Technical Mathematics:
Electronics, Telecommunications, and Semiconductor Technology[1]

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Summary

All three of these professions work in the electronics area, but the level of mathematics needed by the different fields varies considerably. Semiconductor technicians need little more than ratios, elementary algebra and elementary statistics. Electronics and telecommunications technicians need more mathematics, including complex numbers, trigonometry, elementary geometry, elementary statistics, and differential and integral calculus. All areas want their students to be able to use technology, such as spreadsheets, to analyze mathematics.

Better communication between mathematics and technology departments was considered vital. Suggestions included having mathematics instructors take courses in technical areas to gain a better feel for how mathematics is used, and having technical mathematics faculty be members of technology department advisory boards.

Students need more instruction on how to learn and analyze: scenarios, word problems, and open-ended situations should be used. They need to know which type of equation to select as the method for solving a particular problem. Team projects and collaborative learning activities are essential. Teaching with calculators might be a disservice if students meet only computers in the workplace.

Narrative

Introduction and Background

Technicians are essentially troubleshooters and repair persons. Some work in the field repairing equipment such as copy machines while others work at a test bench. Some work for manufacturing companies repairing assembly equipment while others work in engineering laboratories.

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Sometimes a technician is the highest trained technical person in the company, while in other situations technicians work for a research engineer. In the past, technicians would locate and replace discrete components of a system, but the troubleshooting of the future is at board level and requires a more systemic approach.

A Workforce Example

Although specific to the Semiconductor industry, the Maricopa Advanced Technology Education Center (MATEC) has provided a clear definition of what SEMATECH and SEMI/SEMATECH consider as a Semiconductor Equipment Technician's job. This definition provides generic guidelines for the skills required for *any electronics oriented technician* in today's job market.

Required Education and Experience: Associate degree in electronics, semiconductor manufacturing, microelectronics *or* related technical field or equivalent experience.

General Job Duties: Monitor, maintain and perform a variety of complex repairs on semiconductor wafer fabrication equipment to ensure uninterrupted production flow. Perform periodic preventive maintenance procedures as defined by specifications.

Provide technical support in the form of troubleshooting, installation, diagnostics, adjustment, repair, modification, assembly and calibration of equipment according to specifications, blueprints, manuals, drawings and verbal or written instructions. Use a structured and comprehensive method to identify the root cause of process or equipment malfunction; implement corrective action after thorough analysis to increase probability of the right fix the first time, based on product quality parameters.

Perform electrical, mechanical software troubleshooting and maintenance for related equipment tools, cable assemblies and fixtures. Check and calibrate tools, equipment and fixtures using test and diagnostic equipment as required. Clean and lubricate shafts, bearings, gears and other parts of machinery.

Assist in the layout, assembly, installation and maintenance of pipe systems and related equipment. Maintain and monitor maintenance parts stack. Maintain accurate records and logs of work performed, modifications, calibrations, and adjustments and parts inventory.

May perform equipment and fixture modifications as directed by manufacturing engineers. Equipment used includes office equipment, power supplies, oscilloscopes, logic analyzers, volt meters, soldering irons, hand tools, power tools and personal computers or other hand/power tools and test equipment. Maintain proficiency in programmable controllers, microprocessors, control circuits, analog/digital circuits, motors and troubleshooting skills.

Understanding and Content

The most important general goals for the mathematical education of students in technical electronics fields are

- development of formula manipulation skills and understanding of the value of the formulas via the use of real problems
- development of problem solving skills and critical reasoning.

Mathematical Problem Solving Skills. Students coming into technical areas need more training in problem solving: the identification and description of problems, the translation of English prose into well-formed mathematical equations, and the interpretation of mathematical solutions in terms of physical reality.

Students must learn how to set up a problem using appropriate units and satisfying the requirements of dimensional analysis. They must learn how to identify relevant information from that which is superfluous or extraneous.

Students must be able to apply logical analysis to solve multi-step problems: identify the individual steps and concepts necessary to reach a solution, then use appropriate mathematical tools to execute the plan. At the most basic level students must know which (theoretical) equations are needed to solve a particular problem and then be able to substitute known values into these equations and solve for the desired unknown.

