ty. (*MET*, p.23) Middle school mathematics teachers need to understand random sampling or random assignment to treatments, explore and interpret data by observing patterns and departures from patterns, and understand what it means to draw conclusions with measures of uncertainty.

Mathematical thinking and communication skills are vital for teachers. Their needs are well served by the guidelines given in Recommendation 2, but in some respects they must go farther to acquire the abilities necessary to recognize and shape the mathematical thinking of their students. In order for prospective teachers to move beyond the attitudes and strategies they often bring from their own experiences, "they need to have classroom experiences in which they become reasoners, conjecturers, and problem solvers." (*MET*, p. 56)

The uses of mathematics should be included in the teaching of mathematics at every level. Recommendation 3 applies to courses for prospective teachers, as it does to all mathematics courses, and, suitably interpreted, it also applies to the teaching of school mathematics. Teachers need a wide repertoire of examples that illustrate the power of mathematics. Without exposure to significant applications a teacher will be effectively crippled, unable to understand accurately—and hence unable to convey—the primary motivations for many K–8 mathematics topics. It is a challenge to develop the solid knowledge of mathematical topics specified in B.4 in just three courses focused on the K–8 curriculum. However, employing applications to motivate and illustrate the material helps sustain students' interest and assists them in building that solid knowledge. Experiences with mathematical modeling afford excellent opportunities for translating between mathematical and verbal description, clarifying assumptions, and interpreting results.

Career-long professional mathematical growth is necessary to achieve and maintain excellence in the mathematics classroom. "In some countries where student achievement is high, teachers, alone and in groups, spend time refining their lessons and studying the underlying mathematics." (*MET*, p. l0) College mathematics faculty have little power to create these opportunities for professional development, but faculty can ensure that prospective teachers' college experiences in learning mathematics prepare and motivate them to take advantage of future opportunities to strengthen their understanding of mathematics.

C. Students majoring in the mathematical sciences

The recommendations in this section refer to all major programs in the mathematical sciences, including programs in mathematics, applied mathematics, and various tracks within the mathematical sciences such as operations research or statistics. Also included are programs designed for prospective mathematics teachers, whether they are "mathematics" or "mathematics education" programs, although requirements in education are not specified in this section.

Although these recommendations do not specifically address minors in the mathematical sciences, departments should be alert to opportunities to meet student needs by creating minor programs—for example, for students preparing to teach mathematics in the middle grades.

These recommendations also provide a basis for discussion with colleagues in other departments about possible joint majors with any of the physical, life, social or applied sciences.

Two premises underlie the recommendations in this section: the number of bachelor's degrees granted in the mathematical sciences—including joint majors— should be increased, and the population of potential majors has changed.

The number of bachelor's degrees granted in the mathematical sciences should be increased.

- Many other disciplines, including computer science, business, economics, the life sciences, medicine, the physical sciences, and engineering, have much greater mathematical content now than in the mid-twentieth century. All undergraduate majors in these areas have significant mathematical needs, as described in Section B. However, the formal mathematical requirements for majors in these areas have not grown in proportion to what is needed to pursue certain mathematically sophisticated sub-areas at an advanced level. Individuals with joint majors or dual majors in the mathematical sciences and a partner discipline are needed to meet these needs.

- There is a severe shortage of qualified teachers of secondary school mathematics. For example, in his 1999 article in the *Educational Researcher*, Richard Ingersoll reports that in 1995 one third of secondary mathematics teachers were teaching out of their field, having prepared to teach in another area. This shortage can be addressed only by graduating more mathematical sciences majors who are interested in secondary teaching.
- Many graduate mathematics departments have difficulty recruiting well-qualified graduates of U.S. colleges and universities to meet their need for teaching assistants and subsequently to fill positions in industry and universities requiring advanced degrees. Less than 50% of mathematics doctoral recipients from U. S. institutions are U. S. citizens[50] and greater numbers of well-prepared undergraduate majors with an interest in studying mathematics at an advanced level must be prepared to address this critical need.
- The fact that mathematics is a cornerstone of modern society implies that the study of mathematical sciences is important for all students, but it also implies that it is important that some leaders in all areas have the broader and deeper knowledge of mathematics conveyed by a degree in the mathematical sciences. Indeed, business, law, medicine and other professional schools seek mathematical sciences majors, and would welcome more.

