Statistics

CRAFTY Curriculum Foundations Project
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Summary

This report addresses the role of undergraduate mathematics in preparing students to study statistics and the role of statistics in the undergraduate mathematics curriculum. Statistics is a partner discipline as well as a client discipline of mathematics. By this we mean that statistics is a part of the mathematical sciences and should be represented within the curriculum as addressed by the MAA; at most undergraduate institutions, there is no separate statistics department and so responsibility for statistics offerings typically falls to the mathematics department.

The two highest priority needs of statistics from the mathematics curriculum are to:

1. Develop skills and habits of mind for problem solving and for generalization. Such development toward independent learning is deemed more important than coverage of any specific content area.
2. Focus on conceptual understanding of key ideas of calculus and linear algebra, including function, derivative, integral, approximation, and transformation.

The following recommendations are necessary to achieving the two recommendations above. They are listed in decreasing order of importance as determined by the focus group, but all are considered key by both focus group and workshop participants.

1. Emphasize multiple representations of mathematical objects and multiple approaches to problem solving, including graphical, numerical, analytical, and verbal.
2. Instruction should be learner-centered and address students' different learning styles by employing multiple pedagogies.
3. Insist that students communicate in writing and learn to read algebra for meaning.
4. Use real, engaging applications through which students can learn to draw connections between the language of mathematics and the context of the application.
5. Instill appreciation of the power of technology and develop skills necessary to use appropriate technology to solve problems, to develop understanding, and to explore concepts.
6. Align assessment strategies with instructional goals.

In looking at this set of recommendations, the workshop participants observed a strong consistency between them and what we consider to be principles of calculus reform. It was not our intention to take sides in what has, at times, been a source of contention within the mathematics community. Rather, this consistency was a natural by-product of our deliberations.

A second set of recommendations was developed to address statistics' role as part of the mathematical sciences:

1. We endorse the 1991 CUPM recommendation that every mathematical sciences major should study data analysis and statistics.
2. We should advocate relaxing the assumption that the first course in statistics for majors must have the calculus pre-requisite stated by the 1991 CUPM recommendation.
3. We can create a wider acceptance of this recommendation by providing compelling arguments that this need is even greater now than a decade ago and by offering examples of a diverse set of successful courses that address this goal. (We include a diverse set of short course descriptions in the appendices.)
4. We endorse the 1992 recommendations of the ASA/MAA Committee on Undergraduate Statistics for teaching introductory courses in statistics— to emphasize statistical thinking through active learning, with more data and concepts, less theory and fewer recipes.
5. We encourage those responsible for the mathematical needs of students majoring in client disciplines to recognize in their curricular offerings and educational requirements that many of these students would be well served by a statistics course that teaches them how to deal with data-oriented problems in their discipline.
6. We encourage those responsible for the general education requirements in quantitative reasoning to recognize in their curricular offerings and educational requirements that many students would be well served by a modern statistics course that meets the 1992 recommendations of the ASA/MAA Committee on Undergraduate Statistics.

Narrative

Understanding and Content

We felt the tension between content and process, between covering lots of topics and making students think and learn how to solve problems. A list of principles emerged from an early discussion and kept recurring in our conversations. (For the purpose of this section, we use the term 'mathematics' to exclude statistics and consider the issue of what mathematical knowledge and skills we would like statistics students to develop, particularly in the first two years of an undergraduate program.) This list became known as George's list (named for the list's founder, George Cobb).

Desired outcomes of mathematics courses

1. Process of abstraction
2. Ability to distinguish the relevant from the irrelevant
3. Ability to go from special cases to generalizations
4. Ability to test and verify conjectures
5. Ability to use abstract ideas in applied situations
6. Ability to make abstract connections
7. Ability to explain oneself logically

We think of mathematical content as a means to these ends. We believe that the outcomes on this list are not attained through one course or even through several courses, but are achieved incrementally. Attention should be paid throughout the undergraduate curriculum (and before), so that content does not drive out the need to expose students to mathematical experiences that will further these outcomes. These outcomes enable more successful the learning and application of statistics, and we can live with variations in content choices.

Still, there are some fundamental ideas from the traditional first two years that would hinder our teaching were they to be absent from our students' backgrounds. This minimal content set includes key ideas of calculus including the concept of function, derivative as rate of change, integral as accumulation, and approximation. Also included are key ideas from linear algebra including linear transformation, projections, and visualizing Euclidean *n*-space. Basic computational skill and student facility with the interplay among graphical, numerical, algebraic, and verbal representations is also essential.

Technology

With regard to the question of how technology affects what mathematics should be learned in the first two years, the group listed three primary areas:

1. Technology enables more emphasis to be placed on developing students' skills of problem solving. This includes encouraging students to establish the mindset that multiple problem-solving strategies (graphical, numerical, algebraic) are possible. Technology also permits students to analyze "real" applications as opposed to contrived ones.
2. Technology can facilitate students' development of conceptual understanding.Visualization is one avenue through which this can occur, for example with concepts such as approximation by Taylor series. The use of dynamic graphics can help students to understand concepts of calculus. The ability of technology to handle some symbolic manipulations can sometimes allow students to focus their attention on understanding concepts.
3. Technology can promote students' exploration of and experimentation with mathematical ideas. For example, students can be encouraged to ask "what if?" questions, to posit conjectures, to answer them, and to use technology to investigate, revise, and refine their predictions. Specific examples include studying the effects of manipulating parameters on classes of functions and fitting functional models to data.