Students must learn to interpret their mathematical solutions in the context of the physical problem with which they began. They should learn how to perform an error analysis. They must also learn to estimate answers and to obtain approximate solutions: this provides an important tool for detecting mistakes and inaccurate answers. In general, to become good problem solvers students must develop a sense of careful time-management, good judgment, and tenacity.

Specific Mathematical Topics. The following are topics that are important for students planning on careers in technical fields of electronics.

Basic Topics

- The metric system
  - Measurements and units
  - Conversions between and within system
  - Dimensional analysis
  - Consistent use of units
- Concepts of perimeter, area, volume
  - "How many atoms are in this cell?"
  - Visualization of inverse-square phenomena
- Estimating and using orders of magnitude

Algebraic Topics and Skills

- Solving equations for particular variables
- Linear equations and slopes
- Quadratic equations
  - Important to test solutions for feasibility.
  - Needed for complex numbers and conjugates
  - Necessary for considering nonlinear effects
- Simultaneous equations (usually two equations)
  - Cramer's rule, Gaussian elimination
  - Matrix methods (for more than two variables)
- Manipulation of inequalities
- Number systems: binary, hexadecimal

Functions

- Definitions of function, dependent variable, and independent variable
- Absolute value functions, step functions, polynomials, linear functions & slopes
- Exponential functions and logarithms (base 10)
- Composite functions
- Limits of functions, zeroes, asymptotic behavior, extrapolation
  - Semiconductor example: where does Ohm's law break down?

Trigonometry

- Basic trigonometric functions: sine, cosine, and tangent
  - Computations with right triangles
  - Relationships with the unit circle
- Only the *most basic* trigonometric identities should be introduced

Radian measure

Graphical analysis of sines and cosines:

Amplitude, phase, frequency, RMS peak, and relationship between RMS and peak.

Vectors

Resultants, addition and subtraction of vectors

Vector products and cross-products (for higher level degrees)

Phasors (vectors in the complex plane representing sinusoidal signals)

Graphs of Functions and Data

Coordinate systems, both rectangular and polar

Use of log and semi-log paper

Curve-fitting techniques (linear regression, etc.)

Graphical analysis

Statistics

Means, medians, standard deviations

Normal distributions

Sigma notation

Know when a process is “drifting” (statistical controls)

Variability in measurements

Difference between population and sample (discrete vs. continuous data)

Descriptive statistics (skew)

Quality control

Differential and Integral Calculus

(This is most appropriate for engineering technicians.)

Difference quotients (“differencing”) as related to velocity, acceleration, etc.

Derivatives and integrals and their meanings

Delta functions

Technology

There should be a balance between students’ utilization of technology versus manual computation and reasoning from theoretical knowledge.

Relying solely on computer-based instruction or video-based instruction is not effective. Use of computer support is more effective. Instructors should demonstrate computational techniques and software during class sessions.

Web assisted or enhanced courses can be very effective. Instructors should consider developing course web sites, including chat groups. Establish e-mail communication and bulletin boards: encouraging students to seek instructional assistance in this manner can be very effective.

Future employers are likely to each have their own preferred computer computation software such as Mathematica, Maple, MathCad, or MathLab. Students should ideally develop proficiency with at least one such program, as well as a working knowledge of spreadsheets. It is also useful for students to develop three-dimensional visualization skills via programs such as AutoCAD.

Instructors should not promote over-reliance on hand-held calculators. In particular, teaching with calculators alone might be a disservice if students ultimately meet only computers in the workplace. If instruction is centered on calculators, then it must be done in such a way that students will be able to carry the knowledge they gain from calculators over to computer applications.

Instructional Techniques

Mathematics instructors should ensure that active learning occurs in their classes. Require students to think on their feet, to present demonstrations in class, and in general to take responsibility for their own education. There should be an emphasis on communicating mathematics, both in written and verbal form; in particular, students should be required to produce written and oral reports. Students need to see instructor derivations in order to appreciate the theory, but they must have classroom activities that develop their own skills at working with the material.

Instructors should accommodate different learning styles. This can be accomplished by using a variety of class session formats: lectures, demonstrations, class participation, and experiments. Lectures delivered in small bursts are more effective than long monologues. Employ drill and practice when necessary: many algorithms and calculational techniques require this. Encourage note-taking: it is beneficial for students to regurgitate what's been said in class. It helps them prioritize information and rethink what they have learned.