While the need for graduates continues to increase, the number of majors has, as already noted, declined. (See Appendix 3 for an analysis of the data on numbers of majors.) At a time of increased need this decline is alarming at the national level; in some cases it also has very negative implications locally. If enrollments in advanced courses fall below threshold values at an institution, the availability of those courses decreases for students who want and need to take them.[51] Without a sufficient number of majors, it is difficult for a department to offer the range of courses and co-curricular experiences that best serve its students.

The population of potential majors has changed.

- Potential mathematical sciences majors—like all post-secondary students—are more diverse than they were even 30 years ago. English is not the primary language for many students. A very small number of students arrive in college and university classrooms well-prepared, highly interested in mathematics and intending doctoral study in mathematics. Most are less intrinsically interested in mathematics and lack confidence in their mathematical abilities; they may choose to major in mathematics because of its applicability in other disciplines or because it offers the promise of employment opportunities.
- Many potential majors begin their studies at two-year colleges. Over one third of recent bachelor's degree recipients in the mathematical sciences had taken courses at two-year colleges.[52]

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[50] In 2001–2002 only 428 of the 962 doctoral recipients were U. S. citizens according to the Annual Survey of the Mathematical Sciences conducted by the AMS. `www.ams.org/employment/surveyreports.html`.

[51] See Table 4-3 in Appendix 4 for data on the declining availability of advanced courses. At an urban university that is a record-setting producer of mathematics graduates who are members of under-represented groups, there are sometimes too few students enrolled to offer differential equations.

[52] See Appendix 4.

- Not only are prospective mathematics majors more diverse in preparation and in career goals than twenty or thirty years ago, their goals are likely to change during their college years. Consequently it's not reasonable for departments and faculty to expect first and second year students to know what they want to study.
- In the 1960s, 5% of freshmen entering colleges and universities were interested in majoring in mathematics and 2% subsequently majored in mathematics, so it appeared departments were "filtering" prospective mathematics majors.[53] However, for the past twenty years, the percentage of entering freshmen intending to major in mathematics has been smaller than the percentages graduating with majors in the discipline, so departments appear to be "recruiting" majors (and many are succeeding).[54]

The recommendations given below for programs for the major are guided by the changing nature of the mathematical sciences, but also by this clear need for more mathematical sciences graduates in the face of the decline in the number of majors and the changing demographics of the student body.

Recommendation 1, that a department should understand the strengths and aspirations of its students and evaluate its courses and programs in light of its students and the resources of its institution, is particularly important for the mathematical sciences major. Admittedly, it would make life easier if these CUPM recommendations could include a list of courses describing the ideal mathematical sciences major. However, such a list is neither possible nor desirable. It is not possible because of the varying demographics and aspirations of students at diverse institutions nationally. It is not desirable because of the varied careers and fields in which mathematical sciences majors are needed and the capacities of different institutions to meet these different societal needs. Even at a single institution, providing a flexible major or a variety of tracks within the major can position the department to meet the diverse needs of its students most effectively.[55]

The recommendations for mathematical sciences majors include all those in Part I, which addressed all students, as well as the following specifically addressing students majoring in the mathematical sciences. The goals expressed in each recommendation are both desirable and attainable. Illustrative Resources contains examples that work toward implementation of the recommendations in a variety of settings.

C.1. Develop mathematical thinking and communication skills

Courses designed for mathematical sciences majors should ensure that students

- ***Progress from a procedural/computational understanding of mathematics to a broad understanding encompassing logical reasoning, generalization, abstraction, and formal proof;***
- ***Gain experience in careful analysis of data;***
- ***Become skilled at conveying their mathematical knowledge in a variety of settings, both orally and in writing.***

Recommendation 2 states that every course should incorporate activities that will help all students progress in developing analytical, critical reasoning, problem-solving, and communication skills. This is particularly important for majors.

