Engineering:
Electrical Engineering

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Summary

This report focuses on establishing the foundation mathematics needed to support the study and practice of electrical engineering with emphasis on the undergraduate level.

To strengthen communication between communities of mathematicians and electrical engineers, we have prepared this document to highlight the areas of mathematics that are most applicable to the study and practice of electrical engineering.

For **outcome objectives**, we propose that the mathematics taught to undergraduate electrical engineering students should help them in developing skills to:

1. Formulate problems in electrical engineering from real life situations,
2. Conceptualize the outcomes of electrical problems,
3. Simplify complex problems and estimate the reasonableness of solutions,
4. Visualize solutions graphically from inspection of their mathematical descriptions,
5. Visualize the form of a time function by inspection of the poles and zeros of its frequency transform,
6. Be able to mathematically model physical reality,
7. Perform rudimentary analysis in electrical engineering,
8. Validate solutions to electrical engineering problems.

Narrative

Introduction and Background

Electrical engineering deals with the manipulation of electrons and photons to produce products that benefit humanity. The design of these products is based on scientific principles and theories that are best described mathematically. Mathematics is thus the universal language of electrical engineering science.

Undergraduate electrical engineering education must provide students with the conceptual skills to formulate, develop, solve, evaluate and validate physical systems. Our students must understand various problem-solving techniques and know the appropriate techniques to apply to a wide assortment of problems. We believe that the mathematics required to enable students to achieve these skills should emphasize concepts and problem-solving skills more than emphasizing repetitive mechanics of solving routine problems. Students must learn the basic mechanics of mathematics, but care must be taken that these mechanics do not become the primary focus of any mathematics course.

More generally, it is vitally important that electrical engineering students recognize the importance and beauty of mathematics in their chosen profession. We feel strongly that students will appreciate the power of mathematics if each mathematics course clearly states its objectives at the outset. Students should be told *what* they are going to study, *why* they are going to study it, and *how* it fits into the engineering profession. This motivation will need to be repeated throughout each course.

Many undergraduate mathematics curricula currently supporting electrical engineering programs could be modified to better meet the needs of these programs. What follows are common weaknesses (from the viewpoint of electrical engineering) seen in many mathematics curricula.

1. Too much time and emphasis are placed on topics that are not widely used while topics that have widespread use often receive cursory treatment. One example is the excessive time and attention spent on various solution techniques for ordinary differential equations. Although understanding the structure of solutions for first- and second-order, constant coefficient differential equations is important for electrical engineering problems, more useful and widely used are Laplace transforms and related techniques. Yet these latter topics are often given cursory treatment in favor of more general structure theory.
2. There is often a disconnect between the knowledge that students gain in mathematics courses and their ability to apply such knowledge in engineering situations. Perhaps, the use of more engineering or real life examples will reduce this disconnect. Based on current learning theory, efforts to focus on underlying principles (not necessarily abstract statements of mathematical concepts) that are applicable in many different contexts are effective in helping students to transfer knowledge.
3. Current mathematics curricula for engineering are front-end loaded. Consequently, as a matter of timing, many topics are presented too early and cannot be reinforced soon enough through engineering applications before students forget the topics
4. Too often, mathematics is taught as a list of procedures or as theorem-proof exercises without grounding the mathematics in reality. While we do not expect mathematics instructors to be well versed in all engineering applications, we would like examples of mathematical techniques explained in terms of the reality they represent. We strongly urge that team taught mathematics courses be considered. Teams would consist of mathematics and electrical engineering professors. We feel that team-teaching could better motivate and enthuse our students.
5. Failure to utilize appropriate technological tools while continuing to focus on mastery of symbolic manipulation often encourages memorization and rote algorithm practice at the expense of conceptual and graphical comprehension. Introducing symbolic manipulation programs, e.g., *MathCAD*, *Mathematica*, *Maple*, would be valuable to subsequent electrical engineering courses whose instructors choose to allow/encourage students to perform routine symbolic and numerical manipulations using such programs.
6. The first two years of mathematics that support instruction in electrical engineering should present students with conceptual understanding of mathematical disciplines other than just single variable calculus, multivariable calculus and ordinary differential equations. Other mathematical subjects that are important for electrical engineering students include linear algebra, probability and stochastic processes, statistics, and discrete mathematics.