Use assignments built around multi-step projects. Such projects are often ideal for collaborative team efforts. Learning how to work effectively in a team is a skill that cannot be overemphasized for future employment. Students should further be encouraged to form their own study groups.

Instructional Interconnections

There needs to be more cooperation and exchange of information and instructional materials between faculty members in mathematics and faculty members in the technical fields. The individual departments are often too isolated and insular. Arrange for interdepartmental visits. Involve faculty members from different departments on advisory boards or in hiring decisions.

Encourage faculty members in the technical fields, as well as faculty in physics and chemistry, to supply meaningful problems to the mathematics instructors. Mathematics instructors should be encouraged to attend the technical courses to gather further useful examples. Everyone wins — students and instructors — because concepts from the various technical and scientific disciplines are then reinforced in the mathematics courses.

Look for ways to more effectively integrate the content of mathematics courses with the other disciplines. In addition to the incorporation of realistic examples, consider having appropriate courses team-taught by faculty from both mathematics and the technical fields. Reach beyond the college or university: invite industry representatives to make presentations in the classroom.

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APPENDIX A: Definitions

MATEC is the Maricopa Advanced Technology Education Center, a Division of Maricopa Community Colleges, Tempe, Arizona.

SEMATECH is Semiconductor Manufacturing Technology. International SEMATECH is a unique endeavor of 12 semiconductor-manufacturing companies from seven countries. Member companies include AMD, Agere Systems, Hewlett-Packard, Hynix, Infineon Technologies, IBM, Intel, Motorola, Philips, STMicroelectronics, TSMC, and Texas Instruments. For more information see `www.sematech.org.`

SEMI/SEMATECH is a consortium of U.S. semiconductor suppliers. Here SEMI is an acronym for Semiconductor Equipment and Materials International. SEMI/SEMATECH and SEMATECH, the sister consortium of U.S. semiconductor manufacturers, were founded in 1987. SEMI/SEMATECH members are suppliers to all areas of semiconductor manufacturing. SEMI/SEMATECH is the primary interface between its members and SEMATECH and International SEMATECH. The purpose of SEMI/SEMATECH is to ensure the health of the U.S. semiconductor supplier infrastructure. As the voice of the U.S. semiconductor supplier infrastructure, SEMI/SEMATECH fosters communication between its members and their customers within and beyond SEMATECH.

Skill standards are the quality standards applied to people. They are specific statements of desired skills and knowledge presented in an observable and measurable form. A standard consists of a condition, a behavior and the standard to which the behavior should be performed.

MATEC has created a document called *Semiconductor Manufacturing Technician Skill Standards*, which can be purchased at `www.matec.org`. This document clearly defines all of the skill standards that have been identified for technicians in the semiconductor industry ranked by importance, proficiency, frequency and difficulty.

Tasks: While it is important to catalogue skills, it is not always possible to determine the skills requisite to each functional area within the semiconductor-manufacturing environment until the tasks required at each level have been ascertained and analyzed. A task, for the purpose of this definition, is an activity or procedure assigned as part of a technician's job in the factory.

Training Levels: Groups of tasks that define the degree of proficiency for a technician's job performance. SEMATECH and SEMI/SEMATECH recognize three general levels of proficiency. These are:

Level 1: Tasks are a **combination of basic tasks** in the process, operations and equipment categories. *The tasks involve limited decision-making and are frequently performed.*

Level 2: Tasks are **intermediate tasks** in each category. *The tasks require decision-making as prescribed in controlled documents (specifications and response flow charts) and are frequently performed.*

Level 3: Tasks consist of **advanced tasks** in each category. *The tasks require decision-making based on individual judgment and are infrequently performed.*

APPENDIX B: Examples and Vignettes

Electronic Examples

These are sample problems from electronics. Not all programs include all of these problems, and not all types of problems are illustrated. This list does not prescribe what teachers of electronics expect math departments to teach; it illustrates applications in electronics.

1. Convert 26 milliamperes to microamperes and to amperes.

   Answer: 26 milliamperes is 26×10^{-3} amperes and is 0.026×10^{-6} amperes which is 0.026 μA or 0.026 microamperes.

