TWO-YEAR COLLEGES AND BASIC MATHEMATICS

The Panel on Mathematics in Two-Year Colleges was formed in 1966 following some preliminary study of the need and potential in this area for the kind of activities which CUPM had successfully pursued in other areas. The members of the Panel were chosen from two-year colleges, four-year colleges, and universities so that extensive experience in various phases of education would be available. The Panel initially sponsored a series of meetings at which representatives of a wide spectrum of two-year colleges provided much detailed information about local variations, supplementing the Panel's studies of the national scene. The Panel also participated in several other activities related to the problem, such as meetings of the National Science Foundation Intercommission Panel on Two-Year Colleges, meetings of various organizations of two-year college mathematics teachers, individual visits to institutions, and a wealth of personal contacts. During this study phase the Panel was divided into subpanels concentrating on three topics: mathematics for general education, mathematics for technical-occupational programs, and mathematics for four-year college transfer programs (in all disciplines). Many two-year college teachers who consulted with the Panel expressed the opinion that guidance was most needed on the first two topics. However, it became increasingly clear as the study progressed that considerable overlap existed in the problems in these three areas and that an initial concentration on the third topic was most natural, both logically and from the viewpoint of CUPM's customary methods of operation. Thus, the Panel decided to concentrate its initial efforts on the construction of a program for university-parallel mathematics courses in two-year colleges. Its report, <u>A Transfer Curriculum in Mathematics for Two-Year Colleges</u>, was issued in 1969.

Concurrent with the decision of the Panel to restrict itself to the university-parallel curriculum, CUPM appointed an <u>ad hoc</u> Committee on Qualifications for a Two-Year College Faculty in Mathematics, whose membership overlapped that of the Panel. The report of this Committee, which appears in the section on TRAINING OF TEACHERS, discusses the qualifications of teachers of university-parallel mathematics courses and makes some general remarks concerning two-year college mathematics faculties.

The <u>Transfer Curriculum</u> report is essentially an adaptation of the first part of the GCMC curriculum [page 33] to the particular circumstances of those students in two-year colleges who intend to transfer to a four-year institution. That report intentionally deferred the consideration of lower-level or nonuniversity-parallel courses as a matter for further study. In 1970 CUPM appointed a Panel on Basic Mathematics to consider the first of these two areas: courses at a level below that of Mathematics A in the <u>Transfer Curriculum</u>. Among the members of this new Panel were persons from the Two-Year College Panel and representatives from developing institutions. The Panel felt that it would be possible to replace many of these courses by a single flexible course which involved a mathematics laboratory and was innovative in its approach. Its recommendations, together with an outline and commentary on the proposed course, appear in the 1971 publication A Basic Course in Mathematics for Colleges.

In 1971 CUPM issued A Basic Library List for Two-Year Colleges. This list was compiled by an *ad hoc* committee, with the assistance of many teachers from two-year colleges, four-year colleges, and universities.

Having offered suggestions for the improvement of university-parallel and basic mathematics programs, CUPM then turned to the much more complicated area of mathematics for technical-occupational programs in two-year colleges. A reconstituted Panel on Mathematics in Two-Year Colleges laid plans for producing materials designed to improve mathematics instruction for students in these fields. Due to lack of funds, it has not yet been possible to bring these plans to fruition.

A TRANSFER CURRICULUM IN MATHEMATICS

FOR TWO-YEAR COLLEGES

A Report of

The Panel on Mathematics in Two-Year Colleges

January 1969

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I. BACKGROUND

1. Two-Year Colleges.[1]

It is impossible not to be impressed, perhaps a little overwhelmed, by the growth and diversity of the two-year college sector of American higher education. To cite only a few facts indicating the rate of growth: new institutions are being added at the rate of one each week to the approximately 900 that now exist; the largest student body in any Florida educational institution is that of Miami-Dade Junior College; 86 per cent of all freshmen entering California public colleges in 1966 enrolled in two-year colleges; Seattle Community College opened in 1967 with an initial enrollment of 12,688 students. The extraordinary rate of growth in number of institutions and size of individual institutions is matched by their diversity, of size, purpose, make-up of student body, type of institutional control, variety of curricula, etc. A glance at a random selection of two-year college catalogs reveals this diversity quite strikingly. Further aspects of it can be noted in the annual directories of the American Association of Junior Colleges, and others are described and documented in a 1967 report to Congress from the National Science Foundation.[2]

Anyone interested in mathematical education who is accustomed to the comparatively stable national scene in either the elementary and secondary schools or in the four-year colleges and universities may find the two-year college picture bewildering. There is an immense variety of programs, including not only those comparable to the first two years in a university, but also a large assortment of technical, occupational, and semiprofessional programs as well as programs for general education and remedial study. The programs offered vary widely from school to school; the two-year college is often much more responsive to the community it serves and somewhat less responsive to tradition than the other institutions.

The student body attracted by these programs or by the convenience and economy of two-year college education exhibits extremes of age and maturity, background and preparation, and ability and motivation. More than two thirds of these students regard themselves

1. The name "two-year college" is intended to tie in with the "lower division" at a college or university. Students at any of these schools may spend more or less than two years on their "lower-division" work.

2. The Junior College and Education in the Sciences, U. S. Government Printing Office, Washington, D. C., 1967.

initially as transfer students,[1] but only about one third eventually proceed to a four-year college.[2] This phenomenon is reflected in the figures on two-year college course enrollments in mathematical science compiled by the Conference Board of the Mathematical Sciences:[3]

> "Of these course enrollments 324,000 were classified by the institutions themselves as being in freshman courses and only 24,000 as being in sophomore courses. This is a much sharper drop than in four-year colleges."

This survey and the NSF report contain revealing data on the faculty. They describe a faculty which includes many part-time teachers, has a broad range of academic preparation, is recruited from a variety of sources (graduate schools 24%, colleges and universities 17%, high schools 30%, other sources 29%)[4] and is highly mobile ("About 25 per cent of all junior college professors were new to their particular campuses in 1964-65.").[5]

The situation sketched here has attracted the attention of many organizations concerned with the improvement of American higher education, of which CUPM is but one.

2. CUPM.

The Mathematical Association of America (MAA) is the national professional organization concerned with the teaching of mathematics on the college level. The Committee on the Undergraduate Program in Mathematics (CUPM) is one of the standing committees of the MAA. Simultaneously, CUPM is one of the eight college commissions in the sciences, supported by the National Science Foundation, "to serve as instruments through which leading scientists can provide stimulation, guidance, and direction to the academic community in the improvement of undergraduate instruction."

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1. NSF Report, p. 92.

2. NSF Report, p. 5.

3. Report of the Survey Committee, Volume I. Aspects of Undergraduate Training in the Mathematical Sciences. [For more recent figures, see Volume IV. Undergraduate Education in the Mathematical Sciences, 1970-71.] Available from Conference Board of the Mathematical Sciences, Joseph Henry Building, 2100 Pennsylvania Avenue, N.W., Suite 834, Washington, D.C. 20037.

4. NEA figures for 1963-64 and 1964-65 in all fields (see CBMS survey, p. 76).

5. NSF Report, p. 71.

The early curriculum recommendations published by CUPM dealt with specific aspects of education in mathematics, such as the training of physical science and engineering students, the training of teachers of elementary and high school mathematics, and the undergraduate preparation of graduate students of mathematics. It eventually became clear that CUPM had a responsibility to show how an overall curriculum could be constructed which was within the capabilities of a fairly small college and allowed for the implementation of its various special programs. A study of this problem led to the 1965 report A General Curriculum in Mathematics for Colleges (GCMC).* This report has received wide publicity through CUPM regional conferences and Section meetings of the MAA. Its major features have met with general approval and are having a growing influence on college textbook and curriculum reforms in tangible as well as in many intangible ways. Regarding this influence, it must be stressed that CUPM does not write curricula in order to prescribe what courses a department should teach, but rather to offer generalized models for discussion and to provide the framework for meaningful dialogue (within schools, between schools, and on a broader scale) on serious curricular problems. These models are meant to be both realistic and forward looking.

The present report is a natural extension of these efforts. In addition it provides, together with GCMC, an aid to articulation of two- and four-year programs.

3. The Present Report.

This initial CUPM report on two-year colleges is aimed at the transfer programs only. Some history of the study and reasons for this choice are outlined in the introduction to this section of the COMPENDIUM. Here we mention the following reasons:

Transfer programs are offered at almost all two-year colleges, and they determine the basic mathematics offerings. Local variations are least in this area.

A workable, imaginative solution to the problem of a transfer curriculum would provide the natural first step and could go far towards solution of the problems in the other two areas. It would answer the needs of the great majority of the students (two thirds intend to transfer), especially if it kept open a variety of options for these students, at least through the first year.

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* The report presents a curriculum which can be taught by a staff of four or five and which includes courses for the various special programs. (An exception is the special course sequence for elementary teacher training; staff for these courses is not included in the estimate.)

The transfer curriculum lends itself more readily to a curriculum study with follow-up conferences. The other two problems (mathematics for general education and mathematics for technical-occupational programs) involve additional considerations, such as pedagogical techniques and the special needs of students with very specific goals, making this general approach less effective. CUPM's wide experience with the GCMC report and conferences (frequently involving two-year college teachers) had clearly indicated a need and a demand for similar efforts suitably adapted to the realities of two-year colleges.

The present report differs from GCMC in several major respects. Among these is the fact that it includes more detailed course descriptions, discussions of the rationale for choices that were made, and frequent comments on how topics might be taught. It also includes comments on implementation; the use and influence of the computer; articulation; etc. (see Chapter 5). Finally, it includes explicit recommendations on teacher training (see Chapter 3).

4. Staff.

The CBMS Survey mentioned earlier reports the following data on staff. The 167 two-year colleges having enrollments of more than 2,000 employed 44 full-time mathematics staff members with a doctorate in some field (not necessarily a mathematical science); 451 with a master's degree plus one additional year of graduate study in some field; and 439 with a master's degree in mathematics. The corresponding figures for the 543 schools of under 2,000 enrollment are 66, 306, and 513 respectively. There are, of course, many additional staff members with less academic preparation or who teach part time.

From these figures we can compute roughly the average number of staff members with academic preparation in the above range: five or six for the larger schools and one or two for the smaller schools. The corresponding figures when part-time faculty are included are eight and two, roughly.

The present report takes account of these facts by outlining a group of basic university-parallel courses that provide the necessary offerings for normal transfer programs (including elementary teacher training). It was prepared with the small department in mind. A larger department would have the potential for offering some additional courses to supplement these. Several possibilities for such courses are suggested in the report.

The important thing is that we believe our basic courses can be taught by the equivalent of approximately two full-time staff members. In Chapter 5, Section 1, we give an illustration of how this can be done. The actual number of teachers required in a particular case will depend, of course, on class size, teaching load, and other such factors.

It is perhaps the most striking common feature of two-year colleges that their faculties are highly student-oriented, much more so than the more discipline-oriented faculties at the four-year colleges and universities.[1] Nevertheless, it is important not to overlook problems of academic qualifications. In particular, a report such as this or the GCMC report would exist in a partial vacuum if there were no accompanying considered statements concerning the academic qualifications desired of the teachers of the programs. Such a report has been written to accompany the GCMC[2] and another report outlining a graduate program to achieve the proposed training has been prepared.[3] Simultaneously with its decision to begin with the university-parallel study, CUPM organized an ad hoc Committee on Qualifications for a Two-Year College Faculty in Mathematics. The report of this Committee[4] will be a companion to this report.

5. The Proposed Programs.

The courses which CUPM proposes are described in detail in Chapters 2, 3, and 4. We summarize them here under the broad classification of Basic Offerings and Additional Offerings, and discuss some of the programs which such offerings permit. Where appropriate, we draw attention to the comparable course in the report A General Curriculum in Mathematics for Colleges. The latter, of course, serves as an alternative to be considered by those two-year colleges that are structured on purely university-parallel lines.

BASIC OFFERINGS

I. Calculus Preparatory

(a) Mathematics 0. Elementary Functions and Coordinate Geometry.

(b) Mathematics A. Elementary Functions and Coordinate Geometry, with Algebra and Trigonometry.

One or both of these courses should be offered by every two-year college.

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1. See, for example, R. H. Garrison. Junior College Faculty: Issues and Answers. American Association of Junior Colleges, 1967.

2. Qualifications for a College Faculty in Mathematics (1967).

3. A Beginning Graduate Program in Mathematics for Prospective Teachers of Undergraduates (1969).

4. Qualifications for Teaching University-Parallel Mathematics Courses in Two-Year Colleges (1969).

II. Calculus and Linear Algebra

(a) Mathematics B. Introductory Calculus. (An intuitive course covering the basic concepts of single-variable calculus. Similar to Mathematics 1 [page 44].)