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[53] *Models that Work*, page 4.

[54] In 2000, 1% of all bachelor's degrees were awarded in mathematics; in 1998 0.6% of entering freshmen intended to major in mathematics. See Appendix 3.

[55] See the preliminary report from Gilbert Strang at the Massachusetts Institute of Technology in the online *Illustrative Resources*, C.3.

Progress from a procedural/computational understanding of mathematics to a broad understanding. The ability to read and write mathematical proofs is one of the hallmarks of what is often described as mathematical maturity. Some of the pieces that go into this ability include careful attention to definitions, examination of the effects of modifying hypotheses, understanding of logical implication, and appreciation for how different aspects of a problem are related. Although each mathematics class will address the development of these abilities in its own way, some of the foundation for this kind of logical thinking must be laid in every course in which a prospective mathematics major might enroll, including calculus and discrete mathematics.

By the time they graduate, mathematical science majors should be expected to understand and write proofs. This does not imply that most of their coursework should consist of studying polished proofs of theorems. Neither is this the way mathematics is discovered/created nor is it the way to help most students master the underlying mathematical ideas or learn to write their own proofs. Faculty can best help students master the concept of proof by an incremental and broadly based approach; *every* advanced mathematics course can and should make its contribution to the development of students' ability to understand and construct mathematical proofs. Students should learn a variety of ways to determine the truth or falsity of conjectures. They need to learn to examine special cases, to look for counterexamples, and know how to start a proof and how to recognize when it is complete. Significant experience with proof can occur not just in traditional courses such as algebra, analysis, and geometry, but also, as indicated in Illustrative Resources, in courses as varied as cryptography, image compression, and linear models in statistics.

Gain experience in careful analysis of data. The analysis of data provides an opportunity for students to gain experience with the interplay between abstraction and context that is critical for the mathematical sciences major to master. Experience with data analysis is particularly important for majors entering the workforce directly after graduation, for students with interests in allied disciplines, and for students preparing to teach secondary mathematics. A variety of courses can contribute to this experience, including calculus, differential equations and mathematical modeling, as well as courses with a more explicit emphasis on data analysis. For example, students in a calculus course can fit logistic and exponential models to real data or encounter finding the best-fitting line for data as an optimization problem.

Convey mathematical knowledge, both orally and in writing. Careful reasoning and communication are closely linked. A student who clearly understands a careful argument is capable of describing the argument to others. In addition, a requirement that students describe an argument or write it down tests whether understanding has truly occurred. All courses should include demands for students to speak and write mathematics, and more advanced courses should include more extensive demands. Communicating mathematical ideas with understanding and clarity is not only evidence of comprehension, it is essential for learning and using mathematics after graduation, whether in the workforce or in a graduate program.

The 1991 CUPM report states, "Students who complete mathematics majors have often been viewed by industry, government, and academia as being well-prepared for jobs that require problem-solving and creative thinking abilities." The recommendations of the current *Guide* provide a basis to ensure that this reputation is upheld and enhanced.

C.2. Develop skill with a variety of technological tools

All majors should have experiences with a variety of technological tools, such as computer algebra systems, visualization software, statistical packages, and computer programming languages.

Recommendation 5 states that courses at all levels should: 1) incorporate activities that will help students learn to use technology as a tool for solving problems, and 2) make use of technology as an aid to understanding mathematical ideas. This recommendation and the discussion that followed it is applicable to courses designed for mathematics majors.

The first part of the recommendation states that students should be able to use technology as a tool. Learning to use technology effectively is an absolute necessity for students entering the job market immediately after receiving a bachelor's degree. Anyone doing technical work, including many in the teaching professions, will make extensive use of software; mathematical sciences majors need to prepare themselves by acquiring appropriately extensive experience with these tools.

