanding and computational skill be considered essential elements of the overall understanding of concepts. The desired balance between the two aspects is problem/question dependent.

Examples where hand calculation is natural and desirable:
- Manipulating a linear equation by hand should be possible by every student.
- Finding slopes and axis intercepts from a given linear equation should be doable by hand.
- The estimation of the log of a number may require hand calculation using properties of exponential and log functions.
- Some participants mentioned “a familiarity with orders of magnitude” in connection with what should be doable by hand.

Some participants noted that undergraduate biology students in the first two years “may not necessarily know how to solve a differential equation, but they should understand qualitatively what the various terms in the equation mean.” This emphasizes the basic conceptual understanding involved at some expense to the implementation aspect of understanding.

Students should realize (be taught) that conceptual understanding helps guide the search for what needs to be computed as well as how it may be computed, while technology/computation skills help guide how that calculation may be easily implemented.

What are the mathematical needs of different student populations and how can they be fulfilled?
We recommend that these different needs be addressed through the use of the “optional list” of coursework beyond the core curriculum outlined above.

Technology

How does technology affect what mathematics should be learned in the first two years?
Computer technology affects:
(a) How we do things
(b) Ease of implementation of mathematics
(c) Ability to visualize and conceptualize relevant concepts
(d) Ability to design relevant “experiments”

What mathematical technology skills should students master in the first two years?
Students should master a higher level interface, e.g.:
- spreadsheet
- symbolic/numerical computation packages (e.g., Mathematica, Maple, Matlab)
- statistical packages

The specific packages are less important than mastery of the concepts common to each class of package.
What different mathematical technology skills are required of different student populations?
We acknowledge that different technology skills may be required of different student populations.

Instructional Interconnections

What impact does mathematics education reform have on instruction in your discipline?
Biologists are generally unaware of mathematics education reform efforts. If the current recommendations of this conference are enacted throughout the curriculum, then positive change will be facilitated in life sciences curricula.

How should education reform in your discipline affect mathematics instruction?
All areas of biology have moved from observational/descriptive approaches to more quantitative approaches. These changes overall in biology have prompted the current recommendations of this conference.
We recommend that client departments require the core curriculum.

How can dialogue on educational issues between your discipline and mathematics best be maintained?
Participants agreed that dialogue on educational issues between disciplines should be maintained.
In the words of an undergraduate science major whose current expertise is in job placement of bachelor’s degree students in the sciences:

These people need statistics, attention to detail, the ability to think logically, and the ability to master a variety of mathematical tools. Excellent verbal and written communication skills are also needed, since problem solving is usually done in groups of three or more people rather than alone. More extensive exposure to mathematics (beyond minimal core) is desirable.

Effective dialogue will require institutional support (money, time, resources) for workshops and conferences, and convening of interdisciplinary committees (including participation from outside, e.g., industry stakeholders).
Here are some examples of ways to further this dialogue:
• further communication with human resources (e.g., job placement) professionals
• cross-fertilization at scientific meetings
• co-authorship of texts and papers
• joint instruction in courses by mathematics/biology faculty, with more biology concepts and problems in mathematics courses
• national professional society interdisciplinary interactions to gain additional input, both anecdotal and quantitative
• interdisciplinary workshops, e.g., on technology, or conduct a local version of this MAA workshop
• sabbatical exchanges
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