Electrical engineering is an exciting and creative profession. Those engineers possessing an understanding and facility of mathematics have an opportunity to be among the most creative of designers. Students need to know and to feel how important, how useful, and how meaningful mathematics is. Many courses stress the drudgery, not the beauty. This needs to be changed.

Electrical Engineering Subdisciplines

To describe our mathematics recommendations in sufficient detail, the undergraduate electrical engineering curriculum is broken down into the following broad areas:

1. Electrical Circuits
2. Electromagnetics
3. Systems, including Controls, Linear and nonlinear Circuits, and Power
4. Signals
5. Design
6. Microprocessor/Computer Engineering

What follows are summaries of the proposed mathematics requirements for each subdiscipline.

1. Electrical Circuits

The electrical circuits course is the passageway to electrical engineering. Of critical interest are the logical thinking skills to analyze electric circuits. In this course, students are introduced to the application of physical laws, e.g., Ohm's, Faraday's and Kirchoff's, in electrical engineering. Students are also introduced to the electrical engineering foundation elements: resistor, inductor and capacitor, and their response *(voltage, current and power profiles)* to DC, steady state AC, and transient stimuli respectively. In most institutions, the circuits course is a two-part series with DC circuit analyses and transient response offered in the first semester and AC circuits and steady state response offered the second semester.

A. DC Circuits. Typical problems in this section involve the simplification of series, parallel and mesh circuits. Analyses of these circuits require setting up, manipulating, and obtaining solutions to algebraic equations. Subtle mathematical skills in the understanding of the circuit problems also include direct and inverse proportionality to enable students to understand voltage and current divider rules respectively.

In the circuit areas dealing with power, and power transfer, knowledge of integral calculus and basic differentiation is required especially for maximum power transfer analysis.

B. AC Circuits. This part of circuit analysis deals with the response of different circuit configurations and elements to steady state sinusoidal inputs. Different mathematical techniques are necessary to simplify the circuits before gaining understanding of the response. The foundation mathematics necessary for the analyses include:

Concept of functions, especially sinusoidal functions. Students need to understand and visualize profiles of basic functions. Use of common real life examples is strongly suggested in teaching this topic.

Application of trigonometric identities to sinusoidal analyses.

Manipulation and representation of sinusoidal functions in Euler, polar, and rectangular coordinates.

Complex algebra.

C. Transients. This topic deals with response of discrete circuit elements, or combinations thereof, to electrical stimuli. At the DC stage, the typical stimulus is the step. The mathematical background required in the analysis includes exponential functions and introductory differential equations. In the latter subject the primary focus should be on ***standard solutions*** to first and second order differential equations with constant coefficients rather than on more general techniques for solving differential equations. The preferred and more useful approach to solving differential equations in electrical engineering is via the Laplace transform. Laplace transform methods reduce differential equation problems to algebraic formats with which students feel more comfortable. This topic should be presented during the first year of undergraduate mathematics.

At the AC stage, the typical stimuli are sinusoidal, triangular and square functions. Usually, the interest in this setting is steady state rather than transient. The Laplace transform still provides the preferred method of analysis because, in addition to reducing differential equation problems to algebraic equation problems, it also incorporates initial conditions in the solution.

2. Electromagnetics

Study of electromagnetic fields and waves is a crucial area in electrical engineering for which understanding of vector algebra and vector calculus is required. The basic laws of electromagnetics are summarized in Maxwell's equations:

$$\text{Faraday's Law: } \vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

$$\text{Ampere's Law: } \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\text{Gauss' Law: } \vec{\nabla} \cdot \vec{D} = \rho$$

$$\text{No Isolated Magnetic Charge: } \vec{\nabla} \cdot \vec{B} = 0$$

These are partial differential equations that require deep conceptual understanding of vector fields and operations related to vector fields: gradient, divergence, and curl. With the increasing power and availability of software, e.g., *Mathematica*, *Maple*, and *Matlab*, to perform the actual manipulations, it is crucial that students develop a conceptual understanding of vector fields and related operations.