In terms of specific content related to the technology to be taught and learned, the group cited examples which include numerical issues associated with approximations and round-off errors and also applications of linear algebra such as least squares estimation.

Concerning the question of what mathematical technological skills students should master in the first two years, the group listed both general and specific skills. General skills include knowledge of the usefulness of technology, willingness to use technology (without being specifically told to do so), and understanding the importance of choosing appropriate technology tools, including an appreciation of the strengths and weaknesses of different tools. Specific skills include the ability to use technology to:

1. Graph functions
2. Numerically evaluate functions
3. Solve equations, calculate derivatives and integrals (including multivariable functions)
4. Implement algorithms
5. Perform symbolic manipulations
6. Perform matrix operations

Instructional Interconnections and Techniques

For all students:

Addressing questions of interconnections of instructional ideas between statistics and mathematics communities, the group commented that there are many common ideas between statistics education reform and calculus reform efforts. These include emphases on conceptual understanding, active learning, real applications, and the use of technology. One difference, though, is that there appears to be a much broader consensus among statisticians on these principles of statistics education reform than there is among mathematicians with regard to calculus reform. Most of the work in statistics education reform has been directed at the introductory service course and not at courses that have a mathematics prerequisite.

The group also noted that these efforts can support each other. For example, students who learn about fitting models to data in a calculus class can build on that knowledge in a statistics course. Another example is that mathematical statistics courses can build on students' abilities to use technology to perform calculations involving multivariable calculus in the study of multivariate probability distributions.

Addressing the need for collaboration between the mathematics and statistics communities, it was noted that several mechanisms for such collaboration already exist. The MAA and ASA have had an active joint committee devoted to issues of undergraduate statistics for many years, and the MAA has recently established a Special Interest Group devoted to statistics education. These groups need to continue to ensure that communication flows freely between the two groups. The importance of teacher preparation was mentioned as an area in which this communication and collaboration are especially important.

Some other issues discussed with regard to these questions of instructional techniques and connections were that:

1. Multiple techniques of teaching and learning must be employed because of multiple goals and multiple audiences.
2. Students must learn to learn from different instructional techniques.
3. Students must learn to see connections for themselves, for instance between the language of mathematics and the context of an application.
4. Writing should be emphasized as a learning tool for making these connections as well as for the sake of clear communication.
5. Algebra has a role in that students should learn to read algebra for meaning (for example, in the equation for calculating a correlation coefficient based on z-scores of the two variables).
6. Students should learn about the importance of units of measurement. For example, students should understand that the slope and intercept of a least squares line for predicting birth weight from weeks of gestation have different units.

The following passage appears on page 34 of *Everybody Counts* [National Academy Press, 1989]:

> *Mathematical Science is a term that refers to disciplines that are inherently mathematical (for example, statistics, logic, actuarial science), not to the many natural sciences (for example, physics) that employ mathematics extensively. For economy of language, the word 'mathematics' is often used these days as a synonym for 'mathematical sciences', as the term 'science' is often used as a summary term for mathematics, science, engineering, and technology.*

This passage points out the difference between client disciplines (such as physics, biology, and engineering) and what we are calling a partner discipline, namely statistics. Most institutions do not have a separate department of statistics, and we feel that for such institutions the mathematics department should take the responsibility for housing the discipline of statistics. That is, these departments must be mathematical sciences departments. For this reason, the workshop considered the proper place of statistics within a mathematical sciences curriculum and addressed the following two sets of questions:

1. What (if any) statistical concepts and methods are important for all students majoring in a mathematical science, particularly in the first two years? What statistical concepts are essential? What statistical skills are essential? What features should characterize mathematics majors' study of statistics? What role should technology play in this statistics instruction? What impact does reform in pedagogical methods have?
2. What statistical concepts and methods are important for students in quantitative disciplines, particularly in the first two years?

For students majoring in a mathematical science:

The 1991 CUPM Report said:

> *Every mathematical sciences major should include at least one semester of study of probability and statistics at a level which uses a calculus prerequisite. The major focus of this course should be on data and on the skills and mathematical tools motivated by problems of collecting and analyzing data. Any statistics course taught now should use a nationally available software package.*

The workshop considered the question of requirements for the mathematics (mathematical sciences) major. Should every major take a data-centered statistics course? If so, what should it look like?