The second part of the recommendation states that technology should be used as an aid to understanding mathematical ideas. Computer algebra systems, visualization software, and statistical packages can all be incorporated into courses in ways that facilitate exploration of concepts to a much greater extent than is possible with paper and pencil. These tools also enable students to focus on the big picture while working on complex problems. While technology is sometimes used in ways that obscure fundamental ideas and algorithms, this problem can be avoided by careful attention to design of assignments, construction of tests, and other aspects of pedagogy. On balance, the benefits of developing instructional techniques that use technology as an aid to understanding far outweigh the costs.

As described in Illustrative Resources, many departments are requiring majors to become proficient with graphing calculators, computer algebra systems, statistical packages, spread sheets, and programming languages during their first two years of study so that these tools can be used regularly and easily in large numbers of upper-level courses.

C.3. Provide a broad view of the mathematical sciences

All majors should have significant experience working with ideas representing the breadth of the mathematical sciences. In particular, students should see a number of contrasting but complementary points of view:

- ***Continuous and discrete,***
- ***Algebraic and geometric,***
- ***Deterministic and stochastic,***
- ***Theoretical and applied.***

Majors should understand that mathematics is an engaging field, rich in beauty, with powerful applications to other subjects, and contemporary open questions.

Every major should experience the breadth of mathematics. These contrasting points of view can often be encountered in the same course. Most programs include topics that could be described by the first of each of the dichotomies above, but many do not include sufficient emphasis on the second. Therefore, the following clarifications will focus on the second.

Continuous and discrete. Continuous mathematics (calculus, differential equations, analysis) typically is strongly represented in the mathematics major, with many departments requiring four or five courses. Indeed, calculus remains crucially important as the language of science and economics. All majors should study calculus, and many will be interested in areas that require extensive course work in this area. However, some or all of discrete mathematics, matrix algebra, probability and statistics, and other topics are even more important for many other students' career interests. Therefore the continuous and the discrete must both be present for all students. Allowing students to use discrete mathematics courses as prerequisites for higher-level mathematics courses can draw able and interested students into courses such as linear algebra, computational algebraic geometry, abstract algebra, and number theory, and increase the likelihood that they will decide to pursue a joint or double major with mathematics.

Algebraic and geometric. Algebra is typically represented in the mathematics major in many ways, beginning with the use of algebraic techniques in calculus and the study of linear algebra. Indeed, the program of every mathematics major should include linear algebra, and many students are well-served by continuing their study of algebra in courses such as advanced linear algebra, abstract algebra, coding theory, number theory, or linear models.

On the other hand, geometry is typically under-represented in the major. A number of contributors to *CUPM Discussion Papers about Mathematics and the Mathematical Sciences in 2010: What Should Students Know?*[56] call attention to the importance of geometry in the undergraduate curriculum. Herb Clemens points out that the "rush to calculus" in the high schools means students arrive in college with much weaker spatial skills, and yet there is no vehicle for remediating these deficiencies in the first two years. And at the advanced level, he observes, "There is plenty of justification coming from the directions of current research and applications in the very fields (analysis, applied math, probability and statistics, and even abstract algebra) which are the so-called competitors of geometry for time and attention." He also notes the particular needs of majors preparing to teach secondary mathematics, urging that "the fundamental course in geometry for [them should be] a serious course in two-dimensional geometry concentrating on non-trivial results in Euclidean geometry, but also with the three or four fundamental results of spherical and hyperbolic geometry." (pp. 23, 25) Roger Howe writes, "Geometry today is clearly the invalid of the college mathematics curriculum. In many institutions there is no regular offering in geometry, and in others, the main offering is a course specifically aimed at high school teachers.… It should not be so. Geometry was for centuries the heart of mathematics [and] … still is a vital part. As represented in Lie theory, algebraic geometry, dynamical systems, Riemannian geometry and other research areas, it is a central part of the research enterprise." (p. 46)

Several tables in Appendix 4 corroborate the concern about geometry. Table 4-3 shows that from 1995 to 2000 the percentage of departments offering geometry courses fell from 69% to 56%. Also, Table 4-5 shows geometry enrollments falling along with the decline in total enrollments in advanced mathematics courses from 1985 to 1995. Geometry enrollments remained flat (at about half the 1970 enrollment) from 1995 to 2000, while total enrollment in advanced mathematics courses returned to1985 levels.