It is less important to emphasize the actual manipulations. For example, it is less important that a student be given a scalar-valued function of several variables and be asked to compute the gradient. It is more important that students be able to start with a contour plot (topographic map) of a scalar-valued function of several variables and draw the gradient function.

It is less important that students start with a vector field and be able to compute the divergence or curl. It is more important that students be able to interpret verbally and graphically pictures of vector fields.

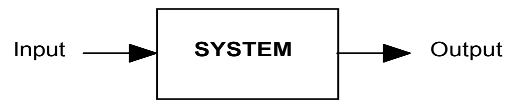
Students should be able to identify regions in which the magnitudes of the divergence or curl will be large or small. To support conceptual understanding, graphical interpretation, and verbal description it is helpful to connect students of vector calculus with applications such as electromagnetic fields, fluid mechanics and heat transfer.

The study of electromagnetics requires a conceptual understanding of partial differential equations and their solutions, and the power and limitations of numerical solutions techniques. The study of specific partial differential equations that permit closed-form solutions is less important than the development of this conceptual picture.

Since most students in electrical engineering do not begin studying electromagnetic fields and waves until their junior year, it is important that the relevant topics of vector calculus and partial differential equations not be taught before the second semester of the sophomore year. Timing of the topics is important to help students connect their studies in mathematics with their study of electromagnetics. Individual schools should encourage conversations between faculty in electrical engineering and mathematics to prepare a mathematics curriculum that is responsive to the specific requirements of the electrical engineering department.

3. Systems

Control Systems—Linear and NonLinear Circuits. One purpose of systems analysis is to represent reality mathematically. At the undergraduate level, linear time-invariant systems are discussed, studied and designed. The systems may be continuous or discrete and may have one or more inputs and one or more outputs. The system is modeled as a "box," a device that modifies the signals entering it resulting in an output according to the transfer function of the system:



Within electrical engineering, the systems problem has the following forms:

1. Find the transfer function of a SISO (single input-single output) system,
2. Find the transfer function of a MIMO (multi input-multi output) system,
3. Given the transfer function of a SISO system, what is the output of the system if the input to the system is a specified function?
4. Given the transfer function of a SISO system, how must the system be modified to satisfy given specifications?
5. Given the state equation matrices for a MIMO system, what are the outputs of the system if the input to the system is a specified vector function?
6. Given the state equation matrices for a MIMO system, how must the system be modified to satisfy given specifications?

In undergraduate courses the systems studied are linear and time-invariant. Continuous systems can be modeled by ordinary differential equations although the order of these equations might be quite high. Software packages such as MATLAB are used extensively in most systems courses.

For **continuous systems** the mathematical tools needed consist of:

1. Laplace transforms and techniques such as partial fraction expansions and residues
2. State variable techniques including eigenvalues/eigenvectors, interpretation of the matrices, etc.
3. Basic differential equations, focused on standard solutions to common problems.

For **discrete systems** difference equations are used instead of differential equations and the discrete-time state model is used. The mathematical tools needed are:

1. Difference equations
2. The state transition matrices and solutions to discrete-time state models
3. Z-transforms
4. Discrete Fourier transforms
5. Fourier analysis.

For both continuous and discrete systems it is important to be able to use the poles and zeros of transformed time functions to visualize the system's time response to various inputs. For continuous systems the s-plane is important. Students should be able to plot the poles and zeros of the transfer function and from this plot know the form of the impulse response by inspection. They should understand how the locations of the poles affect the output of the system. They should see how the locations of zeros affect the various modes of the system. In other words, they need to see the time response in the s-plane and understand the physical realities encoded in the poles and zeros. For discrete systems the same holds true for the z-plane.

The mathematical courses supporting systems are primarily linear algebra and ordinary differential equations. As stated above, ordinary differential equations courses tend to overemphasize the development of numerous solution methods for first- and second-order differential equations. In truth, systems deal with higher order differential equations, and for SISO continuous systems Laplace transforms are almost always the preferred method of solution. Discrete systems use the method of z-transforms. Both continuous and discrete MIMO systems use state variable techniques.