2. Find the total resistance of three resistors in parallel. The formula is $R_t = 1/(1/R_1 + 1/R_2 + 1/R_3)$ where $R_1 = 150\ \Omega$, $R_2 = 300\ \Omega$, $R_3 = 900\ \Omega$

   Answer: 90 Ω

3. Find the resonant frequency of a parallel RC circuit containing a capacitor of 20μfd and an inductor of 30 μH. The formula is $1/(2\pi[LC]^{1/2})$

   Answer: 6497 Hz

4. A series RLC circuit contains 100Ω inductive reactance, 60Ω capacitive reactance, and 30Ω resistance. Find the polar resultant. The inductance is a vector up while the capacitance is a vector down and the resistance is to the right. The inductance and capacitance are on the imaginary axis while the resistance is on the real axis.

   Answer: The solution is a vector 50Ω in magnitude at a positive angle of 53.1 degrees as shown in Figure 1.

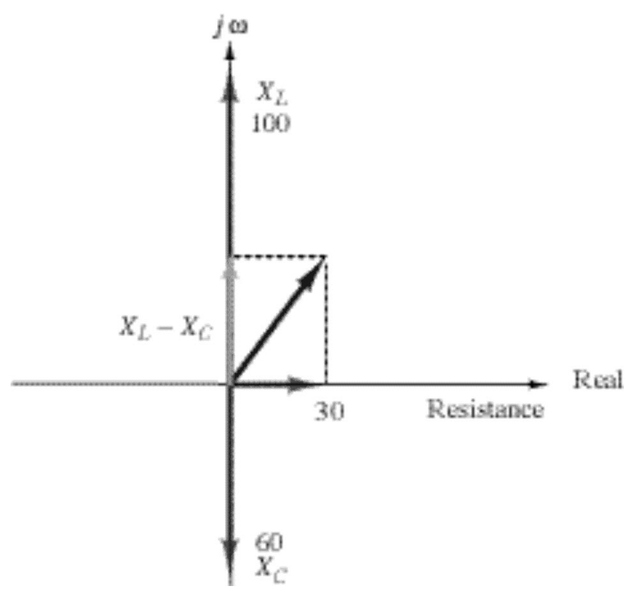

Figure 1

5. A capacitor of 0.01μF is connected in series with a resistor of 100 KΩ and a battery of 100 volts and a switch as in Figure 2. Graph the voltage across the capacitor in response to closing the switch. The

response is described by the equation:

$$V_c = V_F + (V_o - V_F)\varepsilon^{-t/\tau}$$

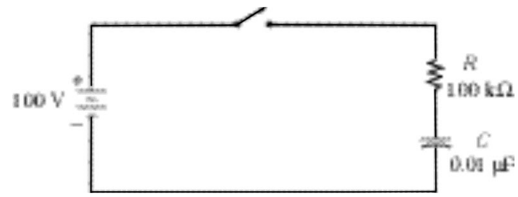

Figure 2

Solution: The full charge voltage is 100 V, which is the 100% level (1.0) on the graph. 75V is 75% of the maximum, or 0.75 on the graph in Figure 3. You can see that this value occurs at 1.4 time constants. One time constant is 1 ms. Therefore the capacitor voltage reaches 75 V at 1.4 ms after the switch is closed.