Attention to geometric thinking should not be confined to geometry courses. It is also important in other courses, such as calculus and linear algebra. Illustrative Resources includes descriptions of a variety of geometry courses appropriate for majors with varying interests as well as of other courses—such as multivariable calculus, complex variables, linear models in statistics, and number theory—that emphasize geometric thinking and visualization and/or make effective use of graphics software to enhance understanding and as an aid to problem solving.

Deterministic and stochastic. Deterministic models are among the glorious achievements of mathematics, and every major should study them. Such models are currently well-represented in programs for majors in the mathematical sciences, as they should be.

But stochastic methods are also valuable and are typically less well-represented in the major program. Students should see examples of probabilistic methods in courses such as calculus and linear algebra as well as in the analysis of data. In particular, the *CUPM Guide 2004* supports the 1991 CUPM recommendation that every mathematical sciences major should study statistics or probability with an emphasis on data analysis for reasons including the following:

- Data analysis is crucial in many aspects of academic, professional and personal life;
- The job market for mathematical sciences majors lies heavily in fields that need people who can effectively draw conclusions from data;
- The emphasis on data analysis in the 2000 NCTM standards[57] and the growth of Advanced Placement statistics courses in secondary school make a study of statistics necessary for those preparing for secondary school teaching in mathematics.

[56]MAA Report, 2001.

[57]*Principles and Standards for School Mathematics,* National Council of Teachers of Mathematics, 2000. An electronic version is available at `standards.nctm.org/info/about.htm`.

The report of the statistics workshop in the Curriculum Foundations project argues that it is more important that mathematics majors study statistics or probability with an approach that is data-driven than one that is calculus-based, and CUPM agrees. The Guidelines of the American Statistical Association/ Mathematical Association of America Joint Committee on Undergraduate Statistics[58] specify that an introductory course should (1) emphasize statistical thinking: the importance of data production, the omnipresence of variability, the quantification and explanation of variability; (2) include more data and concepts, less theory, and fewer recipes; and (3) foster active learning. CUPM endorses the study of statistics for majors following an approach that satisfies the ASA/MAA guidelines. Such study also could serve as an alternative entry point for the major or minor.

Theoretical and applied. The theoretical ideas of mathematics must have an important place in the program of every major. Indeed, majors who have not seriously engaged rigorous theoretical mathematics will not fully understand the coherence, the beauty, and the power of the discipline.

However, recent advances in the mathematical sciences and in technology have implications for the balance between the theoretical and the applied in the undergraduate major. New applications of mathematics and new kinds of mathematics have become both important and possible to teach at the undergraduate level. A list of the mathematics used in different occupations appears on the careers webpage assembled by three of the professional societies in the mathematical sciences.[59] The list includes probability and statistics, ordinary and partial differential equations, Monte Carlo methods, optimization, control theory, game theory, discrete mathematics, splines, information theory, image compression and wavelets, computational geometry, and computational algebraic geometry. Applications in these areas are accessible to undergraduates, and selected topics should be included in the undergraduate program of all mathematical sciences majors. Illustrative Resources includes examples of successful undergraduate courses incorporating several of these topics.

Programs that emphasize discrete, geometric, stochastic, and applied topics along with the continuous, algebraic, deterministic and theoretical will enable students to better understand the emerging nature of mathematics. Majors also need experiences in which they grapple with the powerful applications to other subjects and encounter contemporary open questions.

C.4. Require study in depth

All majors should be required to

- ***Study a single area in depth, drawing on ideas and tools from previous coursework and making connections, by completing two related courses or a year-long sequence at the upper level;***
- ***Work on a senior-level project that requires them to analyze and create mathematical arguments and leads to a written and an oral report.***

The 1985 Association of American Colleges (AAC) report *Integrity in the College Curriculum* "views study in depth as a means to master complexity, to grasp coherence and to explore subtlety. The AAC goals for study in depth are framed by twin concerns for intellectual coherence intrinsic to the discipline and for development of students' capacity to make connections…."[60] This captures well the rationale for requiring work in depth of majors in the mathematical sciences. The question is, by what means is depth achieved?