The 1991 CUPM recommendation on statistics has been widely ignored. One institution that has taken it seriously is Gustavus Adolphus College, which instituted a breadth requirement for mathematics majors that is satisfied by either statistics or computer science. A description of Gustavus Adolphus's course is given in Appendix A. The CRAFTY Statistics Workshop affirms the 1991 CUPM recommendation concerning statistics. The arguments for mathematics majors to gain substantial experience working with issues of data and chance are even more compelling now than a decade ago:

1. Data analysis plays a crucial role in many aspects of academic, professional, and personal life.
2. The job market for mathematics majors is largely in fields (e.g., business) that use data.
3. Future teachers will need knowledge of statistics and data analysis to be current with the new NCTM Standards and with the new and very popular AP statistics course.
4. The study of statistics provides an opportunity for students to gain frequent experience with the interplay between abstraction and context, which we regard as critical for mathematical science students to master.

Requiring a statistics course is good for it can also help the mathematics department in that it broadens the appeal of the department to students and potential majors. The group felt strongly, however, that we should not mandate a calculus prerequisite for the first statistics course. While it is certainly preferable for mathematically inclined students to take a statistics course in which they make use of their mathematical knowledge and ability, not all institutions are able to offer a data-centered statistics course with a calculus prerequisite. The group believes that this requirement may have contributed to the minimal impact of the recommendation and argues that a statistics course lacking a calculus prerequisite can still be an important educational experience for a mathematics major.

We recommend that the course should emphasize the collection and analysis of real data, to show students in this first exposure what the discipline of statistics is primarily about. We recommend that this course adhere to the fundamental principles articulated by Cobb (1992) on behalf of the ASA/MAA joint committee:

1. More data and concepts, fewer derivations and recipes; automate calculations using a modern statistical package
2. Emphasize statistical thinking: the omnipresence of variability and the importance of data production
3. Foster active learning: student projects, group work, activities, writing, oral presentations

A required course for all mathematical science majors should embody these principles, which have the potential of presenting an authentic view of statistics. These broad principles leave room for a variety of first statistics courses for students, and in the appendices we have included descriptions of several examples that are diverse and suggest the possibilities. Courses include:

1. Carolyn Dobler's introductory course at Gustavus Adolphus (Appendix A)
2. A first statistics course in experimental design developed by George Cobb at Mt. Holyoke (Appendix B)
3. A new archaeometrics course, developed by Don Bentley at Pomona (Appendix C)

4. A new data analysis course at the post-calculus level designed by Rossman, Chance, and Ballman (Appendix D)
5. A case studies based mathematical statistics course developed by Deb Nolan and Terry Speed at Berkeley (Appendix E)
6. A time series course developed by Robin Lock of St. Lawrence (Appendix F)
7. A course emphasizing Bayesian statistics developed and taught at Duke University by Dalene Stangl and colleagues (Appendix G)

The course descriptions make it clear that the three principles articulated by the Focus Group are central to those courses and at the heart of each is the focus on data.

Exposing students to such a statistics course as undergraduates allows them to see an area of mathematics that they are probably unfamiliar with (although this may become less true as new NCTM standards are implemented) and this broadens the appeal of the mathematics department to students. Notice that most of the examples described in the appendices are courses that are at the 100- or 200-level, so that early exposure enhances the chances that students will want to take more mathematics or statistics, which should improve departmental enrollments.

For students majoring in quantitative disciplines other than mathematics:

We make a distinction between subjects whose content is often mathematical (engineering, physics, genetics, chemistry, economics, environmental science) and subjects that use data a lot (psychology, sociology, business, biology). Non-majors can also benefit from courses described in the previous sections, especially the more quantitatively ambitious ones. Indeed, most of these courses can be taken before students have declared a choice of major. At the same time, we feel a need for courses that serve the direct needs of client disciplines for students who are less quantitatively ambitious and that serve the general educational needs of the institution. As mentioned earlier in this report, the general education introductory statistics course has been described at length in the literature.

A good survey article on the current state of the general education course comes from a position paper presented in August of 2000 at the ASA's Undergraduate Statistics Education Initiative. This article by Garfield, Hogg, Schau, and Whittinghill (2002) has been published in the Journal of Statistics Education. The paper is "Best Practices in Introductory Statistics", by Joan Garfield, Bob Hogg, Candace Schau, and Dex Whittinghill and is available on line at:

`www.amstat.org/meetings/jsm/2000/usei/usei_1st.PDF`

For students who will take only one college course in quantitative or mathematical reasoning, most will be better served, we think, by a course like the one described in this paper than by a college algebra or precalculus course. Many of these students will find careers that require an appreciation and facility with data and all will be citizens in a world where statistical information is pervasive.

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APPENDIX A: Statistics Requirements at Gustavus Adolphus College

(Carolyn Pillers Dobler, Dept. of Mathematics and Computer Science, Gustavus Adolphus College)

In 1993, the Department of Mathematics and Computer Science at Gustavus Adolphus College revised the mathematics major, following the 1991 CUPM report on the undergraduate major. The revised major consists of a basic core (Calculus I, II, III, Linear Algebra, Theory of Calculus), a breadth requirement (Introduction to Computer Science, Introduction to Statistics), a classical core (one course from complex analysis, real analysis, or modern algebra), a depth requirement (an upper level, two-semester sequence), an applied course, and either a capstone course or senior oral exam.

