Part II. Additional Recommendations Concerning Specific Student Audiences

A. Students taking general education or introductory collegiate courses in the mathematical sciences

General education and introductory courses enroll almost twice as many students as all other mathematics courses combined.[32] They are especially challenging to teach because they serve students with varying preparation and abilities who often come to the courses with a history of negative experiences with mathematics. Perhaps most critical is the fact that these courses affect life-long perceptions of and attitudes toward mathematics for many students—and hence many future workers and citizens. For all these reasons these courses should be viewed as an important part of the instructional program in the mathematical sciences.

This section concerns the student audience for these entry-level courses that carry college credit. A large percentage of these students are enrolled in college algebra. Traditional college algebra courses, with a primary emphasis on developing skills in algebraic computation, have a long history at many institutions.

Students enrolled in college algebra courses generally fall into three categories:

1. Students taking mathematics to satisfy a requirement but not specifically required to take a course called college algebra;
2. Students majoring in areas or studying within states or university systems that specifically require a course called college algebra;
3. Students intending to take courses such as statistics, calculus, discrete mathematics, or mathematics for prospective elementary or middle school teachers and who need additional preparation for these courses.

Unfortunately, there is often a serious mismatch between the original rationale for a college algebra requirement and the actual needs of the students who take the course. A critically important task for mathematical sciences departments at institutions with college algebra requirements is to clarify the rationale for the requirements, determine the needs of the students who take college algebra, and ensure that the department's courses are aligned with these findings (see Recommendation A.2).

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[32] According to the CBMS study in the Fall of 2000, a total of 1,979,000 students were enrolled in courses it classified as "remedial" or "introductory" with course titles such as elementary algebra, college algebra, pre-calculus, algebra and trigonometry, finite mathematics, contemporary mathematics, quantitative reasoning. The number of students enrolled in these courses is much greater than the 676,000 enrolled in calculus I, II or III, the 264,000 enrolled in elementary statistics, or the 287,000 enrolled in all other undergraduate courses in mathematics or statistics. At some institutions, calculus courses satisfy general education requirements. Although calculus courses can and should meet the goals of Recommendation A.1, such courses are not the focus of this section.

Because many students taking introductory mathematics decide not to continue to higher level courses, general education and introductory courses often serve as students' last exposure to college mathematics. It is important, therefore, that these courses be designed to serve the future mathematical needs of such students as well as to provide a basis for further study for students who do continue in mathematics. All students, those for whom the course is terminal and those for whom it serves as a springboard, need to learn to think effectively, quantitatively and logically. Carefully-conceived courses—described variously as quantitative literacy, liberal arts mathematics, finite mathematics, college algebra with modeling, and introductory statistics—have the potential to provide all the students who take them with the mathematical experiences called for in this section.

A common feature of many effective courses and programs that have been developed for these students is the leadership provided by key faculty members. It requires committed and talented faculty to understand the needs of these students and the opportunities inherent in these courses. Continuing leadership is needed and special training must be provided for instructors—including graduate assistants and part-time faculty—to offer courses that will meet the needs of these students.

A.1. Offer suitable courses

All students meeting general education or introductory requirements in the mathematical sciences should be enrolled in courses designed to

- ***Engage students in a meaningful and positive intellectual experience;***
- ***Increase quantitative and logical reasoning abilities needed for informed citizenship and in the workplace;***
- ***Strengthen quantitative and mathematical abilities that will be useful to students in other disciplines;***
- ***Improve every student's ability to communicate quantitative ideas orally and in writing;***
- ***Encourage students to take at least one additional course in the mathematical sciences.***

Different institutions will develop and offer different sets of courses designed to fit the characteristics of their student body. In recent years a large number of engaging and challenging courses have been developed and successfully offered to meet the needs of these students. However, regardless of the curriculum chosen, all general education and introductory courses should strive to meet the goals outlined in this recommendation. Each of these course goals will now be described in more detail.