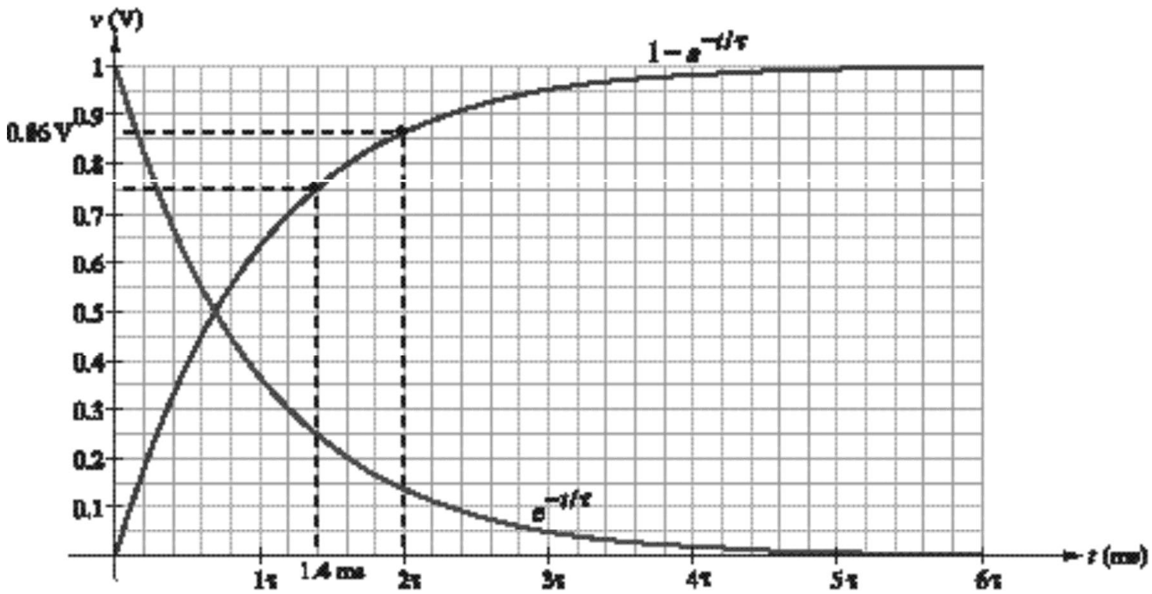

Figure 3

6. Given the circuit in Figure 4, write loop equations and use Cramer's rule to solve for the current in each branch.

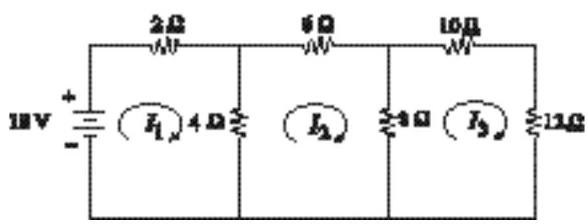

Figure 4

Solution: In standard form the three loop equations are:

$$\begin{aligned} -6I_1 - 4I_2 &= 10 \\ -4I_1 + 18I_2 - 8I_3 &= 0 \\ - 8I_2 + 30I_3 &= 0 \end{aligned}$$

Cramer's rule can be used to find the current in each branch. For example, using Cramer's rule, the current I_2 is given by

$$I_2 = \frac{\begin{vmatrix} 6 & 10 & 0 \\ -4 & 0 & -8 \\ 0 & 0 & 30 \end{vmatrix}}{\begin{vmatrix} 6 & -4 & 0 \\ -4 & 18 & -8 \\ 0 & -8 & 30 \end{vmatrix}}$$

$$= \frac{0-(-1200)}{3240-(480+384)^2} = \frac{1200}{2376}$$

Therefore, I_2= 0.505 A.

General Semiconductor Manufacturing Overview

Use Figure 5 to work Examples 7–9.

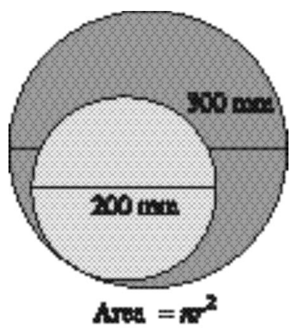

Figure 5

7. What is the manufacturable area of a 200 mm wafer?

   Solution: The diameter of the wafer is 200 mm, so its radius is 100 mm. This means that the manufacturable area is $\pi r^2 \approx 3.14(100 \times 100) = 31{,}400$ square mm.

8. What is the manufacturable area on a 300 mm wafer?

   Solution: The diameter of this wafer is 300 mm, so its radius is 150 mm. Thus, the manufacturable area is $\pi r^2 \approx 3.14(150 \times 150) = 70{,}650$ square mm.

9. How much more manufacturability does a 300mm wafer provide over a 200mm wafer?

   Solution: There are two ways to respond to this question.

   Ratio method: The ratio of the two areas is $\frac{70{,}650 \text{ mm}^2}{31{,}400 \text{ mm}^2} = 2.25$. This means that a 300 mm wafer has an area that is 2.25 times the area of a 200 mm wafer.

Difference method: This method just subtracts the two areas.

$$70{,}650\ \text{mm}^2 - 31{,}400\ \text{mm}^2 = 39{,}250\ \text{mm}^2.$$

The area of a 300 mm wafer is 39,250 mm^2 more than the area of a 200 mm wafer.