Consequently, we would prefer courses that place more emphasis on Laplace transforms, z-transforms, and state variable techniques for solving ordinary differential and difference equations.

Linear algebra courses usually attempt to teach state variable techniques. We recommend that these courses further develop the concepts taught in the “new” differential/difference equations course. State transition matrices and their properties could be studied. Eigenvalues and eigenvectors could be explained. The relation between the roots of the characteristic equation and eigenvalues could be stated. Other topics might include:

1. Techniques for computing the matrix exponential and its integral,
2. Eigenvalue-eigenvector methods for computing matrix exponentials,
3. Decomposition of time-invariant state matrices,
4. Jordan forms,
5. Singular value decompositions and state space applications.

The elements of linear systems could be taught during the sophomore year.

Power Systems comprise the study of the transmission and distribution of electric power. The study of power systems depends upon a firm mathematical grounding in the use and manipulation of trigonometric functions as well as algebraic manipulation of complex numbers. The use of phasor notation (an application of polar co-ordinates) plays a central role in power systems analysis. Students must also know Euler’s formula and be facile in going from polar to rectangular co-ordinates and vice-versa.

Power systems analysis requires not only algebraic manipulation but also recognition of the changes a signal undergoes and the form of the signal that results. While a solution having the form

$$f(t) = 5e^{-2t} - 5e^{-3t}(\cos t + \sin t) \quad \text{volts,}$$

may be correct, the form

$$f(t) = 5e^{-2t} - 5e^{-3t}[1.414\cos(t - 45°)] \quad \text{volts}$$

is more useful. The student can visualize a sinusoidal wave with an amplitude of 1.414 volts that lags the input signal by 45. Next, the student can visualize a sinusoid that decays exponentially. Thus, the waveform with its phase angle can be easily visualized, whereas the sum of sinusoids gives little information about the phase angle. Power systems may be taught as early as the student’s 5th semester of undergraduate studies.

4. Signals/Communications

One of the most fundamental applications in electrical engineering is the transmission, modification and reception of signals. Communication systems is concerned with:

1. The transmission of signals through electric networks
2. The modulation and demodulation of signals
3. Sampling
4. Noise
5. Statistical methods of information transmission systems

Digital signal processing is an important area within electrical engineering. The digitization, modulation, transmission, demodulation, and reception of signals is vital to modern communications. Image processing and pattern recognition techniques fall within the purview of digital signal processing.

Communications and digital signal processing are taught in depth usually during the last two or three semesters of the student’s undergraduate studies. The understanding of mathematical concepts is essential within the communications area. Of particular importance are:

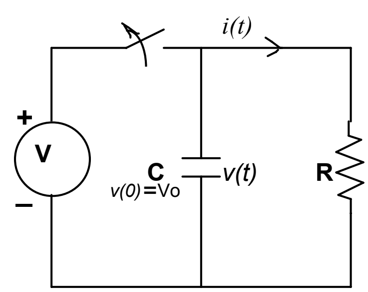
1. Basic algebraic techniques
2. Basic trigonometric identities
3. Integration techniques, including partial fractions and integration by parts
4. Taylor Series Expansion (i.e., linear approximation, expansion out to two or three terms)
5. The Fourier Transform and its use
6. Fourier Series
7. The use of the Laplace Transform
8. The use of the Z Transform
9. Probability and Stochastic Processes

5. Design

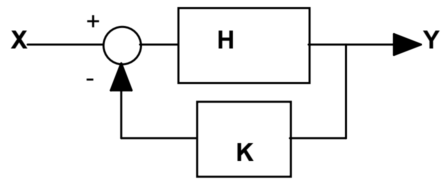
Design and modeling are two generic tasks in which engineers participate after completing their undergraduate degrees. In addition to preparation in mathematics for the other disciplines within electrical engineering, there are additional areas in mathematics that are necessary to support learning and growth in design and modeling. Such areas include statistics, empirical modeling, parameter estimation, system identification, model validation and design of experiments. Demand from industry for expertise in these areas appears to be much stronger than demand within electrical engineering curricula. That may explain why these areas are not prerequisites for courses in electrical engineering. However, expertise in these areas is increasingly important for electrical engineering graduates.