Engage students in a meaningful intellectual experience. Students must learn with understanding, focusing on relatively few concepts but treating them in depth. Treating ideas in depth includes presenting each concept from multiple points of view and in progressively more sophisticated contexts. For example, students are likely to improve their understanding more by writing analyses of a single situation that combines two or three mathematical ideas than by solving half a dozen problems using each idea separately. In addition, students should encounter some meaningful ideas of mathematics. College students study the best paintings, the most glorious music, the most influential philosophy, and the greatest literature of all time. Mathematics departments can compete on that elevated playing field by offering and making accessible to all students intriguing and powerful mathematical ideas. Mathematical modeling, data analysis, quantification of the uncertain and the unknown, analysis of the infinite, cryptography, fractals—these topics and many more can compete well with any other subject for depth and fascination.[33] Indeed, these courses

[33] For example, if students are using geometric sequences and their sums to model a real world situation, it is natural to help them encounter the notion of infinity through an examination of the area and perimeter of the stages of a Koch snowflake.

should be developed and offered with the philosophy that the mathematical component of every student's education will contain some of the most profound and useful ideas that the student learns in college.

Increase students' quantitative and logical reasoning abilities. Departments should encourage and support their institutions in establishing a quantitative literacy program for all students, with the primary goal of developing the intellectual skills needed to deal with quantitative information as a citizen and in the workplace.[34] This program should ensure that all introductory and general education mathematics courses make a significant contribution toward increasing students' quantitative reasoning abilities. Courses in the program should be coordinated with courses in other disciplines in order to reinforce skills and place quantitative reasoning in a broader context. Clearly, no one course can enable an individual to become quantitatively literate, but introductory and general education mathematics courses can help students see how quantitative methods can be used to help answer questions that they find meaningful. Students in these courses should also have the opportunity to use a variety of mathematical strategies—seeking the essential, breaking difficult questions into component parts, looking at questions from various points of view, looking for patterns—in diverse settings. Informing the pre-college community of the institution's commitment to quantitative literacy can help work toward this goal.

Strengthen mathematical abilities that students will need in other disciplines. As reported in "A Collective Vision: Voices of the Partner Disciplines,"[35] faculty representing other disciplines emphasized the importance of mathematical modeling. Students should be able to create, analyze, and interpret basic mathematical models from informal problem statements; to argue that the models constructed are reasonable; and to use the models to provide insight into the original problem. Many of the disciplinary reports cite the importance of conceptual understanding and data analysis. A study of these reports and the textbooks and curricula of courses in other disciplines shows that the algorithmic skills that are the focus of computational college algebra courses are much less important than understanding the underlying concepts. Disciplines outside mathematics rarely ask their students to find the equation of the line that passes through two given points. But social scientists will expect students to recognize a linear pattern in a set of data, interpret the parameters of the best-fitting line, and use the equation of the line to answer questions in context. Mathematical sciences departments need to know which concepts will be needed in subsequent courses. They need to know how they will be needed. And they need to adjust their general education and introductory courses accordingly. Faculty also need to incorporate strategies to help students transfer skills learned in mathematics courses to other subjects. For instance, students should be required to attach units to answers when appropriate and to practice translating algebraic expressions into verbal ones.

Improve students' ability to communicate quantitative ideas. "A Collective Vision: Voices of the Partner Disciplines" reports that nearly every discipline promotes the importance of having students communicate mathematical and quantitative ideas—both orally and in writing. Communication skills are related to logical reasoning: if you can't explain it, you don't understand it. It is imperative that communication be emphasized in all mathematics courses. In general education and introductory courses, communication experiences should focus heavily on increasing students' quantitative reasoning abilities and making connections with the mathematics encountered in other disciplines.

Encourage students to take other courses in the mathematical sciences. On the one hand, general education courses provide the last formal mathematics experience for most students and so must stand on their

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[34] A useful description of quantitative literacy is provided in *Mathematics and Democracy*: *The Case for Quantitative Literacy*, L. Steen, ed., National Council on Education and the Disciplines, 2001.

[35] The introductory article in *The Curriculum Foundations Project: Voices of the Partner Disciplines.*

own intrinsic merits. On the other hand, they should be designed to serve as gateways and enticements for other mathematics courses. No course should ever be constructed as a final mathematics course. Natural subsequent courses include other general education courses and statistics and can even include discrete mathematics or calculus. Depending upon the nature of a subsequent course, modest changes to the general education course might make it more useful as a preparation.