The 1991 CUPM report specifies a course in probability and statistics having a calculus prerequisite, but with the major focus on data, including data collection, data description, and data analysis. In the initial discussions, the department hoped to create a new course, designed specifically for mathematics majors, but staffing restrictions precluded this. Much discussion ensued over which existing course would be appropriate to meet the CUPM guidelines. The upper-level probability and mathematical statistics course was too theoretical, whereas the lower-level introductory statistics course might not be rigorous enough. One option would be to include a data analysis component in the theoretical course, which had been attempted unsuccessfully in the past. Another option would be to make the appropriate calculus connections in the introductory course. Further, the department considered the secondary education mathematics majors. Teacher licensure in the State of Minnesota requires secondary mathematics teacher to have a course in probability or statistics. The department felt that the data-oriented course was much more appropriate for the future teachers, particularly since many may eventually be teaching statistics in a high school. The department decided to include the lower-level introduction to statistics course (with a Calculus I prerequisite) as the required course in the mathematics major.

The course, Introduction to Statistics, is taught each semester to about thirty students, most of whom are mathematics or science majors. The text is Introduction to the Practice of Statistics by David S. Moore and George P. McCabe. Although the text does not specifically use calculus, through supplementation, connections to calculus are made through the topics of the normal distribution, least-squares regression, and probability. The course follows 1991 Statistics Focus Group recommendations to emphasize statistical thinking, to include more data and concepts (less theory, fewer recipes), and to foster active learning. The course is data-driven, emphasizing analysis and interpretation. Students gain experience in technical writing through a semester-long group project and an individual data analysis project. Although there is no formal (weekly) laboratory component, computer lab exercises are interspersed throughout the semester as appropriate.

Mathematics majors typically take the course in their sophomore year, often when they are also taking a more proof-oriented course such as Linear Algebra or Theory of Calculus. Although the student may be taking two mathematics courses in a semester, a balance is achieved between theory and application.

Overall, the department believes the requirement of introductory statistics has been beneficial to most mathematics majors, particularly to those in secondary education. In addition, there has been an increase in the number of students who have selected the upper-level probability and mathematical statistics sequence as their depth requirement after being exposed to statistics in the introductory course. Because most of the students in the upper-level sequence have had the introductory course, the theoretical material can be connected to the applications with which the students are familiar.

APPENDIX B: An Introductory Course that Emphasizes Design of Experiments

(George W. Cobb, Department of Mathematics and Statistics, Mount Holyoke College)

The course described here provides an applied introduction to the design and analysis of experiments for students with no previous background in statistics. At Mount Holyoke, a liberal arts college for 1800 women undergraduates, the course has served as an alternative first course in statistics for our few statistics majors and some of the many students majoring in biology, psychology, and environmental science. Although no statistics or calculus is required or assumed of students who take the course, we do require a semester of (any) 100-level work in mathematics or statistics, and accordingly, the course is listed as 200-level. The next few paragraphs lay out the goals and sketch the approach of the course. Voluminous additional details can be found in a book *An Introduction to the Design and Analysis of Experiments* (Springer, 1997), which evolved, sometimes painfully, over the first 15 years the course was taught at Mount Holyoke.

Goals

Successful students learn to:

- to choose sound and suitable design structures;
- to recognize the structure of any balanced design built from crossing and nesting;
- to explore real data sets using a variety of graphs and numerical methods;
- to assess how well the standard assumptions of analysis of variance (ANOVA) fit a data set, and if the fit is poor, to choose a suitable remedy such as transforming to a new scale;
- to decompose any balanced data set into "overlays" (components corresponding to the factors in the design) and to find the parallel decompositions of the sums of squares and degrees of freedom;
- to construct the interval estimates and F-tests of formal inference; and
- to interpret numerical patterns and formal inferences in relation to the relevant applied context.

Approach

Four main features characterize the approach of the course: modest technical demands, a focus on a small set of recurring basic ideas, an emphasis on examples and context rather than derivations, and a preference for active learning.

- *Modest technical demands*. To make the material accessible to students who do not have strong algebraic skills, the course presents ANOVA visually, without formulas, by representing factors of a design as partitions of the data, and computing the usual summaries by "sweeping out" the averages that correspond to the partitions. (This approach is an elementary analog of taking orthogonal projections into subspaces of Rn.)
- *Small set of recurring ideas*. Our aim is for students to develop a flexible ability to apply a few broad principles. One instance is the approach to decomposition of the data via partitions and averages, described above. Another is an approach to design based on two principles (randomization and blocking) for assigning treatments to experimental units, and one principle (factorial crossing) for choosing a treatment structure.
- *Examples and context*. Students begin by learning, via a series of concrete examples, four specific designs (completely random, complete block, Latin square, split plot/repeated measures) built from the three basic principles. Later, these same principles are used to extend each of the designs to a family of similar designs. Over the course of the semester, specific data sets return again and again, so that students get to know them and rely on them to learn the general principles and structures.