(b) Mathematics C. Mathematical Analysis. (A more rigorous course completing the standard calculus topics, as in Mathematics 2, 4 [page 51].)

(c) Mathematics L. Linear Algebra. (An elementary treatment similar to Mathematics 3 [page 55], but parallel to, rather than preceding, the last analysis course.)

Categories I and II constitute the basic pre-science offerings and should be offered by every two-year college with a transfer program.

III. Business and Social Science

Mathematics PS. Probability and Statistics. (An introductory course stressing basic statistical concepts.)

IV. Teacher Training

Mathematics NS. Structure of the Number System. (A year course as recommended by the Panel on Teacher Training for the preparation of elementary school (Level I) teachers. The second year of preparation, algebra and geometry, should also be offered whenever possible (see Chapter 3).)*

This completes the minimal set of offerings envisioned for a two-year college that embraces the usual range of university-parallel programs.

The courses A, C, and NS are full year courses. If we consider a semester system and use the obvious notation, the basic program is represented by the following diagram.

* Since this report was written, the recommendations of the Panel on Teacher Training have been revised. See the 1971 report Recommendations on Course Content for the Training of Teachers of Mathematics, page 158.

BASIC OFFERINGS

Calculus Preparatory → Calculus-Linear Algebra

$$\left[\; 0 \text{ or } A_1 \longrightarrow A_2 \;\right] \longrightarrow \left[\; B \longrightarrow C_1 \longrightarrow \begin{cases} C_2 \\ L \end{cases} \;\right]$$

Business and Social Science

$$\boxed{PS}$$

Elementary Teacher Training

$$NS_1 \longrightarrow NS_2 \longrightarrow \begin{cases} \text{Algebra} \\ \text{Geometry} \end{cases}$$

ADDITIONAL COURSE OFFERINGS (Optional)

1. Mathematics FM. Finite Mathematics. (A course of considerable interest and utility, especially for students of non-physical sciences.)

2. Mathematics DE. Intermediate Differential Equations.

3. Mathematics DA. Differential Equations and Advanced Calculus.

4. Mathematics PR. Probability Theory. (A calculus-based course adapted from Mathematics 2P [page 76].)

5. Mathematics NA. Numerical Analysis.

Selections from this list are suggested to round out a department's offerings according to its special needs and interests.

PROGRAMS*

We see this combination of courses as providing a variety of tracks to meet the needs of students with different educational goals and mathematical backgrounds and abilities. Some of the possibilities are described here.

* See page 224, Pace and manner of presentation.

(i) For the student in physical science, engineering, and mathematics (including future secondary school teachers) the program should include B, C, and L. A sequence for well-prepared students is provided by the semester courses

$$B \longrightarrow C_1 \longrightarrow C_2 \longrightarrow L,$$

plus possible additional courses. A slightly less well-prepared student can achieve this by taking the sequence of courses

$$0 \longrightarrow B \longrightarrow C_1 \longrightarrow (C_2 \text{ and } L).$$

The student who is not prepared for Mathematics 0 at the outset requires an extra semester or summer session to complete calculus at the two-year college, as indicated by the program

$$A_1 \longrightarrow A_2 \longrightarrow B \longrightarrow C_1 \longrightarrow (C_2 \text{ and } L).$$

(ii) Students of the biological, management, and social sciences probably will take Mathematics PS. Many of them will want a calculus course. Assuming adequate preparation for Mathematics 0, a reasonable sequence for such students is

$$PS \longrightarrow 0 \longrightarrow B,$$

to which Mathematics FM may be added in the third or fourth semester.

Although Mathematics L is thought of as following half of Mathematics C, it is possible for a bright student to take it after Mathematics B. The resulting sequence

$$0 \longrightarrow B \longrightarrow L$$

would be of great value as a mathematically stronger alternative to the more standard

$$0 \longrightarrow B \longrightarrow FM,$$

or

$$0 \longrightarrow FM \longrightarrow B.$$

(iii) If the primary interest of a student is the inclusion of some mathematics as a part of his general education, then any of the courses PS, A, 0, FM or the sequences already mentioned will, depending on his previous preparation, serve the purpose effectively.

II. THE BASIC UNIVERSITY-PARALLEL COURSES

In this chapter we describe in some detail the courses we propose for the basic offerings and discuss reasons for the choices that have been made.

1. Calculus Preparatory: Mathematics 0 and Mathematics A.

CUPM has taken the position that precalculus mathematics properly belongs in the high school but that many colleges may need to continue teaching courses at this level. In view of this need, a course, Mathematics 0, is described in A General Curriculum in Mathematics for Colleges. The same course is proposed for the two-year college.

In addition, recognizing the presence in two-year colleges of many students whose high school preparation is in need of reinforcement, the Panel suggests another course, Mathematics A. The subject matter of Mathematics A includes that of Mathematics 0, but the intended pace is much slower so that reviews of topics from arithmetic, algebra, and geometry can be introduced and pursued at appropriate stages of the Mathematics 0 outline.

It is hoped that each two-year college will be able to adapt the basic idea of this course to its own needs, perhaps offering both Mathematics 0 for its well-prepared students and a version of Mathematics A to meet the needs of the rest of its students who hope to complete some calculus. The courses, although designed for students who plan to take calculus, should carry credit for general education requirements. Most schools, however, have special general education courses and would advise their use in preference to Mathematics 0 or A for the purpose.

Mathematics 0. Elementary Functions and Coordinate Geometry.

Discussion: The prerequisites for Mathematics B (Introductory Calculus) include the following two components, (a) and (b):

(a) Three years of secondary school mathematics. The usual beginning courses in algebra (perhaps begun in eighth grade) and geometry account for two of these years. The remaining year should include: quadratic equations; systems of linear and quadratic equations and inequalities; algebra of complex numbers; exponents and logarithms; the rudiments of numerical trigonometry; the rudiments of plane analytic geometry, including locus problems, polar coordinates, and geometry of complex numbers; and arithmetic and geometric sequences.

(b) A course such as Mathematics 0 (or Mathematics A below): A course outline for Mathematics 0 is given on page 75.

Note: The proposed first course in calculus (see page 226) treats topics in analytic geometry only incidentally, as the calculus throws new light on the subject. Thus, rather than an integrated analytic geometry and calculus, it is more in the nature of a straight calculus course. It assumes as a prerequisite sufficient command of coordinate geometry for the study of single-variable calculus and some familiarity with the elementary functions and general concepts of function as outlined above.

Mathematics A. <u>Elementary Functions and Coordinate Geometry, with Algebra and Trigonometry</u>.

Discussion: This special course is designed for those students who, because of a weak mathematics background, are not prepared to begin an intense calculus preparatory course such as Mathematics 0. Many two-year colleges are currently meeting the needs of these students with courses in intermediate algebra, trigonometry, college algebra, and analytic geometry. It is felt that the contributions of all such courses to the overall two-year college offerings would be achieved better through the medium of this single course, adapted to local conditions.

A two-year college might choose to offer only Mathematics A or only Mathematics 0 or both, depending on the needs of the student population.

When teaching courses of high school level in a two-year college, it is customary to repeat the material in essentially the same form as it was presented in high school. For students who were not successful in high school, this approach is often no more fruitful the second time than the first. The virtue of Mathematics A is its fresh approach to old topics. Instead of requiring courses that repeat early high school mathematics and are then followed by a course on elementary functions, it is suggested that a repetition of the high school material be interwoven with the topics of the latter course.

The notion of function is given a central and unifying role in Mathematics A. Functions and their graphs serve as a peg on which to hang the review of elementary material. They also provide a new perspective, as well as a method of illustrating the old material. In this way, it is thought that the necessary review can be presented in a manner sufficiently fresh and interesting to overcome much of the resistance that students may carry with them from high school. (Moreover, this new approach should make Mathematics A more pleasant for the faculty to teach.) The development of a sound understanding of the function concept provides a solid cornerstone on which to build additional mathematical concepts in later courses.

This course should serve at least the following three purposes:

(i) To prepare the student for calculus and other advanced mathematics by including the material of Mathematics 0. Rather than being concerned with precisely which topics should be included, however, the emphasis should be on developing the ability to understand and use mathematical methods at least at the level of Mathematics B.

(ii) To review and remedy deficiencies in arithmetic, algebra, geometry, and basic logic:

(a) by means of assignments and class drill associated with Mathematics 0 topics. (Every opportunity should be seized to expose and stamp out abuses of logic and notation, and such common atrocities as $1/2 + 1/3 = 1/5$, $1/2 + 1/3 = 2/5$, $(a + b)^2 = a^2 + b^2$, and $\sqrt{a + b} = \sqrt{a} + \sqrt{b}$, familiar to all teachers.)

(b) by interjecting discussions of review topics as the occasion arises in Mathematics 0 topics (see illustrations, page 221).

(iii) To develop mathematical literacy. By this we mean the ability to read and understand mathematical statements and the ability to translate into mathematical language (making proper use of logical connectives) statements and problems expressed in ordinary English. Continual practice should be given in solving "word problems" and in analyzing mathematical statements, with particular emphasis on developing the ability to understand and to use deductive reasoning.

Because of the limited time available, some material must be slighted. In this instance classical synthetic plane geometry is not covered in the detail or to the extent that is common in high school. However, we compensate for this by including enough analytic geometry to provide sufficient geometric preparation for calculus. Care has been used in choosing the topics in Mathematics A so as to include those topics which the student needs in order to progress successfully to calculus or to calculus-related courses. It nevertheless seems likely that five hours a week will be needed for Mathematics A during the first semester and possibly also during the second semester. It should be a slow course that includes a great deal of problem solving and also lengthy excursions as suggested by student interests and needs. Although Mathematics A serves to prepare students for calculus, it should also make students aware of the power and beauty of mathematics. The teacher should exploit every opportunity to provide examples and applications appropriate to the age and maturity of the students. Extensive use of "word problems" can be very effective in developing the ability to think mathematically and to use mathematics. We have attempted to give some indications along these lines in the course outline. In our view it is far more important to stir and develop the interest of the students than to cover each suggested topic.

It is essential that the students be forced throughout the course to translate English sentences into mathematical ones and vice versa, in order to improve their mathematical literacy.

Unfortunately, there exists at this time no single textbook completely appropriate for a course such as Mathematics A. However, by drawing upon several existing texts, the teacher should be able to find suitable textual material, illustrative examples, and exercises with which to build the course and achieve the remedial objectives.

Finally, we believe that a student who is not capable of handling Mathematics A upon entering a two-year college will in all likelihood be unable to pursue mathematical subjects in a four-year college with profit. Such a student should not be considered a transfer student in mathematical subject fields.

COURSE OUTLINE FOR MATHEMATICS A

Concept of function. Introduction as a rule associating to each element of a set a unique element of another set. (There shouldn't be very much worry about the definition of the word "set." Many examples should be given, with special attention devoted to functions that cannot be easily represented by formulas: for instance, the number of birds in a given locality or on a given tree as a function of the time or of the individual tree; the number of female students at the college as a function of the year; a function whose domain is the set $\{A, B, C\}$ and whose range is the set $\{D, E\}$; etc.) Cartesian coordinate systems and graphs of functions. Review of relevant geometry, such as basic facts concerning perpendiculars and directed line segments. Review of negative numbers, decimals, and the arithmetic of fractions, keyed to the problem of representing functions graphically. Decimals reviewed in this vein. Relative sizes of numbers illustrated on the x-axis. Motivation by means of examples of the definitions of sum, product, difference, and quotient of functions. Review of rational operations with algebraic expressions. Graphing of inequalities. Review of fractions and decimals, with special emphasis on comparison of sizes. Simple probability as a set function, i.e., a brief discussion of possibility sets, truth sets, and the assignment of measures to sets and their subsets. Conditional probability.

Polynomials of one variable. Linear functions of one variable and their graphs. Slope of a straight line. Various forms of the equation of a straight line. The graph of a linear equation as a straight line. Quadratic functions of one variable and their graphs. Completing the square. Use of completion of the square to solve some simple maximum and minimum problems. (This provides an opportunity to introduce some interesting "word problems.") Definition of polynomial functions of one variable. Sum and product of polynomials. Laws of exponents for integer powers. Division of polynomials, with the process first taught purely as an algorithm (stress should be given to the similar process for integers, whereby one integer is divided by another and a quotient and remainder obtained; the formula $m = qn + r$). Examples, pointing out the desirability of finding zeros of polynomial functions and ranges where the functions are positive or negative. Review of factoring: common factor, quadratics and the quadratic formula, difference of squares, sum and difference of cubes. The factor and remainder theorems. Theorems on rational roots. The need for complex numbers. Complex numbers and their algebra (introduced as a natural extension of the real number system). Geometric interpretation of addition of complex numbers, multiplication by real numbers (i.e., the vector operations) and of multiplication (see illustrations, page 221).