Each department should consider its offerings to students taking general education or introductory collegiate mathematics courses in terms of the criteria in Recommendation A.1 and its own institutional situation. It is understood that each department has unique opportunities and boundary conditions within which it must operate. These include the mathematical preparation of students, institutional admission standards, institutional transfer guidelines and articulation agreements, mathematical needs of other disciplines with large numbers of majors, institutional general education requirements, level of commitment of the institution to high quality general education, average class size, mandated state curriculum requirements and exams, interests and areas of specialization of faculty, and the extent to which instruction is provided by graduate teaching assistants and/or part-time faculty. However, every mathematics department should strive to implement the spirit of this recommendation in ways appropriate to its individual circumstances.

A.2. Examine the effectiveness of college algebra

Mathematical sciences departments at institutions with a college algebra requirement should

- ***Clarify the rationale for the requirement and consult with colleagues in disciplines requiring college algebra to determine whether this course—as currently taught— meets the needs of their students;***
- ***Determine the aspirations and subsequent course registration patterns of students who take college algebra;***
- ***Ensure that the course the department offers to satisfy this requirement is aligned with these findings and meets the criteria described in A.1.***

Clarify the rationale. College algebra courses are specified for students in a variety of professional fields and by a number of states. To determine the actual needs of these students, mathematics faculty should consult with colleagues in disciplines requiring college algebra to determine whether this course—as currently taught—meets the needs of their students. In such discussions it is helpful to provide several textbooks representing both traditional and nontraditional approaches available because faculty in other disciplines may be unaware of the variety of materials that can be used to teach the kind of course described in A.1. Recommendations of representatives of the partner disciplines reported in *The Curriculum Foundations Project: Voices of the Partner Disciplines* indicate that a traditional, computational college algebra course does not address the needs of students in these disciplines. They recommend a course meeting the criteria in A.1. Many such courses and programs already exist. They are offered at a wide array of institutions and under a variety of names, such as college algebra with modeling, introduction to contemporary mathematics, quantitative reasoning, liberal arts mathematics, contemporary college algebra, and pre-calculus and modeling. Short descriptions of sample courses are provided in Illustrative Resources, along with a list of possible textbooks.

Determine student needs. This is a restatement of Recommendation 1 for the special case of students taking college algebra. What are the mathematical backgrounds of these students? What are their intended majors? Do they take courses for which college algebra is a prerequisite? Do they actually *use* what they learned in college algebra in subsequent courses? Are they successful in those courses? If, for example, the addition of rational expressions is never used, why is it appropriate for these students? Is it an idea of intrinsic interest and value?

Align the course with these findings. Students should indeed be equipped with the specific mathematical skills they need for their academic programs. But they should also be equipped with mathematical knowledge that will remain with them and be useful in their careers and in their lives as contributing citizens. The criteria in A.1 are an appropriate basis for the design of this course.

A.3. Ensure the effectiveness of introductory courses

General education and introductory courses in the mathematical sciences should be designed to provide appropriate preparation for students taking subsequent courses, such as calculus, statistics, discrete mathematics, or mathematics for elementary school teachers. In particular, departments should

- ***Determine whether students who enroll in subsequent mathematics courses succeed in those courses and, if success rates are low, revise introductory courses to articulate more effectively with subsequent courses;***
- ***Use advising, placement tests, or changes in general education requirements to encourage students to choose a course appropriate to their academic and career goals.***

Determine whether students succeed in subsequent courses. For example, students who will subsequently study statistics should be prepared to undertake a data-driven course that emphasizes statistical thinking and contributes to their quantitative literacy. According to the recommendation from the statistics report in *The Curriculum Foundations Project: Voices of the Partner Disciplines,* students who are taking mathematics to prepare for the study of statistics need to "develop skills and habits of mind for problem solving and for generalization" (p. 125). The workshop participants recommended that mathematics courses for students preparing to study statistics should "emphasize multiple approaches for problem solving,… insist that students communicate in writing,… [and use] real, engaging applications through which students can learn to draw connections between the language of mathematics and the context of the application" (p. 125).