Topics from these areas that will be valuable for engineering graduates include the concept of a random variable, analysis of sets of data, concepts of sample means, sample variances and other sample statistics as random variables, and hypothesis testing. To illustrate why these topics are important here are some examples of applications.

First Example, Simple Parameter Estimation: Construct a circuit containing a resistor (resistance = R) and a capacitor (capacitance = C). If the capacitor is initially charged and then discharged through the circuit, voltages and currents decay exponentially. Data on a particular voltage can be taken at various points in time. Students then must estimate the time constant (= RC) using the accumulated data. There are a variety of techniques through which estimates of the time constant can be obtained. Students need to be familiar with the techniques as well as the supporting concepts and broader applications.



Second Example, Design of Experiments: Design a feedback controller that meets several specifications and minimizes percent overshoot. There are a number of parameters that may be adjusted. Students should be able to design a set of experiments that will help determine narrow intervals for the parameter values in order to optimize the design.



Third Example, Model Validation: Develop an empirical model for a complex physical process. Once the model is produced, students should be able to develop a set of experiments to help them understand the validity of the model.

6. Computer Engineering/Microprocessors

Digital logic design and microprocessors require a mathematical background that is fundamentally different than the background necessary for the areas discussed above. Circuits, electromagnetics, signals, and systems require mathematics in which the variables can be any real number, i.e., *continuous mathematics*. Digital logic design and microprocessors require mathematics in which the variables can only assume values in a finite set, so-called *discrete mathematics*. Students need instruction that emphasizes the fundamental difference between continuous and discrete mathematics.

More specifically, students need Boolean algebra and finite state systems. For Boolean algebra, they need to understand truth tables for the basic operators: NOT, AND, OR, NAND, and NOR. They need to analyze combinational networks constructed from these basic operators and methods by which the networks may be simplified. Examples that help students relate combinational networks to actual applications will help build motivation and understanding.

For finite state systems, students need to understand the concepts of a finite state machine and a state transition diagram. Understanding these concepts can be strengthened by examining Mealy and Moore realizations and the equivalence between the two realizations. In addition, connections between finite state machines and regular expressions should be explored. Finally, students need to start with a description of a physical situation, synthesize a state transition diagram, and then design the combinational logic that together with memory can realize the state transition diagram.

Understanding of these concepts from Boolean algebra and finite state machines will provide students with the necessary mathematical background to study computer engineering and microprocessors.

Understanding and Content

What follows are brief summaries of our responses to the specific questions posed by the Curriculum Foundations Organizing Committee.

What conceptual mathematical principles must students master in the first two years?

The mathematics required for electrical engineering students should emphasize concepts and problem solving skills more than emphasizing repetitive mechanics of solving routine problems. Students must learn the basic mechanics of mathematics, but care must be taken that these mechanics do not become the primary focus of any mathematics course.

What mathematical problem solving skills must students master in the first two years?

There is often a disconnect between the knowledge that students gain in mathematics courses and their ability to apply such knowledge in engineering situations. Perhaps, the use of more engineering or real life examples will reduce this disconnect. Too often mathematics is taught as a list of procedures or as theorem-proof exercises without grounding the mathematics in reality. While we do not expect mathematics instructors to be well versed in all engineering applications, we would like examples of mathematical techniques explained in terms of the reality they represent.

Students of electrical engineering need to be skillful at mathematically modeling physical reality. They need to be able to simplify complex problems, estimate the reasonableness of solutions, and visualize solutions graphically from inspection of the mathematical descriptions.

What broad mathematical topics must students master in the first two years?

The first two years of mathematics that support instruction in electrical engineering should present students with conceptual understanding of mathematical disciplines other than just single variable calculus, multivariable calculus and ordinary differential equations. Other mathematical subjects that are important for electrical engineering students include linear algebra, probability and stochastic processes, statistics, and discrete mathematics.

Listed below are the most important mathematical topics that we believe students in electrical engineering should learn during the first two years of undergraduate studies. All of these topics were discussed earlier in this report. The Appendix provides another summary of topics, this one organized by the six sub disciplines of electrical engineering that were identified in the previous section.