Arithmetic and geometric sequences. Definitions and sum formulas. Approximation (noting error term) of $\frac{1}{1 - x}$ by $\sum_{k=0}^{n} x^k$ for $-1 < x < 1$ (both by using the sum formula and by repeating the division algorithm). Intuitive discussion of the meaning of limit. (Application can be made to the fractional representation of an infinite repeating decimal. The limit concept can be illustrated by using both repeating and nonrepeating infinite decimals.)

Combinations and the binomial theorem. Finite sets, their unions and intersections. Combinations and permutations of members of a set. The binomial coefficients $\binom{n}{k}$ as the number of combinations (subsets) with k members chosen from a set with n members. Additional simple probability, using the formulas for numbers of permutations and combinations. More applications and "word problems" using poker, dice, baseball lineups, etc. Review of the manipulation of integer exponents. Direct evaluation of $(x + y)^n$ for $n = 2, 3, 4$. The binomial theorem for natural number exponents. Proof using the formula for number of combinations.

Rational functions and polynomials of more than one variable. Rational functions and their graphs, with particular attention given to factoring the denominators and to the concept of asymptote. Use of asymptotes to strengthen further the intuitive notion of limit. Linear inequalities. Simultaneous linear equations in two or three unknowns. Applications to elementary linear programming. Polynomial functions of two variables. Quadratic polynomials of two variables. Definitions and resulting equations of conics, but only for the axes parallel to the coordinate axes (no discussion of rotation of axes). Simultaneous solution of a linear and a quadratic equation in two variables, of two quadratics, and geometric interpretation of the solutions. (The student should be made aware of the importance of functions of several variables; he should know that the initial restriction to one variable is only for the purpose of handling the simpler problem first.)

Exponential functions. Review of the laws of exponents, with explanations of why, with $a > 0$, a^0 is defined to be 1, a^{-x} to be $1/a^x$, and $a^{p/q}$ to be $\sqrt[q]{a^p}$ or $(\sqrt[q]{a})^p$. Real numbers reviewed and presented as infinite decimals. Discussion of the need to give meaning to arbitrary real power. Discussion of approximations to specific numbers such as $10^{\sqrt{2}}$ or $(1.2)^{\pi}$ and how this concept might be used along with the limit concept to define such exponentials. (This provides a good opportunity to contribute another step toward the understanding of the limit concept. If a computer is available, convergence can be made to seem plausible by actually computing approximations to a number such as $(1.2)^{\pi}$, using the first few places in the decimal expansion of π.) General exponential functions. Sketch of the theory of binomial series for negative and fractional exponents, with more discussion of the limit concept. Use of these in approximations of roots and powers. Applications of exponential functions to selected topics such as growth, interest, or electricity.

Logarithmic functions. Discussion of the general notion of the inverse of a function. Application to the definition of the logarithm to base a as the inverse of the exponential to base a. Review of the use of common logarithms in calculation. The slide rule (only a few lessons; see illustrations, page 221). Some discussion of the idea of approximating exponentials and logarithms by polynomials; no proofs given. (If a computer is available, use it to verify that the approximating polynomials give values of the exponential and logarithm agreeing with those in tables. If time and interests permit, consider $\left(1 + \frac{x}{n}\right)^n$ as n increases, to relate compound interest and exponentials and to motivate polynomial approximations of e^x.)

<u>Trigonometric functions</u>. Review of simplest geometric properties of circles and triangles, especially right triangles (very few formal geometric proofs). Definition of trigonometric functions as ratios (illustrate, but do not belabor, problems of triangle solving). Trigonometric functions defined on the unit circle as functions of a real variable. Graphs of trigonometric functions. Symmetry; even and odd functions. Review of ratios, fractions, and decimals once again. The following trigonometric identities: the elementary identities $\sin x/\cos x = \tan x$, $\sin (x + \frac{\pi}{2}) = \cos x$, etc.; the Pythagorean identities $\cos^2 x + \sin^2 x = 1$, $\sec^2 x = 1 + \tan^2 x$, $\operatorname{cosec}^2 x = 1 + \operatorname{cotan}^2 x$; formulas for $\cos (x \pm y)$, $\sin (x \pm y)$, $\cos x \pm \cos y$, and $\sin x \pm \sin y$. Law of sines and law of cosines. Trigonometric (polar) representation of complex numbers. Roots of complex numbers. Graphs of sums and products of trigonometric and exponential functions. Periodicity. Applications to periodic motion and other periodic phenomena. Inverse trigonometric functions using the general notion of inverse function. Their graphs.

<u>Functions of two variables</u>. Introduction to 3-dimensional coordinate systems, functions of two variables, and graphs of functions of two variables for a few simple cases.

<u>Some illustrations</u>. 1. Elementary probability theory provides an excellent opportunity to develop both the ability to reason deductively and the ability to translate "word problems" into mathematical language. Examples can be chosen from horse racing, tossing

of dice or coins, drawing colored balls from bags, etc. The following is one of an endless variety of problems that are useful for developing the ability to analyze a problem and to assign probability measures to sets:

A man tosses two coins and then informs you that the outcome includes at least one head. Determine the probability that the outcome consisted of two heads, given that the rule determining the man's statement is the following (two separate cases are considered):

Case (a). He says, "There is at least one head," precisely when this is true; otherwise he says nothing.

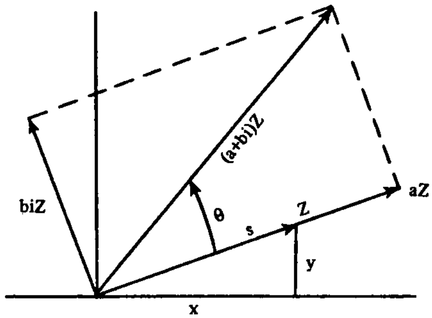
(The fact that he spoke means the outcome TT is eliminated from the four possibilities, leaving three equally likely cases and an answer of 1/3.)

Case (b). He says, "There is at least one head," when the outcome is HH. He says, "There is at least one tail," when the outcome is TT. In the remaining cases, he chooses which of these two statements to make on the basis of a coin toss.

(Under these rules, the outcome TT has probability zero in view of the statement actually made. The remaining elements of the sample space have probabilities 1/2 for HH and 1/4 each for HT and TH. The answer is 1/2.)

2. The student will easily follow the interpretation of complex numbers as vectors and their addition as vector addition. He should also be taught to interpret this addition as a transformation of the plane. Thus $(a + bi) + (x + iy)$, where x and y are variables, defines a <u>translation</u> of the entire plane in the fixed direction of $a + bi$. This remark is preliminary to the more difficult step (for the student) of interpreting $(a + bi)(x + iy)$ as an "operator" on the plane, a point of view which is of prime importance in many technologies.

This is broken down into steps: $a(x + iy)$ extends the vector $(x + iy)$ in the ratio $a:1$ [might be called an "extensor" or "amplifier" (one considers also the case that a is negative)]. Next, $i[x + iy] = ix - y$ is a rotation (of the variable vector and the entire plane) through $+90^{\circ}$ (i.e., counterclockwise), and $bi[x + iy]$ extends this new vector in the ratio $b:1$. From this the geometric meaning of $(a + bi)(x + iy)$ appears as a vector sum. The length of this sum vector is $s\sqrt{a^2 + b^2}$, where s is the length of $Z = (x + iy)$, its polar angle is as shown in the figure (where θ, the polar angle of $a + bi$, occurs as shown by reason of similar triangles). Hence, multiplication by $a + bi$ is an operator which rotates the plane through the angle θ and stretches all vectors in the ratio $\sqrt{a^2 + b^2}$ to 1. This gives the geometric interpretation of a product of complex numbers (multiply lengths and add angles); later in the course it gives the addition formulas for sine and cosine (simultaneously). Of course, this geometric interpretation could also follow a discussion of trigonometry, as an application.



3. Word problems involving conjunctions and disjunctions are useful in sharpening logical thinking. For example:

Thirty students are given grades 3, 2, or 1. If at least 10 got 3's and at most 5 got 1's, what can be said about the average score S for that class? [$3 \geqq S \geqq 65/30$]. What can be said about the number N of students who scored 2? If at most 10 got 3's and at least 5 got 1's, what can be said about S? [$1 \leqq S \leqq 65/30$]. What can be said about the number N of students who scored 2?

Other (similar) problems can be introduced in a variety of contexts.

4. A brief discussion of the slide rule is of value in reinforcing the student's understanding of logarithms. Conversely, such an understanding can be the basis for learning the principles upon which the instrument is designed and can give the student an adequate foundation for self-taught skill in its use.

Also, the student's interest in the slide rule can be exploited to great advantage in order to develop through practice his skill at estimating the magnitude of the result of an arithmetic computation (the problem of placing the decimal point).

2. Calculus and Linear Algebra: Mathematics B, C, and L.

This group of courses constitutes the proposed standard offerings for students expecting to major in mathematics, engineering, or the physical sciences, but it is structured in such a way as to serve a variety of other needs. In particular, this is achieved by the course Mathematics B (similar to Mathematics 1 [page 44]). The main feature of this course is the inclusion of the main concepts of calculus (limit, derivative, integral, and the fundamental theorem) in the setting of a single variable, dealing with the elementary functions studied in the earlier course and including computational techniques and applications of these ideas and methods. Thus the course provides a meaningful and usable study of the calculus for students who are unable to pursue the full sequence, as well as a desirable introduction for those who are.

Mathematics B can also be regarded as the calculus of elementary functions and, as such, forms a natural unit with Mathematics 0 or A: a thorough study of the elementary functions.

The remainder of the usual calculus topics, including series, functions of several variables, and elementary differential equations, are described here as a single one-year course, Mathematics C.

Finally, we describe a course in linear algebra, Mathematics L, which is similar to Mathematics 3 [page 55], only slightly less ambitious and employing a strong geometric flavor.

We consider it very important that the two-year college student who starts Mathematics C should complete it at the same school, rather than risk the probable discontinuity of study and loss of time attendant upon transfer before completion of this material. For this reason we have abandoned the feature of GCMC which proposes that linear algebra precede the study of functions of several variables and which allows a somewhat deeper study of the latter. This feature constituted only one of the reasons for the inclusion of linear algebra in the lower-division offerings. The remaining arguments apply as well to the two-year college situation, where the course can be taken simultaneously with the last half of Mathematics C in a two-year sequence starting with Mathematics 0, and so will serve an important group of students.

Pace and manner of presentation.

Without intending to prescribe the exact format in which the proposed courses are to be offered, we nevertheless find it convenient to assume occasionally a system of semesters (roughly 14 weeks of classes each) in order to state relative intensities of presentation in the familiar terms of 3-, 4-, and 5-hour[1] semester courses. In these terms we imagine the basic sequence of courses as embodying an increase of pace, the pace of the first courses taking realistic account of the wide range of student abilities and the last courses matching in pace the comparable courses at the transfer institutions.[2]

To illustrate, a distribution which fits our image of the average, multi-purpose community college is:

Mathematics A_1, A_2, 0, and B--5 hours each

Mathematics C_1--4 hours

Mathematics C_2 and L--3 hours each

Such a progression of pace will, it is hoped, allow for enough additional drill and reinforcement of topics in the earlier courses and for enough attention to mathematical literacy to provide for the student's growth in these areas to a degree of maturity which, at the end of the two-year college experience, puts him on equal standing with his fellow students at the best transfer school, where more severe screening may have taken place.

It is essential that this nurturing and growth process be achieved without weakening the content of the courses to the detriment of the better students. Indeed, for the better of the science-oriented students, it is important that these courses at the two-year college achieve as deep a penetration of the subject matter as comparable university courses so as to prepare the student for more advanced courses in mathematics and related disciplines. A calculus course that fails to teach students to understand and analyze mathematics, that gives too few and too careless demonstrations of mathematical facts, will make later work very difficult for the student who attends a four-year college whose upper-division curriculum presupposes more: students who have learned in high school to think mathematically may become disillusioned; students who have passed such a course may have false impressions of their mathematical knowledge; for all students, such courses mean the postponement of involvement with the true meaning of mathematics.

This, however, most emphatically does not mean that the course should strive for complete "rigor" as exemplified by, say, a detailed

1. These refer to the number of class meetings, not to the amount of credit granted, which we consider a local problem.

2. To be sure, four-year colleges have students of varying abilities also, but there is more of a sink-or-swim philosophy than at many two-year colleges which regard the careful nurturing of students as a greater part of their function.

development of the real number system from axioms, preceded by formal discussions of logic and followed by careful epsilon-delta foundations for the calculus. It is possible to be intuitive and still be correct, simply by following intuitive motivations and plausibility arguments with precise statements and proofs where possible, and with honest omissions or postponements of these when necessary.