- *Active learning*. Applied statistics lends itself naturally to discussion: for example, on the merits of completely random versus complete block designs to see whether carpeting raises levels of airborne bacteria in hospital rooms, on the choice between logarithms and reciprocal roots for analyzing the effect of day length on the concentration of a neurotransmitter in the brain of a hibernator, or on the way a few mildly deviant observations should qualify conclusions from the F-test in a study of babies learning to walk. Discussions of these and other issues arise easily when the course emphasizes real data sets and their applied context. For the same reason, writing assignments are easy to incorporate into the homework. Both the discussions and the writing help prepare the way for a substantial term project that students complete over the second half of the semester, and present first orally to the rest of the class and then in a final paper.

APPENDIX C: Archaeometrics — An Interdisciplinary Introduction to Statistics

(Don Bentley, Department of Mathematics, Pomona College)

Archaeometrics, an interdisciplinary course, was first offered at Pomona College during the Spring semester of 1996–97. As the name indicates, the topics addressed in the course are applications of statistical models to the field of archaeology. There are no prerequisites for the course, and it satisfies the general education requirement of a course in statistical reasoning. It was designed to be attractive to students in the humanities who feel less than secure in their quantitative skills and need a course to meet this General Education requirement.

Archaeology is an inductive science. Information gathered in excavations is used to make inference about the history and culture of the societies that occupied the studied locations. The discipline of statistics provides tools to assist in inductive reasoning. Dever, in writing on the "New Archaeology"[1] points out that "… the *general* influence of the explicitly scientific school is seen in the deliberate development of research design, in the emphasis on problem solving, and in the testing of hypotheses in general, which increasingly characterized the more sophisticated American project of Syro-Palestinian archaeology …." The structure he describes falls right in the center of the field of statistical inference.

While attending a course titled Archaeology of the Lands of the Bible, this author was continuously amazed at the number of instances in the assigned readings where statistical modeling could have been applied to assist the archaeologists in making inferences from their data. In some instances the models would have suggested further questions to ask of the data, or provided direction for seeking additional data. In other instances, the inferences made were logically invalid. By the end of the semester he was convinced that there was a great need for the new field, archaeometrics, and it would provide a great opportunity for an exciting interdisciplinary course.

In considering the structure of the course it was decided that it should be directed towards students with an interest in areas in which archaeology is an important tool, such as religion, anthropology, and history. To attract this audience precludes setting any mathematics or statistics prerequisite. But the course should also appeal to students in sciences who are seeking areas of application for their concentrations. For this reason there is no prerequisite for the course.

The statistics topics presented in the course are different from those found in the traditional introductory statistics course. They emphasize those models and methodologies that are appropriate for the data and problems encountered in archaeological investigations. As an example, the Poisson distribution is extremely important for archaeological studies as it is, in many instances, the appropriate model to describe the behavior of random data. It is quite common that only a small percentage of a tel is excavated (on the order of 5%). Thus excavations are restricted to extremely localized areas which causes a great deal of dependence to exist within the data. Students must gain a good understanding of the consequences of this dependence. Sampling bias is also very common in archaeological studies since the decision as to where to search for data is made by determining the most likely areas to produce the type of information desired. Another extremely common problem is that inferences are frequently made on the basis of sparse data. Therefore, the concept of power of a test needs to be emphasized.

Above are examples of topics in statistics that are important for the archaeologist to understand when making inferences from collected data. Another area of statistics that is not covered in introductory statistics courses, but could be of benefit to biblical archaeologists, is logistic regression. This methodology would assist archaeologists in determining which specific potential excavation site is most likely to yield particular types of artifacts.

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[1]William Dever, "Syro-Palestinian and Biblical Archaeology." The Hebrew and its Modern Interpreters, eds. Knight and Tucker, (Chico, CA: Scholars Press, 1985),49.

APPENDIX D: A Data-Oriented, Active Learning, Post-Calculus Introduction to Statistical Concepts, Methods, and Theory

(Allan Rossman, Department of Mathematics, Dickinson College; Beth Chance, Department of Statistics; Cal Poly-San Luis Obispo; Karla Ballman, Mayo Clinic)

We have each taught and are continuing to develop a course that provides post-calculus students with an alternative introduction to probability and statistics that focuses on data and active learning. The course incorporates many of the features that have emerged in "Stat 101" courses (analysis of genuine data, focus on conceptual understanding, student exploration and hands-on activities, use of computer tools) and provides a more balanced introduction to the discipline of statistics than the standard sequence in probability and mathematical statistics. The course differs from the algebra-based introductory course by utilizing students' calculus knowledge and mathematical abilities to explore some of the mathematical framework underlying statistical concepts and methods. We have taught this course to mathematics, statistics, computer science, economics, science, and engineering majors. Previously, we utilized a text that focused on data analysis concepts, such as Moore and McCabe, while developing supplementary activities to encourage student explorations of the mathematical underpinnings. We are now incorporating these activities into a stand-alone text.