An area in the Fab has eight tools, which process 11 different layers for the factory. Each time a wafer passes through the area it counts as one wafer. The area can produce 65,000 wafers a week; at 100% tool availability and 100% tool utilization. **Tool availability** is the measure of how much time during the week that the tool is available for running wafers. **Tool utilization** is the measure of how much time during the week that the tool is working on product. Knowing this please answer the Exercises 10–14.

10. How many wafers will the area have to run on each tool in order to maintain 65,000 wafers a week?

    Answer: 65000/8 = 8125

11. Assuming 100% tool availability and 100% tool utilization, what is the processing time for one lot? (Assume 25 wafers per lot).

    Answer:

$$\begin{aligned} 65000/25 &= 2600 && \text{lots/week per area} \\ 2600/7 &\approx 371.43 && \text{lots/day per area} \\ 371.43/8 &\approx 46.43 && \text{lots/day per tool} \\ 46.43/24 &\approx 1.935 && \text{lots/hour per tool} \\ 60/1.935 &\approx 31 && \text{minutes/lot} \end{aligned}$$

    It will take about 31 minutes to process one lot.

12. Assuming 80% tool availability and 100% tool utilization, how many wafers can the area produce?

    Answer: Using the lot run time of 31 minutes, with the tool available only 80% of the time during the week, we get the following:

$$\begin{aligned} 60 \times 24 \times 7 &= 10080 && \text{minutes in the week} \\ 10080 \times 0.8 &= 8064 && \text{adjusted time for processing wafers} \\ \frac{8064}{31} &\approx 260.13 && \text{lots possible to produce on one tool} \\ 260.13 \times 8 &= 2081.04 && \text{lots possible to produce in area} \\ 2081 \times 25 &= 52025 && \text{wafers possible to produce in area} \end{aligned}$$

    Thus, with 80% tool availability and 100% tool utilization, it is possible to produce about 52,025 wafers in a week (allowing for rounding errors).

13. Assuming 80% tool availability and 80% tool utilization, how many wafers can the area produce?

    Answer: $80\% \times 80\% \times \left(\frac{60 * 24 * 7}{31}\right) \times 8 \times 25 = 41620$

    The area can produce about 41,620 wafers.

14. What would the lot run time have to be in order to meet the goal of 65,000 wafers a week?

Answer: $\frac{0.8 \times 0.8 \times 60 \times 24 \times 7 \times 8 \times 25}{65,000} \approx 19.85$. It will take about 19.85 minutes per lot.

Sample Etch Math Problems

15. When you measure oxide on a wafer before etch, it is 5000 Angstroms thick. After two minutes of etching, it has 2000. Etch rate is defined as the rate by which material is removed from a wafer. What is the etch rate?

Answer:

$$\text{Incoming Thickness} - \text{Outgoing Thickness} = \text{Amount Etched}$$

$$\frac{\text{Amount Etched}}{\text{Etch Time}} = \text{Etch Rate}$$

The amount etched is

$$5000\text{A} - 2000\text{A} = 3000\text{A}$$

$$\frac{3000\text{A}}{2 \text{ min}} = 1500 \text{ A/min}$$

The etch rate is 1500A per minute.

16. According to your engineer, the etch rate in Problem 15 should be 1.5×10^2 per minute.
(a) Does the etch rate that you measured match this number?
(b) How much faster or slower is the above etch rate?
(c) Why is this important?

Solution:

The etch rate should be $1.5 \times 10^2 = 150$ A per minute. In Problem 15 we found that the etch rate was 1500 A per minute. This does not match the engineer's number.

It is 10 times faster than it should be.

These wafers would be scrapped due to over-etching the wafers.

17. A gas flow of 350 ccm can vary by 5% and not disrupt the process. What is the highest the gas flow can be? The lowest?

Solution:

$$350 \times 0.05 = 13.5$$

$$350 + 13.5 \text{ ccm} = 367.5 \text{ ccm}$$

$$350 - 13.5 \text{ ccm} = 337.5 \text{ ccm}$$

Highest = 367.5ccm, Lowest = 337.5ccm.