It is in this sense that Mathematics B is suggested to be an intuitive course (a certain amount of such omission is certainly indicated for the suggested amount of material to be covered). The gaps left in Mathematics B, the promises for more sound foundations, are intended to be partially filled in Mathematics C where, for example, questions of limit, continuity, and integrability reappear (in the series and several variables settings).

Mathematics C, therefore, is intended to be a more sophisticated course, both in selection and treatment of subject matter, than Mathematics B. For example, we feel that the inclusion of a study of sequences and series in Mathematics C not only is important in itself and for its applications but also is particularly effective for developing mathematical maturity. With the least upper bound property of the real numbers as the principal tool it is possible to give careful proofs of the convergence theorems for monotone bounded sequences and for series of nonnegative numbers with bounded partial sums. Moreover, the study of sequences and series is a mathematical discipline that shows the need for precise definitions and demonstrates the dependence of mathematical understanding and mastery of formal techniques on basic theory. Also, the subject dramatizes the inadequacies of many of the student's prior intuitive ideas. It provides the student with perhaps his first major experience with a mathematical situation for which precise methods are indisputably needed. Indeed, it is necessary that some of the concepts introduced in Mathematics B, for example "limits," be further developed here.

Mathematics B. Introductory Calculus.

Discussion: Mathematics B is designed to follow Mathematics 0 or Mathematics A. The purpose is to introduce the ideas of derivative and integral, motivated by geometrical and physical interpretations, and to study their interrelations through the Fundamental Theorem of Calculus; to develop techniques of differentiation and integration; and to demonstrate the power and utility of the subject matter through frequent and varied applications.

Skills in manipulation need to be stressed in order that the student may learn to solve effectively problems in the applications of calculus, and to facilitate his study of later topics.

Continuing emphasis should be placed upon developing mathematical literacy; the lessons should stress correct interpretations of the written statements that set forth theorems, problem conditions, and proofs.

COURSE OUTLINE FOR MATHEMATICS B

1. Slope of the secant line to a curve. Geometric discussion of the limiting position. Derivative as slope of tangent line. Average rates and instantaneous rates of change as intuitive motivation for derivative. Limit of difference quotient as special case of limit of function. Intuitive discussion of $\lim\limits_{x \to a} f(x)$ including algebra of limits (describe the definition briefly using pictures and mention that the limit theorems hold for this definition). Definition of derivative based on intuitive notion of limit; algebraic properties: sum, product, and quotient formulas with proofs.

2. Derivative of x^n, polynomials, and rational functions. Point out continuity of polynomials (limit obtained by direct substitution). Examples of discontinuities, definition of continuity (illustrate with pictures). Derivatives and continuity properties of trigonometric functions reduced to showing $\lim\limits_{h \to o} \frac{\sin h}{h} = 1$. The existence of $f'(a)$ implies that f is continuous at a.

3. Interpretations and applications: slope, velocity, rates of change, curve sketching--including increasing, decreasing, maxima, and minima (confined to rational functions and simple combinations of trigonometric functions). Need for more differentiation techniques.

4. Chain rule. Its importance to building techniques of differentiation. The obvious "proof"

$$\frac{df(g(x))}{dx} = \lim_{h \to o} \frac{f(g(x+h)) - f(g(x))}{h}$$

$$= \lim_{h \to o} \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \cdot \frac{g(x+h) - g(x)}{h}$$

$$= f'(g(x)) \cdot g'(x)$$

breaks down because $g(x+h) - g(x)$ might be zero for arbitrarily small h. Difficulty is avoided either by a trick or by showing that in this case both sides are zero, yielding a proof (either do it or give a reference). Comments on the use of limit theorems and differentiability hypotheses in this argument.

General differentiation formulas for rational and trigonometric functions. Drill. Implicit differentiation. Derivatives of algebraic functions (general power formula) and inverse trigonometric functions.

5. More applications: word problems, related rates, curve sketching; need for more information on max, min, etc.

6. Intuitive discussion of Rolle's theorem, Mean Value Theorem. Proofs, assuming existence of maximum for continuous function on closed, bounded interval. Intermediate Value Theorem.

7. Higher derivatives and the differential as best linear approximation.

8. More applied problems. Curve sketching, rate problems and maximum and minimum problems with the more complete methods now available, acceleration, concavity.

9. Integration. (Motivate the integral as a limit of sums by observing how the latter arise naturally in several specific practical problems, e.g., area, work, moments, volume, mass, pressure. The integral is an abstract entity which embraces these and other quantities.) Discussion, with reliance on intuitive understanding of the limit involved. Antiderivative and Fundamental Theorem of Calculus. Plausibility arguments for its validity and stress on its meaning for computation. Practice in finding antiderivatives, need for further techniques.

10. Exponential and logarithm. Definition of $\log x$ as $\int_1^x \frac{1}{t}\,dt$. Derivation of properties of $\log x$ from this formula. Exponential function as inverse of $\log x$. Properties. Drill on complete list of differentiation formulas. Problems on applications involving these functions.

11. Applications of integration to finding areas, volumes of revolution, arc lengths, work done in emptying tanks.

12. If time permits: A more careful development of the definition of integral, discussion of properties of continuous functions, and proof of the Fundamental Theorem.

Mathematics C. Mathematical Analysis.

Discussion: Completing the traditional subject matter of calculus and giving an introduction to differential equations, this course serves also as the vehicle for an increasing sophistication, in both topics treated and the manner of their presentation.

Except for the obvious desirability of including the material on vectors prior to the beginning of Mathematics L (where, in any case, this material is briefly reviewed), no attempt has been made to divide the subject matter to fit quarters or semesters. There are two principal reasons for this. First, it seems important that students who intend to complete the entire calculus sequence (Mathematics B and C) should do it all at the same institution. Otherwise, wide variations in the order of topics can make transfer very difficult. Second, we leave to the particular college the problem of determining how many quarters or semesters and how many meetings per week are needed to present the material.

COURSE OUTLINE FOR MATHEMATICS C

Techniques of integration. Integration by parts. Integration by substitution, including use of the inverse trigonometric functions. Explanation of plausibility of general partial fraction expansion; use of partial fractions in integration. Emphasis on the use of tables of integrals. (Drill on techniques of integration provides useful practice in the manipulation of the elementary functions. Stress should be placed on this rather than the development of ingenuity in integration techniques for its own sake.)

Limits. Limit of $f(x)$ as x approaches a, where each deleted neighborhood of a contains a point of the domain of f. Right and left limits. Limits as x (or $-x$) increases without bound. Statement of basic theorems about limits with a proof of one or two of them and careful discussions of why all the theorems are reasonable and important. Continuity defined by the use of neighborhoods and then expressed in the language of limits. Continuity of sums, products, quotients, and composites of functions. Discussion of the Intermediate Value Theorem; zeros of a function; Newton's method. Discussion of the Maximum Value Theorem; proofs of Mean Value Theorems; l'Hôpital's rule. Integral as a limit; the Fundamental Theorem via the Mean Value Theorem; numerical integration and integrals.

Sequence and series. Definition of sequence as a function. Limits of sequences. Definition of series and sum of series. Relation to limit of sequence. Convergence of monotone bounded sequences and of series of nonnegative numbers with bounded partial sums. Comparison, ratio, and integral tests foɩ convergence of series. Conditional convergence. Power series and radii of convergence. Taylor's theorem with remainder. Taylor's series.

Vectors. 3-dimensional vectors: addition, multiplication by scalars, length, and inner (dot, or scalar) product. Parametric equations of lines and planes established by vector methods. Inner products used to find the distance between points and planes (or points and lines in a plane). Vector-valued functions and the position vector of a curve. Parametric equations of curves. Differentiation of vector-valued functions. Tangents to curves. Velocity as a tangent vector. Length of a curve. Polar coordinates. Curves in polar coordinates. Area in polar coordinates. Radial and transverse components of velocity for polar coordinates in the plane, with application to determining the length of a curve whose equation is expressed using polar coordinates. Area of surface of revolution. If time permits: vector products, acceleration, curvature, and the angle between tangential and radial lines.

Differentiation. Partial derivatives. Differentials of functions of several variables. The chain rule for functions of several variables. Directional derivatives, gradients, and tangent planes. Theorem on change of order of partial differentiation. Implicit and inverse functions. Taylor's theorem and maxima and minima of functions of several variables.

Integration. Further use of simple integrals for computing such things as work, force due to fluid pressure, and volume by the shell and disk methods. Multiple integrals. Volume. Iterated integrals. Change of variables for polar, cylindrical, and spherical coordinates.

Differential equations.* Discussion of what an ordinary differential equation is; of ways in which ordinary differential

* Many current calculus texts include a chapter which provides a brief introduction to differential equations, and closely related topics are often interwoven through other chapters (e.g., simple growth and decay problems in connection with logarithmic and exponential functions). Such treatments indicate the time allowance, if not necessarily the spirit, of the treatment which we have in mind.

equations arise; and of what it means for a function to be a solution of a differential equation, with verification of solutions in specific cases. Use of tangent fields for equations of type $y' = f(x,y)$ to build more understanding of the meaning of a differential equation and of its solution curves. Integration of first-order equations with variables separable that arise in studying problems of growth and decay, fluid flow and diffusion, heat flow, etc. Integrating factors for first-order linear equations, with more applications. Second-order homogeneous equations with constant coefficients. Use of undetermined coefficients to solve the initial value problems of undamped and damped simple harmonic motion with forced vibrations. If time permits: more study of second-order nonhomogeneous equations; series solutions.

Mathematics L. Linear Algebra.

Discussion: There are perhaps three main reasons why a course in linear algebra should be offered by a two-year college that is able to do so. Such a course will contain the first introduction both to a kind of mathematical abstraction essential to the mathematical maturity of the student and, of course, also to serious mathematical ideas other than those of calculus. The subject is becoming more and more useful in the physical, biological, management, and social sciences. Finally, it provides the proper setting for a deeper understanding of the basic notions of analytic geometry that the student sees in Mathematics 0, A, B, and C.

In this program the linear algebra course is designed to be a course parallel to the last part of Mathematics C. Of course, vector ideas and methods should still be introduced as much as possible in the calculus sequence since they make possible a more meaningful development of that subject. Even though some basic vector ideas will be introduced in Mathematics C, the proposed linear algebra course can still be taken by capable students who have had only Mathematics B. To this end it begins with an introduction to geometric vectors, i.e., the course below has been designed in such a manner as to be accessible to a student who has not worked with vectors before.

In its 1965 publication A General Curriculum in Mathematics for Colleges, CUPM recommended that the student take linear algebra before the last term of calculus. This recommendation is gaining very widespread acceptance in the four-year college. However, the situation in two-year colleges is quite different; it is probably better there to allow the last part of calculus and the linear algebra to be taught in parallel, rather than to require one before the other. There is little question that most four-year colleges and universities would rather see the transfer student come in with the calculus completed than with some linear algebra at the price of having to complete the calculus. To finish calculus before transferring, the student may not have time to take linear algebra first, although he may well be able to take it simultaneously with the last part of calculus. Moreover, those transfer students who will major in the social or management sciences may take only one term of calculus but almost certainly will find linear algebra more useful than any other kind of mathematics. Here again, it is important that good students be able to take both the last part of calculus and the linear algebra.

The course outline below emphasizes a geometrical outlook and adheres closely to the guidelines for linear algebra suggested by the following quotation from A General Curriculum in Mathematics for Colleges (1965):

> When linear algebra is taught as early as the first term of the second year of college one should examine carefully the choice of topics. We need something intermediate between the simple matrix algebra which has been suggested for high schools and the more sophisticated Finite-Dimensional Vector Spaces of Halmos. The pieces of linear algebra which have demonstrated survival at this curriculum level for many years are: systems of linear equations and determinants from college algebra, uses of vectors in analytic geometry, and the calculus of inner products and vector cross products. More modern ideas call for the introduction of matrices as rectangular arrays with elementary row operations, Gaussian elimination, and matrix products; then abstract vector spaces, linear dependence, dimension, and linear transformations with matrices reappearing as their representations. Finally, of all subjects in undergraduate mathematics after elementary differential equations the one which has the widest usefulness in both science and mathematics is the circle of ideas in unitary geometry: orthogonality, orthogonal bases, orthogonal expansions, characteristic numbers, and characteristic vectors.

COURSE OUTLINE FOR MATHEMATICS L

Geometrical vectors. 2- and 3-dimensional vectors, sums, differences, scalar multiples, associative and commutative laws for these. Dot and cross products. Equations of lines in two and three dimensions and of planes, by vector methods. Normals and orthogonality, direction cosines. Components of vectors and vector operations in terms of them. Mention correspondence between plane (space) vectors at the origin and R^2 (R^3).