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[58] See `www.amstat.org/education/Curriculum_Guidelines.html` and *Heeding the Call for Change*, pp. 3–11.

[59] The careers page of the MAA, AMS and SIAM is at `www.ams.org/careers/mathapps.html`.

[60] MAA-AAC report, p. 189.

Study a single area in depth. To achieve depth in the major, the 1991 CUPM report specified a "two-course sequence in at least one important area of mathematics" with at least a calculus-level prerequisite for the first. Examples included probability and statistics, combinatorics and graph theory, two courses in numerical analysis, or real and complex analysis, as well as the more traditional two semester sequences in algebra or analysis.

These are effective means, but there are other combinations that can work, for instance a pair of courses in which neither is a prerequisite for the other, although both have a common prerequisite. In such a pairing, connections must be drawn at different levels of sophistication for students with different backgrounds. Examples of this kind of pairing include not only real and complex analysis (both with perhaps multivariable calculus as a prerequisite) but also two chosen from abstract algebra, coding theory, computational algebraic geometry, projective geometry (each with a linear algebra prerequisite), or the pair operations research and mathematical modeling (each with a linear algebra and multivariable calculus prerequisite). See Illustrative Resources for details of these and other examples.

Work on a senior-level project. Working on a project provides an experience of depth even when the project doesn't expand the area of study in the curricular sense. Although its focus may be quite narrow, a project requires the kind of integration and synthesis that are the desired consequences of the depth requirement. The kind of project stipulated in this recommendation requires the extended argument and analysis that are also part of what is meant by mathematical depth.

Further, a project for which a student must read, write and orally present significant mathematics develops the communication skills that are vital in the workplace and in future study. Such a project can ease the transition to the world beyond the classroom, where students must learn and use mathematics more independently than ever before and must communicate it clearly to a variety of audiences, including others less mathematically knowledgeable than themselves. Many employers are much more interested in an enthusiastic and lucid explanation of a personal mathematical project than in any list of courses or topics covered. A project also is excellent preparation for the intense independent learning that will be required for students continuing their studies with graduate work.

In some departments, it is feasible to require a capstone course for majors that includes such a significant project. Illustrative Resources contains examples of how such stand alone courses are conducted at a variety of institutions. But other means are also possible. A project can be embedded in a variety of advanced courses, or appended to a course for additional credit. Departments at institutions with January mini-terms can use them for this purpose. Summer research projects and internships also afford opportunities for projects. See Illustrative Resources for examples of projects of these varied kinds. Each department has to choose means appropriate to its students and resources, but ensuring that every major has this learning experience should be a priority, both for its value to the student and for its value to the department in assessing what its majors have learned.

C.5. Create interdisciplinary majors

Mathematicians should collaborate with colleagues in other disciplines to create tracks within the major or joint majors that cross disciplinary lines.

This is a restatement of part of Recommendation 4, placed here for emphasis. Also see the discussion of interdisciplinary majors in B.1. The greatest growth in the number of degrees awarded by departments of mathematics and statistics has been in areas outside the "classic" mathematics major (see Appendix 3). Based on the spring 2001 CUPM sample of mathematics departments, one third of degrees granted that year were to joint or double majors (see Appendix 4). The mathematical sciences major at the University of Michigan, the applied mathematics major at Brown University, and the joint majors in applied science at UCLA are effective programs, as are the joint majors with biological sciences, computer science, economics or business at a number of large and small institutions. See Illustrative Resources for details of these and other examples.

This recommendation highlights the importance of a department knowing its students and understanding the particular strengths of its broader institution. The Curriculum Foundations project found significant support within the partner disciplines for joint, interdisciplinary majors. Every college or university has its own opportunities for fruitful collaboration, and each mathematical sciences department should offer at least one major or one option within a major that encourages students to combine in-depth study of mathematical sciences with an allied field.

