Engineering:
Mechanical Engineering

CRAFTY Curriculum Foundations Project
Clemson University, May 4–7, 2000

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Summary

We have endeavored to answer the questions as posed in order to satisfy the understood needs of the Mathematical Association of America. However, the answers to just the posed questions do not address the bigger picture that was the underlying theme of our discussions. We therefore present an overall context of what we want the students to "be" and then a list of the details within each mathematics topic.

We want students who are able to:

- Learn in context
- Conceptualize
- Set up equations
- Apply problem solving techniques
- Translate mathematical results
- Understand the language of mathematics vs. language of engineering (in all dialects)
- Use modeling techniques
- Understand logic discipline
- Manipulate complex equations

We want students to understand the physics and how equations describe the physics. We want them to then solve problems with whatever tools are appropriate *and* to understand the solutions.

Narrative

Introduction and Background

We have tried to address our answers to the MAA questions with respect to where we think the Mechanical Engineering curriculum is now and where it is going to be in the near future. The different themes we think are important included:

- "Slide rule engineering" and packaged knowledge is insufficient for modern problems
- Smaller scale systems
- Nanoscale-thin layers

- Smarter systems—more intelligent (Mechatronics)
- Modularization
- Precision movement
- New materials—soft/wet, increasingly complex (using tensor notation)
- Biological and medical engineering
- Analytic complexity and interrelatedness of fields
- Three types of systems
  - Static—large scale stresses
  - Dynamic—phenomenon over time
  - Non-deterministic—statistics and probability
- Increasing use of discrete techniques in addition to continuous
- Central roles of calculators and computers
- Massive codes used for problem solving
- Need to understand microprocessors and mechanical systems—Boolean logic

Regardless of what the new foci will be one thing is clear: the way students need to know the material is different than before. Students need to be able to deal in a more complex interface with the different disciplines. They need to understand how to take the fundamentals and use them in different ways. Calculators, computers, and packaged software give students a facility to see the physical reality in a variety of ways.

Understanding and Content

We believe that the basic mathematical foundations have not changed a great deal. It is, however, the implementation of this knowledge and the drive for pedagogical transformation that is of concern in the future. The following are the group's responses to the specific MAA questions, organized as requested by considering different aspects of what is desired in the first two years of undergraduate instruction.

What conceptual mathematical principles must students master in the first two years?

The following table states the specific mathematical principles and where they are used in the mechanical engineering curriculum.

Mathematical Concept	Context for Mechanical Engineering
Algebra and trigonometry Coordinate systems	All areas of mechanical engineering
Geometry Transformation: rotation & translation	All areas of mechanical engineering Notation facility Codes
Estimation Order of magnitude Distinguishing realistic from nonsense	Thermo fluids Engineering design Physical design of experiment Modeling
Statistics Mean and standard deviation	Experimental design and analysis Design Quality Assurance Manufacturing

Mathematical Concept	Context for Mechanical Engineering
Derivatives and integrals	Statics and mechanics Thermodynamics Energy and work Control theory
Taylor series and Fourier series	Vibrations/Instrumentation System Dynamics
Ordinary differential equations: Linear first and second order Homogeneous and non-homogeneous	Heat conduction Fully developed flow Solid state diffusion Dynamic systems
Multivariable calculus Gradients Surfaces and planes	Fluids Conduction
Linear Algebra Vectors and matrices Eigenvalues and eigenvectors	Dynamics and systems dynamics Vibrations Controls Nonlinear finite element analysis Optimization
Vector analysis	Statics, dynamics, and kinematics Mechanics of materials
Partial differential equations	Thermal fluids Soils mechanics
Numerical Methods Discretization Use of software such as *Matlab*	Dynamics Dynamical systems Vibration Solid and fluid mechanics
Complex variables	Controls (bode plots) Vibrations Electronic Circuit Theory

What mathematical problem-solving skills must students master in the first two years?

In an engineering discipline problem solving essentially means **mathematical modeling**: the ability to take a physical problem, express it in mathematical terms, solve the equations, and then interpret the result.

We feel that, especially in the first two years, the students are more comfortable and adept at being able to understand concepts through tackling sample problems. Part of the problem solving must move through Bloom's Taxonomy from mechanics to conceptualization to integration. The final stage could be done by moving some of the mathematics topics into the third year and coordinating the timing of the topics with the engineering program.

What broad mathematical topics must students master in the first two years? What priorities exist between these topics?

We provide a high level of specificity and detail in our presentation of the mathematics topics we consider important to an undergraduate in mechanical engineering. Moreover, rather than limit ourselves to just the first two years, we address all the mathematics skills we believe are necessary for our students, separating them into the three major levels of their educational career:

Secondary education—Major topics for prospective engineering students in high school include:

- algebra
- trigonometry
- geometry

A student should develop skills in these areas prior to entering an engineering program.