Matrices and linear equations. Matrices, row and column vectors defined as rectangular arrays and n-tuples of real numbers. Sums of matrices and vectors, products of matrices. Examples and applications to elementary economics, biology, analytic geometry. Linear equations written in matrix form. Gaussian elimination. Row echelon form of matrix. Existence and uniqueness theorems for solutions of homogeneous and nonhomogeneous systems of linear equations. Numerical examples for 2, 3, 4 variables.

Vector spaces. Need for unifying concept to subsume n-tuples and geometric vectors of the previous paragraphs. Definition of abstract vector space over the reals. Examples of vector spaces from the above, including the null space of a matrix (solution space of a system of linear equations), function spaces, spaces of solutions of linear differential equations, space of polynomials of degree $\leq n$, of all polynomials. Linear combinations of vectors. Geometric applications. Linear dependence and independence. Theorem: If $m \geq n$, then m linear combinations of n vectors are dependent. Definition of dimension and proof of uniqueness (finite-dimensional case only). Subspace. Geometric examples and function space examples. Dimension of subspaces: $\dim(S+T) = \dim S + \dim T - \dim(S \cap T)$, with appropriate definitions and examples.

Linear transformation. Definition of linear transformation. Matrices associated with linear transformations with respect to different bases. Sums and products of linear transformations defined. (Show how this leads naturally to definitions of sums and products of matrices given in the second paragraph above.) Range and kernel of linear transformation. dim (kernel) + dim (range) = dim (domain). Elementary matrices. Rank of linear transformation and matrix. Row rank = column rank. Linear equations re-examined from this point of view. Inverses of linear transformations and matrices. Calculation of inverse matrix by elimination. $PAQ = I_r$ if r = rank of A. Matrix of same linear transformation with respect to different bases. Similarity.

For the remainder of the course a choice may have to be made between the following two topics.

Determinants. The determinant as a function from matrices to the reals which is alternating and multilinear on the rows and columns, i.e., the determinant of a matrix is unchanged by adding a multiple of a row (column) to another row (column), the determinant of a diagonal matrix is the product of the diagonal entries, etc. (No attempt at proving existence in the general case.) Derivation of explicit formulas for 2×2 and 3×3 determinants. Calculation of several explicit $n \times n$ determinants for $n > 3$, for example, van der Monde 4×4 and 5×5 determinants. Expansion in terms of cofactors. If A, B are $n \times n$ matrices, then determinant (AB) = determinant A · determinant B. (Last two topics with only indications of proof.) A very brief discussion of the formula $A \text{ adj } (A) = I \det (A)$. Cramer's rule.

Inner product spaces. Definition. Orthogonal and orthonormal bases. Gram-Schmidt process. Distances. Orthogonal complements. Orthogonal expansions. Examples in function and polynomial spaces where inner product is defined via an integral. Applications to 3-dimensional analytic geometry.

3. Probability and Statistics: Mathematics PS.

The discussion and list of topics given here represents tentative recommendations arrived at jointly by some members of the CUPM Panels on Statistics and Two-Year Colleges, in order to elaborate the latter Panel's recommendation that a course of this kind be included in the basic offerings.

Historically, a proliferation of introductory statistics courses, differing not only in content but also in emphasis and method of approach, has developed. Some of the factors responsible for this are the desire by different disciplines to have courses tailored to their needs, the varied backgrounds of persons teaching statistics, and differences in objectives for the basic statistics courses. The Panel on Statistics of CUPM is undertaking a full study of the difficult problem of developing models for such courses. The study will look to the future. It will investigate the role of computers, teaching devices, and laboratories. It will consider special approaches, such as those stressing decision theory, nonparametric methods, Bayesian analysis, data analysis, and others. A major aspect of the study will be consultation with representatives of the various fields whose students are served by courses of this kind.

The study may well indicate that no single statistics course is best suited to fill the variety of existing and anticipated needs; the Panel may eventually recommend a choice from among several different types of statistics courses (hopefully providing at the same time the impetus for the appearance of any new texts they may require).

The projected recommendations will, of course, supersede the suggestions given here in that they will be more explicit and will be based on a more extensive study of the problem.*

Discussion: An important element in the education of transfer students in many fields is a one-term introductory course in statistics, including an adequate treatment of the needed topics from probability. Such a course is often required of students majoring in particular fields: to cite a few examples, business administration, psychology, sociology, forestry, industrial engineering. In addition, it serves as an excellent elective subject for other students.

A course of this kind can be intellectually significant although based on minimal mathematical preparation. In what follows a prerequisite of two years of high school algebra is assumed.

The main function of a first course in statistics which will be terminal for many students and, for others, will provide a basis for studying specialized methods within their majors, is to introduce them to variability and uncertainty and to some common applications of statistical methods, that is, methods of drawing inferences and making decisions from observed data. It is also desirable, as a second objective, for the students to learn many of the most common formulas, terms, and methods.

Faculties of some departments served by this course stress the latter objective and, in addition, wish their students to be knowledgeable in certain specialized statistical techniques. We suggest that it is preferable for the students to learn basic concepts in the first course. (If a specialized technique is needed, it can be taught effectively in the subject matter course in the context in which it is used, at a small cost in time.)

It is quite plausible that an imaginative presentation which develops only a minority of the most common topics could be very effective in providing the students with an intelligent and flexible approach to statistical problems and methods through the development of basic concepts. Therefore, we regard the second objective of the course as secondary in importance.

Since the main objective of the course is understanding of the basic statistical concepts, proofs and extensive manipulations of

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* The report mentioned in these paragraphs appeared in 1972 and is entitled Introductory Statistics Without Calculus. See page 472.

formulas should be employed sparingly. While statistics utilizes these, its major focus is on inference from data. By the same token, the course should not dwell upon computational techniques. The amount of computation should be determined by how much it helps the student to understand the principles involved.

We urge the use of a variety of realistic problems and examples, to help motivate the student and to illustrate the approach to such problems by statistical methods. (The use of a laboratory in conjunction with the courses offers attractive possibilities for these purposes.) Examples drawn from disciplines of common interest to the class members should be sought. Also, the generation of data within the class (e.g., heights, birth dates) can be very useful.

Despite the diversity of basic courses, there has been a common set of topics included in most of them. The list below corresponds in an approximate fashion to this common set. A number of topics have been omitted from the list--some because we feel they would detract from more important topics: combinatorial aspects of probability, skewness, kurtosis, geometric mean, index numbers, and time series, for example. (These need not be avoided if they serve as pedagogical devices in attaining the major objectives of the course.) Other topics have been omitted because their incorporation would make it difficult to cover enough of the topics on the list. These include Bayes' theorem, Bayesian analysis, experimental design, sequential analysis, decision theory, and data analysis (insofar as this refers to an as yet emerging body of knowledge devoted to techniques for detecting regularities from data). These are potentially excellent topics; some of the possibilities for innovation in the basic statistics course are either (1) to stress some of these at the expense of omitting other (perhaps many) topics included in the list, or (2) to make one of these topics the basic focus for the entire course and discuss other topics in the framework thus established.

Finally, we urge that this course be a sound intellectual experience for the students. It should not be just a compendium of terms and techniques.

LIST OF TOPICS FOR MATHEMATICS PS

Note: This is not a course outline. It is not intended that this list of topics be necessarily covered in the order presented or in its entirety; an extremely skillful presentation would be required for a great majority of them to be combined satisfactorily in one course.

1. Introduction. Data: collection, analysis, inference. Examples of some uses of statistics emphasizing data collection, summarization, inference and decisions from data.

2. Probability: sample space, probability of an event, mutually exclusive events, independent and dependent events, conditional probability. (Keep enumeration to combinatorial notation at most. Use tables.) Random variable, expected value, mean and variance of a random variable.

3. Discrete random variable: binomial, use of binomial tables, Poisson (optional).

4. Summarization of data: grouping data. Central tendency: arithmetic mean, median, mode. Dispersion: range, standard deviation (others optional). Graphical analysis: histogram, frequency polygon, cumulative frequency polygon, percentiles.

5. Continuous random variables and distributions. Normal distributions. Use of tables. Normal approximation to binomial.

6. Sampling theory: random sampling, Central Limit Theorem, normal approximation to distribution of $\bar{x}$.

7. Point estimation: estimation of mean of the normal and mean of the binomial.

8. Interval estimation: confidence intervals on the mean of the binomial, confidence intervals on the mean of the normal using the t distribution. Large-sample approximate confidence intervals.

9. Hypothesis testing. Null hypothesis, alternate hypothesis, Type I and Type II errors, power of the test. Test of hypothesis on the mean of the normal using the t test. Test on the mean of the binomial. Large-sample approximate tests.

10. χ^2 tests: goodness of fit, contingency tables.

11. Comparison of two population means. Paired and unpaired cases using the t test (nonparametric or range tests may be substituted).

12. Nonparametric methods. Sign test, Mann-Whitney test, Wilcoxon test. Confidence interval on median.

13. Regression and correlation. Least squares, correlation coefficient. Estimation and tests.

14. One-way analysis of variance. F test, simultaneous confidence intervals.

III. TEACHER TRAINING AT TWO-YEAR COLLEGES

One of the major concerns of CUPM since its inception has been the proper mathematical training of prospective teachers of school mathematics. Although much progress has been made over the years on this serious problem, a great deal remains to be done, and the two-year colleges will play an increasingly important part in these efforts. For this reason we have included in this report a statement from the CUPM Panel on Teacher Training, together with some excerpts from their current recommendations.

[Editor's note: The original report contained a statement from CUPM's Panel on Teacher Training and a course outline for Mathematics NS (The Structure of the Number System). Rather than reprint these, we refer the reader to the latest report of the Panel on Teacher Training, Recommendations on Course Content for the Training of Teachers of Mathematics (1971), and to the course outlines contained therein. See page 177.]

IV. OPTIONAL ADDITIONAL OFFERINGS

Many two-year colleges offer courses for transfer students other than a basic set of courses such as those described in Chapters 2 and 3. With the growing trend for well-qualified and highly motivated students to attend two-year colleges, the demand for such additional courses will increase. The opportunity to teach such courses will certainly be attractive to faculty members and will offer them stimulating challenges. In this chapter we offer some suggestions for the kind of additional courses which we think deserve prime consideration, including sample course outlines and discussions of the circumstances under which the course is deemed desirable.

1. Finite Mathematics: Mathematics FM.

This course is designed primarily for students interested in business, management, social, and biological sciences. Its purpose is to introduce a variety of mathematical topics, showing how these are related to problems in the areas mentioned above. It would be a valuable component of the curriculum only if a substantial amount of time, at least 20% of the course, were devoted to applications of the material developed. Simply taking up mathematical topics of potential relevance is not enough; indeed, unless it includes a substantial unit on applications it would be better not to offer the course.

In finite mathematics the student is exposed to mathematical topics of a different nature from those he has previously studied--topics that apply to new types of problems and topics that are perhaps more closely related to his special interests. It is expected that some students, even if a small proportion, will be attracted to further study of mathematics by this course, either because they find these new topics interesting in themselves or because they recognize the need to learn more for further applications.

The course described, while not requiring all of the topics in Mathematics 0, can profitably build on the mathematical sophistication and maturity of that level of mathematical education in order to develop the ideas needed for the applications. A course certainly can be designed for students who have completed less, although it may then be necessary to have the course meet an extra hour per week in order to have enough time at the end of the course for the desired substantial unit on applications. A course assuming Mathematics PS could proceed somewhat further with applications, either in greater depth or in greater variety. However, we believe finite mathematics will serve a greater need and a larger audience if statistics is not a prerequisite.

The topics suggested are chosen because of their applicability; they include finite sets, probability, and linear algebra. The elements of logic may be, but need not be, treated as a separate topic. All that is needed is an ability to analyze and interpret compound statements formed by the usual connectives "and," "or," "not," "if and only if," "if ... then ...," and familiarity with what is meant by a logical argument. These ideas can be woven into the course. We prefer the latter procedure since it will help to insure time for applications.

Although it is not indicated in the outline, an attractive way of beginning is to present one or two applied problems that will be solved by the end of the course (e.g., one of the several Markov chain examples). As the course progresses and partial solutions are obtainable, the student should be apprised of the progress being made toward the ultimate solution to be given at the end of the course.

COURSE OUTLINE FOR MATHEMATICS FM

Sets. Subsets and set inclusion. The algebra of sets: union, intersection, complementation, difference. The relation with logic of compound sentences constructed from the connectives "and," "or," "not," "if and only if," "if ... then ... ." Tree diagrams as a systematic way of analyzing a set of logical possibilities. The number of subsets of a set. Cardinality of finite sets; card $(A \cup B)$ = card A + card B - card $(A \cap B)$. Partitions, binomial and multinomial theorems and related counting problems.