Course Principles and Goals:

The guiding principles of the course are for students to:

- Develop understanding by conducting investigations of statistical concepts and properties.
- Learn about probability models in the context of statistical ideas, applied to real data.
- Utilize their mathematical skills such as their knowledge of functions, graphically and analytically, as well as counting techniques and calculus optimization methods.
- Develop an assortment of problem-solving skills as they approach data from graphical, numerical, and analytical perspectives.
- Use technology as a tool for simulations, graphical displays, exploration, etc.
- Analyze data from a variety of fields of application, including data from scientific studies, popular media, and self-collected.

By the end of the course, we hope students will have:

- acquired proficiency with a variety of specific data-analytic techniques, including exploratory data analysis, confidence intervals, tests of significance, t-tests and intervals, regression analysis, contingency table analysis, analysis of variance;
- gained an understanding, at both conceptual and mathematical levels, of fundamental statistical ideas, such as variability, randomness, distribution, association, transformation, resistance, sampling, experimentation, confidence, significance, power, model.

We also aim to help students develop skills of:

- reading technical material
- using computer software
- working productively with peers
- expressing themselves in written and oral presentations

Content:

Students are introduced to statistical ideas at the conceptual level in a genuine context, then to methods of analysis, and finally to the theoretical foundation. Students then continually revisit important ideas in new settings. For example, an early unit on randomization and significance begins with the results from a scientific study to motivate the question, uses simulation to explore the likelihood of obtaining such sample results by randomization alone, and then develops the combinatorial tools necessary to determine the exact p-value. The focus throughout this unit is on the interpretation of p-value and the proper conclusions that can be drawn from an experimental study. The next unit then compares these results to the analysis and conclusions that can be drawn from an observational study. A tentative outline of the course syllabus includes:

- Variation, randomization, comparisons, significance (analysis of two-way tables)
- Observational studies, confounding, association vs. causation
- Sampling, binomial model, tests of significance, power
- Large-sample approximations, normal distribution
- Confidence intervals for categorical variables
- Analysis of quantitative variables (including t-tests and intervals)
- Analysis of bivariate data (regression analysis, analysis of variance)

Use of Technology:

The course is taught in a computer-equipped classroom and makes fairly extensive use of technology. The computers play an important role in this course in three ways:

- performing calculations and creating graphics necessary for analyzing data
- conducting simulations to approximate long-run behavior of random phenomena
- addressing "what if" questions, allowing students to explore probability and& statistics concepts

A statistical software package is essential for teaching the course; we use Minitab. We also use the spreadsheet package Excel for facilitating students' investigations. Java applets created specifically to help students to visualize and explore statistical concepts are also used.

Assessment:

Student work in this course is evaluated on the basis of regular assignments, quizzes, fairly standard exams, and more extensive written assignments and projects. The homework assignments are a combination of interpretation and presentation of results, practice on techniques, and mathematical extensions. Students are also expected to be able to utilize a statistical package and interpret its output, and conduct simulations.

More information about this course and project can be found at `www.rossmanchance.com/scmt/`.

APPENDIX E: A Case-Studies Based Mathematical Statistics Course

(Deb Nolan, Department of Statistics, University of California at Berkeley)

Nolan and Speed have developed a course and accompanying text (Stat Labs) that teaches undergraduate upper-level mathematical statistics through the use of in-depth case studies. They call these case-studies "labs."

It is via the labs that mathematical statistics is introduced, which leads to an integration of statistical theory and practice in a way not commonly encountered in an undergraduate course. The labs raise scientific questions that are interesting in their own right, and they contain datasets for use in addressing these questions. The context of the scientific question is the starting point for developing statistical theory.

The labs are central to the course. Each lab is a substantial exercise with nontrivial solutions that leave room for different analyses. They serve to motivate and to provide a framework for studying theoretical statistics, and they give students experience with how statistics can be used to answer scientific questions. An important goal of this approach is to encourage and develop statistical thinking while imparting knowledge in mathematical statistics. Through the labs students develop their quantitative reasoning and problem-solving skills in a broad multidisciplinary setting. They become practiced in communicating their ideas orally and in writing, and they become versed in the use of statistical software.

Audience and Course Prerequisites

The labs are designed for use in a mathematical statistics course for juniors and seniors. They have been used by Nolan and Speed for two such courses, one for mathematics and statistics majors, and one for engineering and computer science majors. Both courses require students to have studied calculus for two years and probability for one semester. There is no statistics prerequisite for the course. The course enrollments range from 20 to 60 students. Classes meet three hours a week with the instructor and one hour a week with a teaching assistant.

In a typical semester, roughly one week is spent on summary statistics, and about three weeks are spent on each of the following four areas: sampling, estimation and testing, regression and simple linear least squares, and analysis of variance and multiple linear least squares.

Lab Organization

The material for a lab is divided into five main parts: an introduction, data description, background material, investigations, and theory. In the introduction, a clear scientific question is stated, and motivation for answering it is given. The question is presented in the context of the scientific problem, and not as a request to perform a particular statistical method. Documentation for the data collected to address the question is provided next. It includes a detailed description of the study protocol, as appropriate. Material to put the problem in context is provided in the background section. Information is gathered from a variety of sources, and is presented in non-technical language. The idea is to present a picture of the field of interest that is accessible to a broad college audience.