First two years of undergraduate education – Major topics to include are:

- geometry
- functions and graphs
- statistics
- integration and differentiation
- linear algebra
- ordinary differential equations
- partial differential equations
- numerical methods
- complex numbers and functions

Third Year—Advanced topics or topics focused on certain sub disciplines rather than across the curriculum are placed in this category.

The Appendix lists specific topics for each of these three educational levels. Moreover, instead of simply listing together the first two undergraduate years we suggest what the students should learn during the first year and what they should learn during the second.

What is the desired balance between theoretical understanding and computational skill? How is this balance achieved?

Students need enough of a conceptual foundation—not based on formal derivations—so that they get a basic understanding of the mathematical principles. They need to tie these with computational skills. Then computing will allow an effective application of the mathematics, yielding an understanding of the physical implications of the system under study. The emphasis is on physical understanding without the axiomatic structure.

What are the mathematical needs of different student populations and how can they be fulfilled?

We differentiate between students who terminate with a bachelor's degree and those who continue on for advanced degrees. The more theoretical mathematics courses should be made available during the last two years, when a student knows if they wish to go to graduate school.

Technology

How does technology affect what mathematics should be learned in the first two years?

Computers and appropriate software allow work on more complex problems earlier in a student's educational career. They also allow for visualization of the effect of varying parameters.

The internet gives access to vast amounts of information, allowing solution of more interesting complex problems.Many crank turning skills—like integration by parts, complicated integration substitutions, and manual manipulative skills—can be handled by technology. Instructors can thus spend more time teaching conceptual understanding of the important skills.

What mathematical technology skills should students master in the first two years?

Students need to be familiar with software applications for numerical computation and symbolic computation. They also need to be introduced to spreadsheets.

Instructional Techniques

What are the effects of different instructional methods in mathematics on students in our discipline?

Mathematics is the entry point for engineering students. We get our students because they "were good in math," and often because some instructor told them so. An ineffective mathematics program will drive our students away. The students can be turned off by poor instruction.

What instructional methods best develop the mathematical comprehension needed for your discipline?

Instruction is not effective when material is presented in an overly theoretical way, divorced from application. Topics should be presented in the context of physical concepts. Interdisciplinary team teaching/active learning/collaborative learning should be utilized as effective instructional methods. The use of student teams enhances instructional effectiveness.

Is it pedagogically more effective to teach mathematics to engineering students as a homogeneous group or as part of heterogeneous groups with students in other disciplines? We recommend that the MAA seriously consider which method of student grouping is the most conducive to student learning.

What guidance does educational research provide concerning the mathematical training in your discipline?

Several educational research insights are valuable in guiding mathematical training in our discipline. Specific examples include teaming and Bloom's taxonomy. Active learning has become an important activity for our students, both as preparation for the job market and as an effective way to deal with open-ended problems.

We propose that mathematics departments develop small projects that require teaming and active learning. This would help students learn fundamental mathematical principles, avoiding the mere memorization of algorithms that can arise from over-concentration on examples. There could be one or two projects during a semester, handled by teams of two to four students. Examples of subjects could be differential equations whose complexity increases from project to project. The applications could cut across different engineering disciplines so that students could develop a sense of the nature of each discipline.

Instructional Interconnections

What impact does mathematics education reform have on instruction in your discipline?

Some workshop participants felt that students are being trained in symbolic manipulation and memorization in ways that negatively impact their ability to attack more complicated problems. Our students seem to have a decreased ability to conceptualize. They are too dependent on methodology and not on principles.

How should education reform in your discipline affect mathematics instruction?

Educational reform in engineering, being driven by ABET. EC2000 has added more active learning, problem-based, team learning experiences. This results in more open ended, complicated problems for students to tackle. The paradigms for teaching in engineering are evolving from solely derivation and lectures to more varied methods, addressing a diversity of student learning styles. If mathematics is still taught in ways that do not address differing learning styles or teaching pedagogies, it will lose its effectiveness in preparing engineering students.

How can dialogue on educational issues between your discipline and mathematics best be maintained?

- Establish sites that implement change in the education paradigm.
- Organize workshops with mathematics and engineering faculty to advance this discussion.
- Establish joint meetings or sessions between MAA and FIE/ASEE.
- Recognize and encourage interdisciplinary cooperation via changes in the merit system.
- Encourage engineers to learn more about reforms under consideration by the MAA.
- Encourage books to be written jointly by mathematicians and engineers.

Funding needs to be found for such activities. The NSF is one appropriate funding source.