Probability. The concept of a probability measure: the axioms, and the realization of these axioms for finite sets. (The intuitive basis for this model discussed and illustrated by several examples.) The Law of Large Numbers. Equiprobable measure. Probabilistic independence. Repeated occurrences of the same event and binomial distribution. Approximation of the binomial distribution by the normal distribution as a means of making predictions about the likelihood of outcomes of repeated independent trials. (If time permits, also discuss Poisson distribution as approximation of the binomial.) Introduction to Markov chains (particularly if the course is begun by a discussion of a problem involving Markov chains; otherwise this topic may be delayed).

Linear algebra. Vectors and matrices introduced as arrays of numbers. Addition of vectors and matrices. Products (motivated by such examples as: given a vector of prices of certain commodities and a matrix giving the quantity purchased on each of a set of days, find the total expense on each day). Solution of linear equations, including the cases where the number of equations and number of indeterminates differ, and including the cases of multiple solution and no solution. The definition of the inverse of a matrix. Its computation without using determinants. Applications to Markov chains; classification of chains, with the emphasis on absorbing chains and regular chains. Fixed vectors and eigenvectors. Examples such as: for regular chains, analyze a problem in social mobility, probability of change of job classification, change of residence, change in brand preference among choices for a particular article, probability of change of political party allegiance, genetic heredity; for absorbing chains, many of these same examples with a change in the probabilities, e.g., brand preferences, genetic heredity, as well as matching pennies, learning processes.

Applications. One of the largest classes of applications involves Markov chains, including the long-term development of proportions of dominant and recessive genes, learning models, models on the judgment of the lengths of lines, the spread of a rumor in a housing development, or an extension of any of the items mentioned above.

Another attractive example is linear programming, including: an introduction to convex sets; the fact that a linear function defined on a convex set assumes its maximum and minimum at a vertex; some simple examples that can easily be solved by hand, with indication of the magnitude of the task in case the convex set has many faces and vertices. If time permits: a discussion of means of solving such problems, including the simplex method.

Still another example is the theory of games. Here one can introduce the idea of a zero-sum two-person game, especially a 2×2 matrix game, the idea of pure and mixed strategies, pure strategy solutions both for strictly determined games and for those that are not strictly determined. Mixed strategies and the min-max theorem (proved for 2×2 matrix games). Examples: one of the simplified poker games; competition between two businesses.

2. Differential Equations and Advanced Calculus: Mathematics DE and DA

Many two-year colleges now offer courses in differential equations. This practice is expected to continue because of the large number of students specializing in fields for which some familiarity with differential equations is important. Some students who take such a course do not transfer to four-year colleges and some of those who do transfer plan to major in fields which do not require many upper-division courses in mathematics at the four-year college. A two-year college might well offer a course in differential equations as an elective for such students, with Mathematics C and L as prerequisites. However, such a course has limited value if it trains students only in formal techniques for special cases without developing understanding of the general nature and meaning of differential equations and their solutions.

We regard linear algebra as being a more basic and important tool than differential equations for most students. For students who will take only one of the courses we strongly recommend linear algebra. Since the concept of vector space is very important when discussing general linear equations, linear algebra should be a prerequisite for differential equations. If students do not have this preparation, the treatment of systems of differential equations will, of necessity, be quite minimal.

The content of a differential equations course should be planned to meet the needs and interests of the students. Therefore, we have outlined a course, Mathematics DE (<u>Intermediate Differential Equations</u>), that has considerable flexibility and can be varied to meet such needs and interests. Certain important topics have been listed first. Other topics are listed from which choices can be made, either as additional topics or as substitutions. For example, the study of existence and uniqueness theorems might be substituted for the study of systems of equations, or numerical and operator methods might be introduced throughout the course.

For some students, further study of calculus is more important than the study of differential equations. This is true for many students planning to take mathematics after transfer. For such students we recommend a course which develops some concepts of advanced calculus and also provides an extension of the study of differential equations begun in Mathematics C. This course, Mathematics DA (Differential Equations and Advanced Calculus), would follow Mathematics C and L. Recognizing varied student and faculty interests and time limitations, we describe the core material and some additional topics from which choices should be made as time permits.

The two courses whose outlines follow are designed to develop good understanding of the mathematics involved and to improve the mathematical maturity and ability of the students. These courses are suitable only for students who have gained from courses such as Mathematics C and L the ability to understand and to readily assimilate mathematics. In particular, it is assumed that the students have been given an introduction to differential equations such as that described in the outline of Mathematics C.

COURSE OUTLINE FOR MATHEMATICS DE

Review. First-order equations with variables separable; integrating factors for first-order equations. Second-order linear homogeneous equations with constant coefficients. The method of undetermined coefficients, with applications to undamped and damped simple harmonic motion with forced vibrations.

Linear differential equations. Superposition of solutions. Homogeneous equations of n^{th} order with constant coefficients; existence and uniqueness theorems for initial value problems. Linear dependence and independence; vector spaces of solutions. Relations between solutions of a nonhomogeneous equation and the corresponding homogeneous equation. Methods of undetermined coefficients and variation of parameters. Applications to initial value problems.

Series solutions. Discussion of term-by-term differentiation and integration of a power series within the interval of convergence; proof that the interval of convergence does not change, with at least heuristic justification of the procedure. Use of series for solving some first-order and other simple cases for which convergence can be verified easily and the procedure justified. Discussion of both the method of undetermined coefficients and the use of Taylor series for determining series of solutions. General theory of series solutions about regular points. Discussion of the indicial equation and of the nature of series solutions about regular singular points, with most results stated and only heuristic justification given. Applications to classical equations such as those of Bessel and Legendre. Some simple nonlinear equations, such as $y' = x^2 + xy^2$.

Systems of equations. Equivalence of general systems to systems of first-order equations. Vector representation of a system of first-order equations. Use of matrices to solve systems with constant coefficients. Description of the nature of the solution when the coefficient matrix can be reduced to diagonal form, e.g., when the eigenvalues are distinct or the matrix is symmetric; description and application of the Jordan canonical form. Discussion of applications to mechanical and electrical problems.

Additional or alternative topics chosen from among the following:

1. Numerical methods. Difference equations and interpolation. Runge-Kutta method. Elementary considerations of stability and error analysis.

2. General linear equations. Wronskians; linear dependence and independence; number of linearly independent solutions of an ordinary linear differential equation.

3. Existence and uniqueness theorems. Convergence of power series solutions. Existence and uniqueness proofs for first-order equations using Picard's method. Generalization to systems of first-order equations.

4. Operator methods. The operator D for linear equations with constant coefficients; factoring and inversion of operators; partial fraction techniques. The Laplace transform applied to linear differential equations with constant coefficients and to simultaneous linear first-order equations with constant coefficients.

5. Nonlinear differential equations. Special nonlinear equations which are reducible to linear equations; local stability; simple phase-plane geometry of trajectories. Self-sustained oscillations of

a nonlinear system. Forced oscillations of a nonlinear system (e.g., $x'' + k^2(x-cx^3) = A \cos ct$) and the corresponding resonance phenomenon.

COURSE OUTLINE FOR MATHEMATICS DA

Topology of the real line. Brief description of the construction of the real numbers from the rational numbers. Open and closed sets. Least upper bound property shown to imply Bolzano-Weierstrass and Heine-Borel properties.

Continuity. Preservation of connectedness and compactness by continuous maps; maximum value, mean value, and intermediate value theorems; continuity of inverses. Uniform continuity.

The above topics should be accompanied by numerous examples and by counterexamples showing the need for the hypotheses.

Linear differential equations. Extension of the methods developed for second-order linear equations in Mathematics C. Linear dependence and independence; number of linearly independent solutions of an ordinary linear differential equation; vector spaces of solutions. Relation between solutions of a nonhomogeneous equation and the corresponding homogeneous equation. Methods of undetermined coefficients and variation of parameters. Applications to initial value problems.

Convergence. Review of tests for convergence of series and sequences of constant terms. Algebraic operations with series and power series. Uniform convergence of sequences and series. Term-by-term integration and differentiation of sequences and series. Existence and uniqueness proofs for first-order differential equations using Picard's method.

Additional or alternative topics chosen from among the following:

1. Series solution of differential equations. Power series. Use of power series to obtain solutions of differential equations. General theory of series solution about regular points. Discussion of the indicial equation and of the nature of series solutions about regular singular points, with most results stated and only heuristic justification given. Applications to classical equations such as those of Bessel and Legendre.

2. Systems of differential equations. Equivalence of general systems to systems of first-order equations. Vector representation of a system of first-order equations. Use of matrices to solve systems with constant coefficients. Description of the nature of the solution when the coefficient matrix can be reduced to diagonal form, e.g., when the eigenvalues are distinct or the matrix is symmetric; description and application of the Jordan canonical form. Discussion of applications to mechanical and electrical problems.

3. Riemann integration. Area and integrals. Properties of definite integrals. Existence of integrals of continuous and monotone functions.

4. Transformations. Review of partial differentiation and of linear transformations and matrices. Jacobians. Inverse transformation and implicit function theorems. Change of variables in multiple integrals. Cylindrical and spherical coordinates.

3. Probability Theory: Mathematics PR.

The importance of probability theory makes it a prime candidate for inclusion early in the program of many transfer students. The usual treatment of this subject requires some background in calculus. Sufficient background for this course is provided by Mathematics B. The course itself will provide additional calculus material as the need arises.

This course should lay stress on problems and, in particular, on problems which provide motivation and develop interest in the conceptual aspects of probability theory. Problems of this kind can be found, for example, in Fifty Challenging Problems in Probability by F. R. Mosteller (Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1965).

The course outline below is an adaptation of Mathematics 2P [page 76]. The outline is brief. It is intended that the topics be treated in depth.

COURSE OUTLINE FOR MATHEMATICS PR

(a) Probability as a mathematical system. Sample spaces, events as subsets, probability axioms, simple theorems. Intuitively interesting examples. Special cases: finite sample spaces, equiprobable measure. Binomial coefficients and counting techniques applied to probability problems. Conditional probability, independent events, Bayes' formula.

(b) Random variables and their distributions. Random variables (discrete and continuous), density and distribution functions, special distributions (binomial, hypergeometric, Poisson, uniform, exponential, normal, etc.), mean and variance, Chebychev inequality, independent random variables.

(c) Limit theorems. Poisson and normal approximation to the binomial, Central Limit Theorem, Law of Large Numbers, some applications.

4. Numerical Analysis: Mathematics NA.

A two-year college should consider offering a course in numerical analysis provided it has

(a) a curriculum of courses including all the "Basic Offerings";

(b) someone on the staff qualified to teach the course;

(c) access to computing facilities with reasonably short turn-around time. A time-sharing system is ideally suited to this purpose, as is a calculator-computer that students can use "hands on."

The first two provisions above are not likely to be challenged. To fulfill the second it may be possible for some institutions to find a part-time teacher from local industry who is interested in the mathematical issues raised. As for computing facilities, while numerical analysis can be studied effectively without a digital computer, the actual computational processes involved can be tedious, and the course would be much less likely to attract students. If the only accessible computing equipment involves long delays between starting a program and getting the answers, the result is frustration and a discontinuity of interest in the problem being solved.

This course has several purposes. It is intended to capitalize on the interest students have in computers in order to spur their interest in mathematics and to kindle in them a desire to study further topics in mathematics, for in it they should find that turning a problem over to a computer eliminates some mathematical problems while raising others. In addition, it provides an introduction to the important techniques of numerical solution to a variety of problems and introduces ways in which digital computers can be helpful in problem solution.

It should be recognized that this is not a course in computer science, but rather it is a mathematics course. It contains material closely related to the curriculum of any computer science program, however, and hence is appropriate for those who wish to pursue a major in computer science at a four-year college.

The course begins with a discussion of some of the problems that will be treated. Most of these will be problems with which the student is already acquainted in some form: definite integrals that cannot be expressed in closed form, differential equations that he has not learned how to solve, as well as large systems of linear equations with coefficients containing more than two significant figures, and nonlinear equations in a single variable which in theory he may know how to solve, but which in practice are quite impossible. The process of obtaining numerical solutions to these problems will stimulate the study of additional topics in analysis, linear algebra, probability, and statistics. The student will become concerned with approximations, rapidity of convergence, and error analysis.

Selection of topics from the outline will depend on how much, if any, attention to numerical techniques appeared in the calculus course, and on the tastes and interests of the instructor. It should in any case include examples of interpolation, approximation, and solutions of systems of equations.

COURSE OUTLINE FOR MATHEMATICS NA

Prerequisite: At least one year of calculus, including infinite series.