Suggestions for answering the question posed in the introduction are provided next in the investigations section. These suggestions are written in the context of the problem, using very little statistical terminology. The ideas behind the suggestions vary in difficulty, and are grouped to enable the assignment of subsets of investigations. Also included are suggestions on how to write up the results, e.g., as an article for a widely read magazine; as a memo to the head of a research group; or as a pamphlet for consumers. Often included in the report will be an appendix containing more technical material.

The theoretical development of the statistical concepts and methodology appear after the problem is introduced, at the end of the lab handout. The material includes information on general topics in statistics, as well as topics more specific to the individual lab.

Student Work

The labs are chosen by the instructor according to the topic (theoretical or practical) and the background of the students. They are divided between those labs that are discussed primarily in lecture and those that require students to do extensive analyses outside of class and to write short papers containing their observations and solutions. Typically students write reports for four labs, about one for each of the five main topics in the course. The students find this work very challenging, and they typically work in groups of twoor three on their lab assignments.

Bringing the computer into the theoretical course enables us to go far beyond the traditionally small, artificial examples found in textbooks. But care is taken to keep the demands made upon students at an appropriate level. Assistance on how to use statistical software is provided in the weekly section meeting, and handouts with sample code are also provided. Often the section meets in a laboratory room, where students double up at workstations to work on the assignment and the teaching assistant provides advice as needed.

Nolan and Speed have found that when using these labs several fundamental changes take place in the classroom. The format of lectures changed. More time is spent on determining how to answer general scientific questions using statistical analyses and on deriving a statistical method from its application to a specific problem. Less time is spent covering many small examples constructed to illustrate a single statistical technique, yet all the basic material traditionally covered in the course is still covered.

Lectures also include: discussion of the background to a particular problem, where students who have taken courses in related fields can bring their own expertise to the discussion; and motivation of the theoretical material through discussion of how to address a problem from a lab. In addition, for labs on which students are to write reports, we hold regular question and answer periods where students raise concerns about their work. Roughly about one class period in three is spent on these types of activities. The remainder of time is spent in a more traditional presentation of theoretical results.

More information about Stat Labs can be found at `www.stat.berkeley.edu/users/statlabs`.

APPENDIX F: Time Series Analysis—An Alternative Introduction to Applied Statistics for Mathematics Students

(Robin H. Lock, Mathematics Department, St. Lawrence University)

Time Series Analysis is offered at St. Lawrence University every other year in the spring semester. The course is dual listed under mathematics and economics so students may use it as an upper level elective towards either major. The primary audience is juniors and seniors with an occasional sophomore and mostly mathematics, economics, or combined math/econ majors and statistics minors. Prerequisites are just two semesters of calculus. Although students may have already seen some statistics through our introductory service course, mathematical statistics, or an econometrics course, we assume no particular background in statistics. Thus the course serves as a first exposure to statistical ideas for some mathematics students and, fortunately, the content is sufficiently disjoint from other offerings that students with previous experience will see lots of new material. This flexibility increases the pool of students for the course.

A course in time series analysis offers a number of unique opportunities for introducing mathematically oriented students to the applications of statistics. The data structures encountered in the study of time series are typically very straightforward—often just a single univariate series of historical values. Yet the statistical methodology used in the analysis can require a broad range of mathematical sophistication while still illuminating fundamental concepts of applied statistics. Real world applications are immediate and compelling. Wouldn't many students today be interested in methods for predicting where the stock market will go?

The general notions of an underlying model for some real world phenomenon, estimation of its parameters from data, and diagnostic checking of the model assumptions are central themes in statistics. The models encountered in forecasting are fairly intuitive, yet can be used to effectively illustrate important statistical principles such as parsimony, variability in parameter estimates, construction of prediction intervals, and criteria for choosing between competing models. The clear interaction among the "identify," "estimate," and "forecast" steps of the Box-Jenkins approach can be fruitfully applied to many other statistical situations. The analysis of residuals to check model assumptions, suggest alternative models, or gauge the accuracy of the fit is a featured part of time series methodology that is often neglected in traditional introductions to statistics. Similarly, statistical graphics—plots such as the time series itself, a differenced or transformed series, residuals, or sample autocorrelations—are used at many points throughout a time series analysis. This integration of graphics into all phases of an analysis is an important part of modern statistics.

What about mathematical content? A key point for attracting mathematically oriented students to such a course is the inclusion of nontrivial applications of mathematics. These must go beyond simply "plugging & chugging" with more complicated formulas than the standard introductory service course. As one example, consider the duality between autoregressive (AR) and moving average (MA) models. The question of stationarity/invertibility of such models, depending on the location in the complex plane of the roots of a characteristic polynomial determined by model parameters, can be quite challenging. In fact, the process of inverting an MA model to get an AR model (of infinite degree) may seem to be only of mathematical interest, until one sees an example where a complicated AR model requiring several parameters is effectively replaced by a more efficient MA model with perhaps just one parameter. The use of the difference operator on a time series provides an interesting analogy to the differentiation process that is familiar to most students from calculus. In fact, the use of operator notation (differencing and backward shifts) in specifying and manipulating many time series models is good experience for mathematics students in dealing with the general notion of an abstract operator. Of course, one can also see the traditional calculus-based derivation of least squares estimates in regression, with a neat special case for students to work on their own when the dependent variable can be assumed to take on the values $t = 1, 2, \ldots, n$. One could even dive into the spectral analysis of a time series, although that would probably be beyond the mathematical sophistication of most undergraduates.