A description of a statistics course that fully meets Recommendation A.1 is provided in a position paper presented in August of 2000 at the American Statistical Association's Undergraduate Statistics Education Initiative.[36] While such a course does not necessarily have any college-level prerequisites, many institutions successfully offer a two-course sequence that includes a mathematics course and a statistics course both of which meet the goals of Recommendation A.1.

For another example, students who are preparing to study calculus need to develop conceptual understanding as well as computational skills. Appropriately designed pre-calculus courses can enable students to be successful in calculus. Often, creation of an effective pre-calculus course requires learning about different curricular and pedagogical approaches and experimenting with how the most promising ones might be adapted for local implementation. Students who are preparing to study the entry-level discrete mathematics course now recommended for computer science students by the computer science societies need essentially the same mathematical background as students preparing to study calculus, except that trigonometry is not normally needed for discrete mathematics.

The discussion here concentrates on preparation for subsequent courses, but it should be remembered that students' plans change, and an introductory course could end up being a student's last formal exposure to mathematics. No course should have value *only* as a preparation for a subsequent course. It should have intrinsic value on its own as well as offering preparation for further study.

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[36] Garfield, Joan, Bob Hogg,, Candace Schau, and Dex Whittinghill, (2002), "First Courses in Statistical Science: The Status of Educational Reform Efforts," *Journal of Statistics Education,* Volume 10, Number 2, `www.amstat.org/publications/jse/v10n2/garfield.html`.

Help students choose appropriate courses. For example, pre-calculus is frequently chosen inappropriately. Ordinarily, students not intending to study calculus should be discouraged from registering for pre-calculus and encouraged to choose courses more appropriate for their future mathematical needs. Carrying out this recommendation requires liaison work with other departments to improve placement and to clarify the true needs of their programs. Similarly, appropriate placement in other introductory courses requires conferring with departments whose students take those courses, educating academic advisors, and perhaps making use of placement tests. In institutions where students' academic plans are very fluid in the first two years, it is especially important that students take introductory mathematics courses that will flexibly equip them for future study of mathematics should their plans change.

B. Students majoring in partner disciplines

Partner disciplines vary by institution but usually include the physical sciences, the life sciences, computer science, engineering, economics, business, education, and often several social sciences.[37] It is especially important that departments offer appropriate programs of study for students preparing to teach elementary and middle school mathematics. Recommendation B.4 is specifically for these prospective teachers.

Many of the courses taken by students majoring in the partner disciplines are also taken by students who may choose to major in the mathematical sciences. (Sometimes they are the same students.) Obviously, the needs of these prospective mathematical sciences majors also warrant careful consideration.

The recommendations in this section are heavily influenced by the findings of the Curriculum Foundations project (see Appendix 2). The resulting reports, published in *The Curriculum Foundations Project*: *Voices of the Partner Disciplines*, can assist mathematical sciences faculty in discussions with faculty from other departments, serving both as a guide and a resource for collaboration.

B.1. Promote interdisciplinary collaboration

Mathematical sciences departments should establish ongoing collaborations with disciplines that require their majors to take one or more courses in the mathematical sciences. These collaborations should be used to

- ***Ensure that mathematical sciences faculty cooperate actively with faculty in partner disciplines to strengthen courses that primarily serve the needs of those disciplines;***
- ***Determine which computational techniques should be included in courses for students in partner disciplines;***
- ***Develop new courses to support student understanding of recent developments in partner disciplines;***
- ***Determine appropriate uses of technology in courses for students in partner disciplines;***
- ***Develop applications for mathematics classes and undergraduate research projects to help students transfer to their own disciplines the skills learned in mathematics courses;***
- ***Explore the creation of joint and interdisciplinary majors.***

The explosive growth and development of scientific disciplines over the past half century have resulted in unprecedented pressures on curriculum. Partner discipline faculty want to provide students with the necessary background and tools to understand current developments, but degree programs ordinarily limit

[37] See Appendix 2 for a list of the disciplines represented at the Curriculum Foundations workshops.