C.6. Encourage and nurture mathematical sciences majors

In order to recruit and retain majors and minors, mathematical sciences departments should

- ***Put a high priority on effective and engaging teaching in introductory courses;***
- ***Seek out prospective majors and encourage them to consider majoring in the mathematical sciences;***
- ***Inform students about the careers open to mathematical sciences majors;***
- ***Set up mentoring programs for current and potential majors, and offer training and support for any undergraduates working as tutors or graders;***
- ***Assign every major a faculty advisor and ensure that advisors take an active role in meeting regularly with their advisees;***
- ***Create a welcoming atmosphere and offer a co-curricular program of activities to encourage and support student interest in mathematics, including providing an informal space for majors to gather.***

Put a high priority on effective teaching in introductory courses. Departments with large numbers of majors have almost without exception made strong teaching in introductory courses a priority. Faculty in introductory courses have a significant influence on both recruitment and retention of majors and minors. Upper-level mathematics majors also can act as mentors to students in introductory courses, visiting classes to share their own experiences in subsequent courses and internships.

Seek out prospective majors. While recruitment and encouragement of mathematics majors can begin as early as middle school, these recommendations are restricted to secondary and post-secondary recruitment. Most colleges and universities sponsor special days when prospective students visit campus and meet with faculty in various departments. Mathematics faculty should be carefully assigned the task of meeting with these students—first impressions mean a lot.

The next place recruitment of mathematics majors can occur is within courses at the college level. As implied by Recommendation B.3 on prerequisites, there are multiple ways to begin the serious study of mathematics. Calculus is just one of them. Faculty should reach out to students in first year courses in discrete mathematics, statistics, number theory, and geometry and in survey courses, as well as calculus. Never underestimate the power of a faculty voice encouraging a student to consider a major in mathematics. Even if only a small percentage of these students go on to become majors, that fraction can represent a sizable increase in a program. A larger fraction may take additional courses in mathematics and add both numbers and vitality to advanced courses. Encouragement and support are particularly important for students from groups that do not traditionally major in mathematics. The websites of the MAA program Strengthening Under-represented Minority Mathematics Achievement (`www.maa.org/summa`) and of the Association for Women in Mathematics (`www.awm-math.org`) include links to a variety of resources.

Two year colleges play an important role in providing introductory courses in mathematics. They enroll 44% of undergraduates in the U.S., 46% of first time freshmen and 49% of undergraduates who identify themselves as members of racial or ethnic minorities.[61] Departments should cultivate relationships with nearby two-year colleges and seek out their students with interests in mathematics.

Students taking mathematics to satisfy the requirements for another major should be encouraged to consider a double or joint major with mathematics or at least a minor in mathematics. The growing presence of mathematics in traditionally non-mathematical disciplines such as biology is a strong selling point for students, but faculty need to make sure that students know of the pervasive uses of mathematics in those disciplines.

Inform students about careers in the mathematical sciences. Career information is vital. The Mathematical Sciences Careers web page (`www.ams.org/careers/`) can serve as an extremely valuable resource as can the MAA document *101 Careers in Mathematics*.[62] Departments also should make certain that advisors have access to relevant information from their own institution's careers office about alumni employment historically, as well as from recent alumni and their employers.

Most students will experience multiple career changes over their working lifetimes, so continuing professional education is increasingly important. There are new master's degree programs offered not only by mathematical sciences departments but also departments such as computer science, statistics, operations research, economics, business, and engineering. Some include BA-MS "articulation programs," connecting undergraduate mathematics to graduate programs in other disciplines. Awareness of these newer options places added responsibility on advisors. Departments should be sure that advisors are well informed about alumni experience in these and other graduate programs. (See the discussion of Recommendation D.3.)

Set up mentoring programs. Department faculty need to focus on the needs of individual students. Only a small minority of mathematics majors will pursue doctoral study in the subject (less than 10% at most institutions—see Appendix 4), and faculty must recognize the different interests of their majors. One successful department maintains a bulletin board (and web page) of brief reports from their graduates that describe their academic programs and current employment.