Sample Thin Films Problem

The etched patterns on a wafer now are called vias. As an insulator or a metal is deposited on a wafer, all patterns that have been etched into the surface are filled (as illustrated in Figure 6).

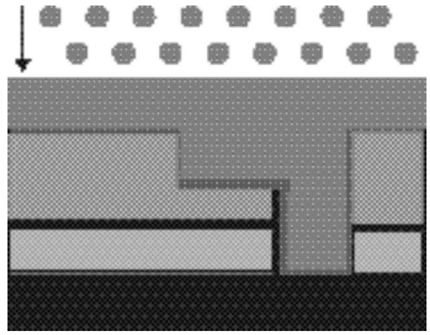

Figure 6

Determine what to set the **Dep Time (DT)** to in order to bring wafer thickness back into specification. Dep Time is the amount of time that an insulator or a metal layer is being deposited on the wafer.

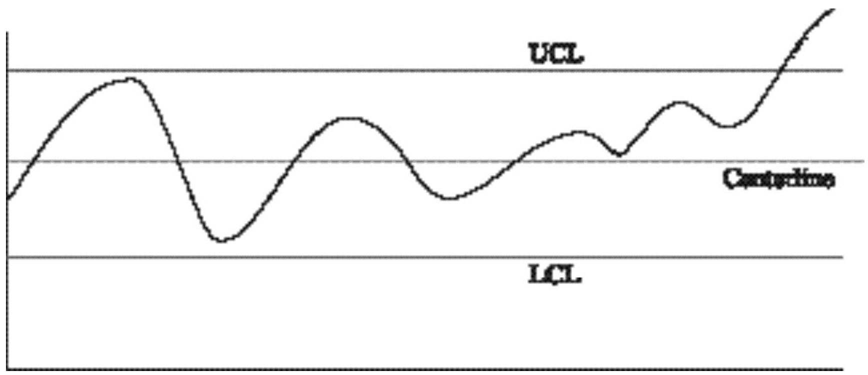

Figure 7

The Upper Control Limit (UCL) is 500 angstroms (see Figure 7) and the Lower Control Limit (LCL) is 400 angstroms. The last test fire was 525 Angstroms. Current Dep Time for the tool is 4.28 minutes. We generally target the tool to centerline to provide for the greatest amount of variability within the process. The Dep Time to Thickness ratio is generally considered to be a linear relationship.

18. What Dep Time will bring the tool back to the centerline thickness?

Answer:

$$\frac{525\text{ Angstroms}}{4.28\text{ min}} = \frac{450\text{ Angstroms}}{x}$$

$$x = \frac{450\text{A} \cdot 4.28\text{ min}}{525\text{A}}$$

$$x \approx 3.66\text{ min}$$

A Dep Time of about 3.66 minutes will bring the tool back to the centerline thickness.

Sample Planarization Problem

After all of the vias in the wafer are filled with a layer, the surface must be planarized (or since it is smoothed it is also called polished) as illustrated in Figure 8.

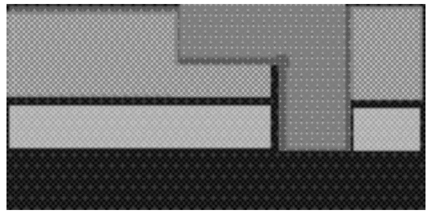

Figure 8

To create such a uniform surface, a precise incoming thickness and outgoing target thickness is required. The tool generally calculates the polish rate automatically.

19. Assuming a planarization rate of 1000 Angstroms/minute, how much time is required on the tool if the incoming thickness is 5000 Angstroms and the desired outgoing target is 3500 Angstroms?

    Solution: First we determine the polish rate:

$$\frac{\text{Incoming Thickness} - \text{Outgoing Target Thickness}}{\text{Required Polish Time}} = \text{Required Rate}$$

    Next we determine the required polish time:

$$\frac{\text{Incoming Thickness} - \text{Outgoing Target Thickness}}{\text{Required Rate}} = \text{Required Polish Time}$$

    Applying these to the given data, we have:

$$\frac{5000\text{ A} - 3500\text{A}}{1000\text{ Angstroms per minute}} = \frac{1500\text{A}}{1000\text{ A/min}}$$
$$= 1.5\text{ min}$$

    So, 1.5 minutes is required to hit the target thickness.