Introduction. Some typical numerical problems; the theoretical aspects of numerical analysis such as convergence criteria and error estimates, versus the algorithmic aspects such as efficiency of an algorithm and error control. (Most of these topics can be nicely introduced through examples, e.g., the solution of quadratic equations or the problem of summing an infinite series.)

Interpolation and quadrature. The linear and quadratic case of polynomial interpolation; basic quadrature formulas; numerical differentiation and its attendant error problems. If time permits, discussion of the general Lagrange and Newton interpolation formulas, the Aitken method, and Romberg integration.

Solution of nonlinear equations. Bisection method, successive approximations, including simple convergence proofs, Newton's method, method of false position. Application to polynomial and other equations and to special interesting cases such as square and cube roots and reciprocals.

Linear systems of equations. The basic elimination step, Gauss and Gauss-Jordan elimination. Roundoff error, the ill-conditioning problem in the case $n = 2$. If time permits: brief introduction to matrix algebra and the inversion of matrices.

Solution of ordinary differential equations. Series solutions and their limitation. Euler's method, modified Euler's method, simplified Runge-Kutta.

V. IMPLEMENTATION

This final chapter is devoted to some comments on various aspects of the problem of implementing the proposals of the earlier chapters. It is never the intent of CUPM that its curriculum recommendations be "swallowed whole," but rather that they be used as a focus for the discussion of curricular problems on a national or regional scale and ultimately that they serve on the local level as a starting point for improvements tailored to local needs. It is in this sense that CUPM wants to see this program implemented. We point out below how it can be considered in small pieces; how it can be staffed economically; how problems of articulation with high schools, four-year colleges, and other two-year programs can be approached; and how such factors as the computer and the existing choice of textbooks may affect implementation.

We aim the following remarks at a generalized audience in full recognition of the fact that the actual readers represent diverse schools and that each will need to interpret them for his own needs.

1. Implementation by Stages.

CUPM does not, of course, recommend a total alteration of current practices, but rather an evolution in certain directions.

The curriculum seems to divide naturally into parts, reasonably independent from the viewpoint that changes in one part need not force major changes in the others. Thus the proposed structure can be approached in stages, beginning wherever a department recognizes the greatest need:

(1) The introduction of Mathematics 0 and (or) Mathematics A in place of current calculus-preparatory courses.

(2) The revision of the calculus courses along the lines of Mathematics B and C.

Remarks:

(a) If current practice includes highly integrated analytic geometry-calculus courses, these two stages are not quite independent. Nevertheless, the problem of proceeding with one of these stages and making appropriate adjustments in the other courses does not seem to be a major one.

(b) If stage 2 has been carried out successfully, the course Mathematics B should serve the students of nonphysical sciences who need an introduction to calculus. Thus, some need for special-purpose courses, such as business calculus, might be eliminated.

(3) Introduction of Mathematics L.

(4) Introduction of Mathematics PS.

Remark: Although courses of this kind are offered in most schools, frequently in departments other than mathematics, a well-designed course given in one department can help avoid duplication of effort within a school.

(5) Introduction or extension of the offerings for elementary school teachers.

(6) Selection of appropriate additional courses.

We believe that none of these changes would create unusual demands in terms of the teaching staff they require. Indeed, that the entire basic program calls for only slightly more than the equivalent of two full-time staff members is indicated by the following sample patterns.

	First Semester			Second Semester	
entry $\longrightarrow$	A_1	(5 hours)*	$\longrightarrow$	A_2	(5 hours)
entry $\longrightarrow$	B	(5 hours)	$\longrightarrow$	C_1	(4 hours)
	C_2	(3 hours)	$\longrightarrow$	L	(3 hours)

This provides the full science sequence with two points of entry and uses approximately the equivalent of one full-time instructor.

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* See pages 224 and 225.

First Semester			Second Semester	
entry → O	(5 hours) →		B	(5 hours)
C_1	(4 hours) →		C_2	(3 hours)
L	(3 hours)	entry →	PS	(3 hours)

This offers the additional natural point of entry and Mathematics PS once each year, again using about one instructor. The addition of the course Mathematics NS (3 hours each semester) and perhaps the second year of teacher-training courses completes the basic offerings.

This simplified model provides considerable flexibility, perhaps even more than is needed by many of the smaller schools where the need to offer each course each semester may not be present. On the other hand, it does not take account of multiple-section courses, of the possibilities for more economy through such devices as a summer school entry to the science sequence, or of the additional flexibility that can perhaps be achieved with the quarter system. Nevertheless, it seems clear that departments working out more detailed models according to their own needs will find this program quite economical of manpower as compared to alternative curricula.

2. Articulation.

Articulation efforts have as their primary goal establishing a procedure of transfer that is as smooth and convenient as possible for the student, thereby enabling him to be cognizant of the required courses in his field and to plan carefully whatever program he is pursuing. Although articulation between two-year colleges and four-year colleges encompasses many areas such as staffing, curricula, administration, and personnel, the main obstacle to be surmounted is that of "communication." Detailed information must be exchanged between the two, and that information must then be disseminated to all concerned.

As indicated in "Guidelines for Improving Articulation Between Junior and Senior Colleges,"* there are five main aspects of the transfer from the two-year college to the four-year college. These are: administration; evaluation of transfer courses; curriculum planning; advising, counseling, and other student personnel services; articulation programs. We have kept these in mind in formulating the present recommendations.

Many states, however, have large, geographically dispersed systems of two-year colleges and four-year colleges in which a slowdown or lack of communication causes some of the items listed in the

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"Guidelines" to become problems. Florida and California, for example, are states having geographically dispersed systems which have experienced or foreseen such problems. They have organized statewide committees to discuss, study, and recommend solutions or guidelines for the solution of these problems.*

The Advisory Council on College Chemistry published a report in January, 1967, entitled "Problems in Two-Year College Chemistry, Supplement Number 20A." This report deals with curriculum and articulation problems in the area of chemistry that are in many ways identical to those in mathematics.

The counseling, guidance, and placement procedures employed by the two-year college serve to minimize the articulation difficulties both between the high school and two-year college and between the two- and four-year college. Complete information about the individual two-year college's mathematics program should be in the hands of local high school counseling staffs. In this manner the high school student, regardless of his background, can be informed as to where he should start his mathematics program in college. For example, a graduating student who suddenly realizes that he needs calculus for his major field, and who is not prepared, should be informed that the local two-year college offers courses such as Mathematics A and Mathematics 0 to correct the deficiency.

In general, the mathematics department is urged to exercise its responsibility for coordinating its work with high schools, general counselors, and transfer institutions. Two particular goals are to improve placement procedures and to establish sound criteria for transfer credit.

Articulation between institutions can cease to be a problem only when complete communication has been established.

3. General Education and Technical-Occupational Programs.

A further study of these topics will be a major part of the continuing work of the Panel. However, because of their great importance in two-year colleges a brief commentary on these subjects is included here.

General Education. A desirable way for a nonscience major to acquire some understanding and appreciation of mathematics may be to take a course or courses in general education mathematics. This is true if the course is carefully planned and taught by skilled and inspiring teachers. Above all else, the course should be so designed

* See Report of the California Liaison Committee, November, 1967, Sacramento, and A Report on Articulation in Mathematics (Florida), June, 1966, Tallahassee.

and taught that the students discuss and do mathematics rather than only hear about mathematics.

No definite outline for such a course is suggested at this time. Each teacher should judge the best type of course to suit his class. CUPM hopes to offer him more specific assistance in this task soon. [See A Course in Basic Mathematics for Colleges, page 314.]

Whatever is done should be meaningful relative to the student's experience and studies; it should be taught in a new light relative to his past experiences with the study of mathematics. In many ways college students are becoming more sophisticated, and hence a new approach is needed to reinforce old mathematical experiences and to develop new ones. It is desirable that the course achieve the minimum levels of understanding, skills, and reasoning needed by all students regardless of their goals or preparation, and that it provide some base for continuation in mathematics for those students who can and should continue. The course should emphasize basic mathematical concepts of recognized importance for an educated person and should be oriented, where possible, toward meaningful applications. The material should cut across the traditional segmentation of arithmetic, algebra, and geometry. The solution of "word problems" should be emphasized throughout.

Mathematics 0, A, PS, and FM as described above can be very effective general education courses for students having the necessary preparation.

Technical-Occupational. Even though the majority of students who enter two-year colleges do so with the intention of transferring, there still exists a large group who seek training and instruction that will prepare them for employment at the end of two years. In order to accommodate the latter group of students, along with the portion of the first group who change their objectives, most two-year colleges provide a wide array of technical, semi-professional, and occupational programs.

The need for mathematics in these career programs varies widely with the program. Some require no more than simple arithmetic while others demand a working knowledge of mathematics ranging from elementary algebra through the calculus and differential equations. We feel that the course Mathematics A, with examples drawn from technical areas, has great potential as a service course for many such programs. If a program requires a more specialized course, it should be taught by a mathematics instructor who is familiar with the curriculum's objectives. The content and sequence in which the topics are taught should be determined by the coordinated efforts of the mathematics and career curriculum faculty members. The "integrated" or "related" mathematics of vocational programs may be taught more effectively by the vocational instructor.

Whenever possible, instruction and practice in the use of the computer for problem solving should be given in engineering- and science-oriented two-year occupational programs and use of the computer in data processing should be taught in business- and health-oriented two-year occupational programs.

4. The Computer in Two-Year Colleges.

The two-year college must be prepared for, and articulate with, the changes taking place in both secondary schools and four-year colleges in the area of computing and in the use of computers in mathematics courses. Computers are rapidly being introduced into secondary schools both physically and as a part of the curriculum. Courses of three types are already being taught.

1) The first type of course serves as an introduction to and emphasizes "computer science" as a distinct discipline incorporating the many facets of modern computers. It is exemplified by the School Mathematics Study Group's course Algorithms, Computing, and Mathematics.

2) The second type of course emphasizes instruction in a computer language. It includes an exposition of the essentials of a programming language and many examples of programs for the solution of various categories of mathematical problems.

3) The third type of course emphasizes the integration of computing into the mathematics curriculum. It uses the computer as a part of the instructional process in selected subject matter areas within mathematics.

To indicate the trend further, we note that the School Mathematics Study Group is preparing a new curriculum intended to go from seventh through twelfth grades which emphasizes the use of algorithmic ideas and computation. Flowcharting, for example, is introduced in the seventh grade. Many colleges have introduced computing into their elementary mathematics courses--calculus, in particular.

The reasons for this far-reaching change appear to be pedagogical, substantive, and social.

Pedagogically, the computer is a mathematical instrument and a laboratory. Computing facilitates, extends, and enriches learning in mathematics; mathematics extends and facilitates and is essential to learning in computing. The mathematics class is the prime place in the two-year college for learning computing. Computing in the mathematics class can aid understanding, help to build concepts, and develop mathematical intuition.

Substantively, the student's education is broadened to include an introduction to computer science and numerical analysis.

Socially, the computer is becoming an essential part of many aspects of the student's life and work and is an excellent means of demonstrating the relevance of mathematics to the needs of society and the individual. In addition, training in the use of the computer is likely to provide the two-year college student with a skill which will open the doors to a variety of employment opportunities otherwise unavailable to him.

Every transfer student, with the possible exception of humanities majors, should at some time in his college career take a course in computer science such as the course C1 in Recommendations for an Undergraduate Program in Computational Mathematics [page 563]. We make this recommendation both because of the direct importance of the computer in many fields of knowledge and because we feel the students should be aware of the capabilities and limitations of the computer.

The two-year college should be prepared to introduce computing into mathematics courses wherever relevant and appropriate. All of our recommended courses can make good use of flowcharts, algorithms, and computer-programmed assignments.

Support for these recommendations can be found in the studies that have been and are being made of the role of the computer in higher education.

5. Implementation of Individual Courses.

(a) Applications.

At all times in teaching these courses, every effort should be made to illustrate and motivate the material in them by showing how it is used in a large variety of applications. Furthermore, the basic orientation should be towards the solution of concrete problems; at all costs the deadly dry "definition, theorem, proof" approach should be avoided. Of course, there should be carefully stated definitions, theorems, and some proofs, especially in Mathematics C and L as well as in the postcalculus courses, but, especially in the earlier courses, PS, O, A, B, and FM, the problem-solving approach should predominate and new concepts should only be introduced after adequate motivation and examples.

One of the main aims of the teacher should be to get as active student participation in these courses as possible, by involving students in class discussions and a great deal of homework, which, of course, must be adequately criticized and discussed. One of the best ways to awaken and stimulate students' interest is constantly to demonstrate the many applications to a large variety of areas of knowledge that their course material has. After all, a great deal of the mathematics in this program originated in attempts at solving very definite real-world problems.