Our time series syllabus typically includes the following topics:

1. **Basics**: models, parameters, estimators, residuals, mean squared errors, probability, normality, expectations, autocorrelation.
2. **Linear regression on t**: least squares estimation, tests for parameters, transformations, diagnostics on residuals, confidence intervals for forecasts.
3. **Exponential smoothing**: simple and double exponential smoothing, choice and effect of smoothing constants, confidence intervals.
4. **Box-Jenkins methodology**: development of the general ARIMA model with emphasis on special cases, patterns in autocorrelations and partial autocorrelations, invertibility, stationarity, differencing, autoregressive / moving average duality, model identification, estimation, and forecasting (with considerable computer assistance).
5. **Seasonal models**: based on cosine trends, ANOVA means, or seasonal ARIMA terms.
6. **Models based on one or more other series**: simple and multiple regression, selection of "predictor" series.
7. **Mixed models**: combinations from among (2), (4), (5), and (6) above.

Some additional topics that we typically do not get to (either due to time constraints or complexity of the material) include Winter's method, formal estimation methods for ARIMA models, matrix notation for regression, Fourier methods and spectral analysis.

The course relies heavily on actual data for motivation and illustration. Fortunately, a wide variety of excellent sources (including websites) for time series data are readily available. In contrast to some other areas of statistical application, published reports involving time series seem to be more likely to include all the data, either explicitly or graphically, rather than just summary statistics. Thus students have relatively little difficulty finding interesting data on their own for projects. We use Minitab as a statistical package for computation and graphics, although a number of specialized time series packages would also work well.

In summary, a course in time series analysis can be used as a mechanism for attracting and introducing mathematics students to the field of applied statistics. They can encounter many of the fundamental principles of statistical thinking in a mathematically interesting setting and, hopefully, be encouraged to pursue further work in statistics.

APPENDIX G: A First Course in Statistics for Mathematics Majors at Duke University

(Dalene Stangl, Institute of Statistics and Decision Sciences, Duke University)

At Duke University, the introductory statistics course for mathematics majors was designed with the belief that real-world applications motivate and drive the field of statistics. With this in mind two faculty, Mike West and Robert Wolpert, have designed a course that balances theory and application. Student curiosity is piqued on day one, when they are presented with two quotes:

Bruno de Finetti, on foundations: *Probability does not exist.*

George E.P. Box, on mathematical and statistical modeling: *All models are wrong, but some are useful.*

The course is an introduction to the concepts, theories and methods of statistical modeling and inference. It focuses primarily on ideas and methods of modern Bayesian statistical science, but it also teaches and contrasts classical methods. Statistics is a vast field, and a first one-semester course must be a broad introduction overlaying key conceptual ideas. The over-arching goal of the course is to provide this introduction, exploring the foundations of scientific reasoning and inference and rousing curiosity via applications in medicine, genetics, policy, astronomy, physics, economics, finance, education, and many other fields. Students use statistical software to explore data and statistical models. Students learn the theory and software that enable scientific learning in the face of uncertainty.

Topics covered:

Reasoning with Probabilities: Scientific Learning and Inference

Learning from data and observation: Pre-data and Post-data probabilities

Bernoulli trials and binomial sampling models

Likelihood and maximum likelihood

Exponential, Poisson, Normal and other sampling models

Simulation of probability distributions for inference and model assessment

Sampling distributions and pre-data probabilities

Perspectives on statistical inference: Classical and Modern

Significance testing and p-values

Honest prediction

Linear regression models, including various special classes
(factor models, multiple regression, time series regression, ...)

Graphical data display and exploration

Statistical computing with *S-Plus*

The course draws on material from the traditional text *Probability and Statistics* by Morris H. DeGroot, published by Addison Wesley (2nd Edition). This book covers basic elements of both Bayesian and non-Bayesian approaches to statistics and has been a standard introductory text for many years. Many other texts cover similar material on the non-Bayesian side. Chapters 1–5 provide in-depth coverage of prerequisite probability theory. Supplementary material beyond the scope of the text, particularly on applied statistical methods is provided in handouts from the website `www.stat.duke.edu/courses/` under STA114. Supplementary texts include *Bayesian Data Analysis* by Andrew Gelman, John B. Carlin, Hal S. Stern and Don B. Rubin, published by Chapman & Hall. This book goes beyond the scope of this course, but it is a truly excellent text for both statistical modeling and applications. More extensive material on the theory side, but less so on the application side, appears in *Bayesian Statistics: An Introduction* by Peter M Lee, published by Arnold UK and distributed in the USA by Wiley (2nd Edition). The software used for the course is *S-Plus*. This software is available in student edition for Windows from Duxbury Press.