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APPENDIX: Specific Mathematical Topics

Secondary School

- **Algebra/Trigonometry**
  Manipulation skills
  Logarithmic and exponential functions
  Trigonometric functions:
    In terms of right triangles and general triangles
    Expressing periodic phenomena
  Understanding variable names and use of symbols
- **Geometry**
  Sketching process for visualization
  Planar objects

First Two Years of Undergraduate Mathematics

- **Geometry**
  3D visualization (1st yr)
  Coordinate systems (1st yr)
  Transformations (1st yr)
    Translations
    Rotations
- **Functions and Graphs**
  Plotting—Cartesian, semi-log, log-log, (1st yr)
  Special functions and their properties—exponentials, step functions (1st yr)
  Parametric graphing (2nd)
- **Statistics**
  Mean/Standard Deviation/Variance (2nd yr)
  Regression Analysis (2nd yr)
    Least Squares
    Linear and Polynomial Regression Analysis
    Multivariable Regression Analysis (nonlinear)
  Probability—(2nd yr)
  Distributions—Gaussian, Poisson, Exponential (2nd yr)
  Examples:
  - Linear regression is used to find the slope of a stress-strain curve (the elastic modulus of a material).
  - Polynomial regression is used to model the rise and fall of hardness with aging time in precipitation hardenable aluminum alloys and predict the peak hardness conditions.
  - ANOVA and/or T-Tests are used to determine if there is a significant correlation between all kinds of data, such as heat treatment conditions and hardness.

- Basic statistics like average and standard deviation are used to describe the average grain size from metallographic measurements.
- The Weibull distribution is used as a model for lifetime and durability phenomena.
- The Exponential distribution is used as a model for reliability data.
- The Binomial distribution is used to model the number of defective items in samples drawn from large lots of items such as mechanical or electrical parts.
- Probability concepts are used in the stochastic design method for mechanical components (e.g., shafts, beams, plates and other solid objects under load). This is the design for reliability approach that is sometimes preferred over the design factor of safety approach.

- **Integration and Differentiation**

  Standard functions—polynomial, exponential, logarithmic (1st yr)

  Special functions—hyperbolic (1st)

  Product/quotient/chain rules (1st)

  Extreme points, points of inflection, and similar analysis of functions (1st)

  Setting up integrals (1st)

  Definite integrals representing area, energy, work, etc (1st)

  Moments of inertia & finding centroids (1st)

  Know how to get to the "answers" (1st)

  Multivariate calculus—derivatives/integrations—curl, divergence, gradient (2nd yr)

  Exact differentials $Mdx + Ndy = dz$ (2nd)

- **Linear Algebra**

  Vectors

  - Properties—magnitude, direction, linear, angular (1st yr)
  - Vector algebra and manipulation (dot product, cross product, possibly gradient) (1st yr)
  - Vectors as coordinate system (1st yr)
  - Linear independence (2nd yr)

  Matrices

  - How to cast equations into matrix form (1st yr)
  - Matrix manipulation (addition, multiplication, inversion, transposition) (1st yr)
  - Special matrices: symmetric, sparse, banded (2nd)
  - Basic dimensional analysis (2nd yr)
    - Singular matrices
    - Determinant
    - Rank

  Solution of system of homogeneous equations (1st or 2nd yr)

  - Eigenvectors and eigenvalues

  Vectors as a "special" matrix (2nd yr)

  Understanding homogeneous vs. non-homogeneous (non-forced vs. forced) (2nd yr)

- **Ordinary Differential Equations**
  1st, 2nd and higher order
    Constant coefficient (1st or 2nd order)
    Linear (1st or 2nd order)
    Boundary and initial value problems (1st or 2nd order)
    $dx/dt + ax = 0$ implies time constant (1st or 2nd order)
    d^2x/dt^2 interpreted as natural frequency and damping ratio (1st or 2nd order)
  Laplace transform
- **Partial Differential Equations**
  What are PDEs?
  How to convert PDEs to ODEs (2nd yr)
- **Numerical Methods** (not necessarily as a separate course)
  Symbolic manipulators (Matlab, etc.) (1st or 2nd yr)
  Discretization concepts (2nd yr)
  Finite differences (2nd yr)
  Numerical integration (2nd yr)
    Euler's Method
    Simpson's Rule
  Root finding (2nd yr)
  Runge-Kutta methods for ODEs (2nd yr)
  Regression and curve fitting (2nd yr)
- **Complex Numbers, Variables, and Functions**
  Arithmetic and algebra of complex numbers (1st yr)
  Representations of complex numbers: polar, Cartesian, exponential (1st yr)
  Laplace functions (2nd yr)
  Fourier series (2nd yr)

Third Year of Undergraduate Mathematics

- **Functions and Graphs**
  Complex plane
  Bode diagrams
- **Matrix analysis**
  Advanced matrix operations: Gauss, LUD (2nd or 3rd yr)
- **Ordinary differential equations**
  Nonlinear equations
  Periodic forcing functions
- **Partial differential equations**
  2nd order linear
  Simple systems
  Numerical methods

- **Statistics**

  Sampling (vs. total population) and estimation

  Factorial design and design of experiments

  Hypothesis testing (e.g., quality assurance)

  Examples

  - Statistical design of experiments (factorial design) is used to determine the main effects in experiments involving multiple variables, like the effects of fuel octane level, rpm, and torque on internal combustion engine efficiency. Multi-variable regression methods are also applied in experiments like this.
  - Control charts (e.g., X-bar and R-bar) are used extensively for continuous quality control decisions about manufacturing processes.

- **Numerical Methods**

  Boundary value problems—2nd order

  Finite element analysis