Manipulation, solution, and analysis of real and complex algebraic equations

Basic differential and integral calculus

Standard solutions for basic differential equations, in particular first- and second-order differential equations with constant coefficients

Laplace, Fourier and Z transforms

Vector calculus

Taylor series

State variables and finite state systems

Difference equations

Probability and stochastic processes

Statistics

Model validation

Parameter estimation—techniques and application

Boolean algebra—analysis and application

Technology

How does technology affect what mathematics should be learned in the first two years?

New engineering and mathematical software only reduce the dependency on routine, excessive and repetitive mathematical computations. Software should not be used to replace the necessity to teach students how to pose and formulate mathematical questions and how to evaluate answers obtained for these questions.

What mathematical technology skills should students master in the first two years?

The use of mathematical software is necessary. In this regard, mathematics departments are strongly encouraged to routinely use common math software tools and to promote students' use of them. Failure to utilize appropriate technological tools while continuing to focus on mastery of symbolic manipulation often encourages memorization and rote algorithm practice at the expense of conceptual and graphical comprehension.

What different mathematical technology skills are required of different student populations?

We did not identify any difference in requirements.

Instructional Techniques

What are the effects of different instructional methods in mathematics on students in your discipline?

We are not aware of differing effects.

What instructional methods best develop the mathematical comprehension needed for your discipline?

Hands-on, interactive engagements and project-based learning methods have been observed to promote students' learning.

Team teaching of mathematics courses should be considered. Teams could consist of faculty members from both mathematics and electrical engineering. We believe that team-teaching could better motivate and enthuse electrical engineering students.

Each mathematics course should clearly state its objectives. Students should be told *what* they are going to study, *why* they are going to study it, and *how* it fits into the engineering profession.

What guidance does educational research provide concerning mathematical training in your discipline?

We are not aware of the relevant educational research.

Instructional Interconnections

What impact does mathematics education reform have on instruction in your disciplines?

We are not aware of or familiar with current directions in mathematics education reform

How does education reform in your discipline affect mathematics instruction?

We did not have a response to this question.

How can dialogue on education issues between discipline and mathematics best be maintained?

One effective method for continuing this dialogue between mathematics and electrical engineering is to have more extensive contact between individuals in each discipline and the major professional associations in each discipline. Professional engineering associations which should be contacted include:

The American Society for Engineering Educators (ASEE)

The National Electrical Engineering Department Heads Association (NEEDHA)

Similar organizations for mechanical, civil and chemical engineering

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APPENDIX: Specific Issues on Understanding and Content

1. Electrical Circuits

Algebraic equations, manipulations and solutions

Differential and integral calculus

Concept of functions

Exponential functions

Sinusoidal functions, representation and manipulation in Euler, polar, rectangular coordinates

Application of trigonometric identities

Algebra of complex numbers

Introductory differential equations - focus on *standard solutions* to problems with basic inputs including step, sinusoids, triangular, square functions *(teach by end of first year)*

Laplace transform (emphasize this topic over differential equation techniques in the first year)

2. Electromagnetics

Vector calculus (Do not teach before the 2nd semester of the sophomore year)

Conceptualization

Operation

Gradients

Divergence

3. Systems

Continuous Systems

Laplace transforms and techniques

Integration by parts

Partial fraction methods

State variables

Eigenvalues and eigenvectors

Basic differential equations—*focus on standard solutions to common problems*

Discrete Systems

Difference equations

Systems of first order differential equations

Z-transforms

Discrete Fourier transform

Fourier analysis and techniques

Power Systems

Sinusoidal functions

Algebra of complex numbers

4. Signals/Communications

Fourier analysis, transform and techniques
Integration by parts
Partial fraction methods
Probability and stochastic processes
Z-transform
Taylor series (linear approximation—interested only in the first two terms)

5. Design

Statistics
- Data collection
- Sampling
- Analysis (distribution, graphical techniques)

Concepts of random variables
Model validation
Parameter estimation
System identification

6. Computer Engineering/Microprocessors

Boolean algebra
Finite state systems