Working as tutors or graders can increase students' interest in continuing the study of mathematics as well as deepening their understanding of the content of the course they are assisting. Some departments invite students who performed well in first year courses to apply for these jobs. Departments need to offer appropriate training and support to assure that the tutorial or grading experience is rewarding both for the students receiving assistance and for those providing it.

Assign every major a faculty advisor who actively meets with advisees. Advising of majors throughout their undergraduate years is an important department responsibility. Unlike an earlier, simpler day when all mathematics majors took the same sequence of courses with only a few electives in the senior year, the typical undergraduate mathematical sciences department today requires students to make substantial curricular choices. Advisors should carefully monitor each advisee's academic progress and changing goals and work with the student to explore the many intellectual and career options available to mathematics majors. For some students, achieving the best choice of courses may necessitate coordination between the major advisor and faculty in another department.

Advisors also should pay particular attention to the need to retain capable undergraduates in the mathematical sciences pipeline, with special emphasis on the needs of under-represented groups. When a

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[61] "First Steps: The Role of the Two-Year College in the Preparation of Mathematics-Intensive Majors," by Susan S. Wood, in *CUPM Discussion Papers about Mathematics and the Mathematical Sciences in 2010: What Should Students Know?*, MAA Reports, 2001, p.101.

[62] *101 Careers in Mathematics*, 2nd edition, edited by Andrew Sterrett, Classroom Resource Materials, MAA, 2003.

department offers a choice of several tracks within the major, advisors have the added responsibility of providing students with ample information even when students do not ask many questions. This individualized approach to advising requires that no advisor be assigned too many advisees.[63]

Create a welcoming atmosphere and offer a co-curricular program of activities to encourage and support student interest in mathematics. Departments should provide space for informal student contact. Students who develop good working relationships with peers are more likely to succeed in their mathematics courses and to sustain interest and motivation. A student lounge equipped with work tables, writing boards and (comfortable) chairs should be available for student/student as well as student/faculty contact. Shelves containing current student-friendly mathematical journals as well as selected mathematics books would add to the atmosphere of such a room.

Departments should also offer opportunities for mathematics majors to interact with one another through a campus organization. This might be a mathematics club, a student chapter of the MAA or a chapter of Pi Mu Epsilon. Such an organization can serve as a catalyst for a number of retention activities — mentoring programs, attendance at local or national meetings, and sponsorship of movies and presentations. Such an organization also gives students a sense of ownership in their major.

D. Mathematical sciences majors with specific career goals

D.1. Majors preparing to be secondary school (9–12) teachers

In addition to acquiring the skills developed in programs for K–8 teachers, mathematical sciences majors preparing to teach secondary mathematics should

- ***Learn to make appropriate connections between the advanced mathematics they are learning and the secondary mathematics they will be teaching. They should be helped to reach this understanding in courses throughout the curriculum and through a senior-level experience that makes these connections explicit.***
- ***Fulfill the requirements for a mathematics major by including topics from abstract algebra and number theory, analysis (advanced calculus or real analysis), discrete mathematics, geometry, and statistics and probability with an emphasis on data analysis;***
- ***Learn about the history of mathematics and its applications, including recent work;***
- ***Experience many forms of mathematical modeling and a variety of technological tools, including graphing calculators and geometry software.***

The teacher preparation recommendations of this *Guide* have been informed by *The Mathematical Education of Teachers (MET),*[64] a recent CBMS report that presents detailed and carefully considered guidelines concerning the education of future teachers of mathematics. Mathematics faculty and departments are advised to study MET in its entirety.[65] Prospective teachers of secondary school mathematics

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[63] This recommendation and discussion are adapted from the 1991 CUPM report, which reappears as Appendix E of *MAA Guidelines for Programs and Departments in Undergraduate Mathematical Sciences,* and from the recommendations in the *MAA Guidelines for Programs.*

[64] *The Mathematical Education of Teachers*, volume 11 of the Issues in Mathematics Education series of the Conference Board of the Mathematical Sciences, AMS and MAA, 2001, available at `www.cbmsweb.org`.

[65] The *MET* authors write, "This report is not aligned with a particular school mathematics curriculum, although it is consistent with the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* as well as other recent national reports on school mathematics."