Courses O, A, and FM offer many opportunities to include applications to elementary probability and combinatorics. In the latter course some mention of questions answered by linear programming techniques can also be made. In the calculus courses the usual applications should, of course, be done but, at all times, the teacher must be on the lookout for unusual applications as, for example, those given in the last part of Calculus in the First Three Dimensions by Sherman K. Stein (New York, McGraw-Hill Book Company, 1967). The technical literature is full of applications of the calculus to subjects such as biology and the social sciences and problems such as traffic flow, pollution, etc. Every calculus teacher has an obligation to try to learn something about them. Mathematics L also has many applications in these areas and the same general remarks apply.

(b) Rigor.

Statements (1) and (2), beginning on page 41 of Commentary on A General Curriculum in Mathematics for Colleges, reflect our views.

(c) Textbooks.

Obviously the textbooks for all the courses will have to be chosen with a great deal of care. The teacher should make a continuing effort to keep abreast of new books as they appear. Moreover, while teaching any given course from a fixed book, the teacher should be consulting several other books for additional problems, extra illustrative material, and different slants on presentation. Clearly, local conditions will often play a determining role in the choice of books, but at all times the instructional staff should be aware of other books existing for the same subjects and should make good use of these additional sources.

The CUPM Basic Library List and Basic Library List for Two-Year Colleges should prove particularly helpful in making choices of texts and reference works for the courses described in this report. Moreover, the book review sections of the American Mathematical Monthly, Mathematical Gazette, Mathematics Magazine, Mathematics Teacher, School Science and Mathematics, and SIAM Review often have helpful comments about new books and should be consulted regularly.

A COURSE IN BASIC MATHEMATICS FOR COLLEGES

A Report of

The Panel on Basic Mathematics

January 1971

TABLE OF CONTENTS

I. INTRODUCTION

1. Background

There is a sizable student population in both two-year and four-year colleges taking courses with titles such as college arithmetic, elementary algebra, intermediate algebra, and introduction to college mathematics. In fact, according to a report issued by the Conference Board of the Mathematical Sciences, approximately 200,000 of the 1,068,000 students enrolled in mathematics courses in four-year institutions in the fall of 1965 were taking courses of this level. In 1966 approximately 150,000 out of 348,000 two-year college mathematics students were enrolled in such courses.* A large portion of these students do not take mathematics courses beyond this level or at best go on to courses of the college algebra and trigonometry type, after which their college-level mathematics training ends. These statements were supported by a survey of a representative sample of two-year colleges conducted by members of CUPM's Panel on Mathematics in Two-Year Colleges in the fall of 1969.

In January, 1970, CUPM formed a Panel on Basic Mathematics to consider the curricular problems involved in mathematics courses of this level. This Panel contained members of the Two-Year College Panel, which had already completed a great deal of preliminary work in this area, as well as additional representatives from two- and four-year colleges and universities. This Panel was charged with making curricular recommendations for the student population described above, whether enrolled in two- or four-year colleges. We will refer to the broad general area of courses below the level of college algebra and trigonometry as basic mathematics.

Many of the students in basic mathematics courses have seen this subject matter in elementary and high school without apparent success in learning it there. It is often the case that a second exposure to essentially the same material, similarly organized, is no more successful even though an attempt is sometimes made to present the subject matter in a more "modern" manner.

There are certainly many complex reasons for this state of affairs. Some of these may be psychological and sociological and may require the work of learning theorists and others trained in the

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* A more recent report of the Conference Board of the Mathematical Sciences (Report of the Survey Committee, Volume IV. Undergraduate Education in the Mathematical Sciences, 1970-71) indicates that approximately 193,000 of the 1,386,000 students enrolled in mathematics courses in four-year institutions in the fall of 1970 were taking courses at this level. In 1970 approximately 272,000 out of 584,000 two-year college mathematics students were enrolled in such courses.

social sciences in order to lessen their influence. However, it is our belief that the type of student described can also be greatly helped by reform in the mathematics curriculum. Evidently there are many ways of doing this; perhaps the most direct would be to allow these students to gain sufficient mastery of arithmetic in a mathematics laboratory by means of various learning aids and only then permit them to begin the elementary-intermediate-college algebra route. We believe that the curriculum described in what follows is another path that promises to be of greater interest and use to the present generation of college students. Of course there are many possible ways of attacking these problems, and we would not want our solution to be interpreted as the only possible one. Nonetheless, we believe that it deserves very serious consideration by the mathematical community and hope that many different kinds of institutions will find our suggestions, wholly or in part, of good use when dealing with the type of students described.

2. The Present Recommendations--Mathematics E

We propose the replacement of some of the currently existing basic mathematics courses by a single flexible one-year course, A Course in Basic Mathematics for Colleges (hereafter referred to as Mathematics E), together with an accompanying mathematics laboratory. The main aim of this course will be to provide the students with enough mathematical literacy for adequate participation in the daily life of our present society.

The adjunct laboratory, which we describe in detail in Section V, will serve to remedy the deficiencies in arithmetic that so many of these students possess. Moreover, the laboratory will offer opportunity for added drill in algebraic manipulation and for instruction in vocationally oriented topics of interest to particular groups of students. This learning should be tailored to each student's individual goals and needs but should not ordinarily require constant supervision by an instructor. It must also be carefully integrated with the material in the course proper.

As a subsidiary aim of the course we hope that the student will gain enough competence in algebraic manipulation, the translation of statements into algebraic formulae, and the careful use of language, to allow him to continue, if he wishes, with courses such as Mathematics A [page 216], intermediate or college algebra, or various vocational mathematics courses, including courses in computing. Thus the proposed course will include material on simple algebraic formulae, handling simple algebraic expressions, the distributive property (common factor), setting up and solving linear equations in one or two variables, the beginning of graphing, and the rudiments of plane geometry. However, we emphasize that the main thrust is to provide the basic literacy spoken of earlier; if there are pressures of time or lack of student interest, then topics should simply be omitted.

This course has been developed to meet a set of circumstances different from those which prevail in the secondary schools. Not only is there the increased maturity due to the higher age level of the students (many of whom may have been out of school for a number of years), but there is the significant fact that the students in question have elected to enter college. Thus, a new approach appropriate to these conditions is needed.

One device for meeting this need is the introduction of flowcharting and of algorithmic and computer-related ideas at the beginning of the course. These ideas permeate the course, encouraging the student to be precise in dealing with both arithmetic and nonarithmetic operations. However, the presentation of computer-related ideas in the course will <u>not</u> depend on the availability of actual machines. Topics of everyday concern, such as how bills are prepared by a computer, calculation of interest in installment buying, quick estimation, analyses of statistics appearing in the press, and various job-related algebraic and geometric problems, are mainstays of the syllabus.

We have tried to make the proposed course coherent; that is, after a topic is introduced, it should be used in other parts of the course and not left dangling. The students must be actively involved throughout the course and should be encouraged to formulate problems on their own, based on their experience. Full advantage should be taken of the playful impulses of the human mind; interesting tricks and seemingly magic ways of solving problems are to be exploited. This point is more fully discussed on page 278.

It should be quite obvious from the foregoing that spirit is more important than content in the proposed course. In order to make as clear as possible the exact intent of the Panel's recommendations, we have included below a detailed discussion of the relationship of the proposed course to certain other courses widely offered at this level, a topical outline of a preferred version of the course, and an extensive commentary on how these topics can achieve our objectives. In addition, we give in Section V a description of the laboratory we propose as a means of dealing with remedial problems and meeting individual goals. In Appendix I there is a brief compilation of problems representative of those that might embody the philosophy of our program; Appendix II contains exercises illustrating the use of flowcharts.

Because the needs of students in the various institutions across the country vary, the Panel believes that a number of different adaptations of the proposed course may be developed. Thus, our topical outline should not be construed as a rigid description of a single course but rather as a flexible model.

II. THE BASIC MATHEMATICS CURRICULUM

There is a large number of well-populated courses in that part of the mathematics curriculum which precedes college algebra and trigonometry. One large group of such courses consists of different versions or repackagings of subjects for special groups of students, especially those in various occupational programs. In addition, there are what might be termed the low-level liberal arts courses, perhaps designed to satisfy a mathematics requirement.

The Panel has gathered anecdotal evidence concerning the reasons students are enrolled in such courses:

1. Some students, despite poor preparation, are ambitious to proceed to more advanced courses in mathematics. We observe that only a small minority of such students do in fact proceed as far as a course in calculus. As described below and in greater detail in Section V, our course would make this possible for the minority of students who will actually go on, even though it may not be as efficient as the traditional route.

2. Many students are advised or required to take certain existing courses in mathematics in the hope that such courses will give the students in some peripheral fashion the kind of mathematical literacy that is the central purpose of our proposed course in basic mathematics.

3. Many mathematics requirements are made in the hope that the mathematics courses prescribed will contain quite specific techniques of value in a student's proposed area of specialization or vocation. Through use of the laboratory, students can be taught such specific material almost on an individual basis.

4. Frequently, there is an institution-wide requirement of one or more courses in mathematics intended primarily for cultural or general education purposes. The basic mathematics course we propose is to be broad and relevant to the actual concerns of students and, therefore, could perhaps serve as a genuine liberal arts course for students of this level of mathematical maturity better than do most of the courses currently taught for that purpose.

Therefore, we believe that the course we propose will meet the objectives listed above better than current courses in the basic mathematics curriculum. Our aim is not to add a new course to the profusion of courses already existing. We wish, rather, to replace several of them by a flexible, more relevant course which will come closer to meeting the genuine requirements of this kind of student. In order to clarify this point, we examine in somewhat greater detail the relationship between the proposed course and certain widely offered courses.

1. Arithmetic. We feel that the courses customarily offered in colleges under this title are subsumed by the course we propose.

2. Elementary Algebra. A student finishing our course should be able to acquire (considering both his work in the classroom and in the laboratory) the computational and technical facility expected of a student finishing a course in elementary algebra. In addition, he should be able to make use of this knowledge in various concrete circumstances. Therefore, it would appear that the introduction of the course we propose could quite properly result in the elimination of courses in arithmetic and elementary algebra.

3. General Education Courses. It is necessary to distinguish between two quite different levels of general education or liberal arts courses in mathematics. The higher-level course may frequently contain material of considerable complexity and depth and might be thought of as an alternative to calculus (or at least to college algebra and trigonometry) for students majoring in the humanities or social sciences. Such courses, because of the relatively high level, are not properly part of the basic mathematics curriculum as defined above and, therefore, lie outside the scope of this report.

The second or lower-level courses, although motivated by much the same philosophy, may be thought of as forming a part of a program of general education for a broader category of students not necessarily restricted to those majoring in the liberal arts. Such courses are sometimes used as a substitute for remedial courses. A general education course is usually intended to develop an interest in and appreciation of mathematics, beginning with the concept of mathematics as an art or as a discipline and working gradually outward to broader issues.

Mathematics E (although it has its remedial aspects) is not primarily a remedial course. From the standpoint of a general education program the proposed course is a broad one; it can be termed the mathematics of human affairs, and as such should be a reasonable alternative to the usual general education mathematics course. Moreover, the prospective students for Mathematics E are likely to be of a pragmatic turn of mind. For them an appreciation of mathematics seems likely to stem from seeing how mathematical ideas illuminate areas in which they have an established interest.

4. Courses for Students in Occupational Curricula. It must not be presumed that all courses designed specifically for students in occupational programs are at a low level. For example, students in physical science related curricula such as Engineering Technology will generally begin their college mathematical training at a higher level and often continue through the calculus. In addition, there are many occupational curricula which by their nature must contain mathematics courses taught in extremely close relationship to the major program. For these categories of students the proposed course, Mathematics E, will not serve. However, there remain large numbers of students in occupational curricula whose mathematical needs are less specialized.

These include students in some programs in business and health professions as well as students in other occupational programs not containing a strong element of scientific training. These programs very likely encompass a large majority of students in occupational curricula. We believe that the principal mathematical need for such students is basic mathematical literacy together with some work in the laboratory directed toward their special needs.

5. Mathematics for Prospective Elementary School Teachers. These students have very special and pressing needs to which our course does not address itself. However, Mathematics E might be needed by some as preparation for the teacher-training courses recommended by CUPM. [See Recommendations on Course Content for the Training of Teachers of Mathematics, page 158.]

6. Mathematics A. This course is described in A Transfer Curriculum in Mathematics for Two-Year Colleges, page 205. Mathematics A is, briefly stated, an improved and extended version of college algebra, trigonometry, and analytic geometry interwoven with certain remedial topics. The Panel believes that Mathematics A should prove as viable in some four-year institutions as in the two-year colleges for which it was originally designed. Mathematics A is the most natural continuation of Mathematics E for the minority of students who continue with further courses in mathematics.

7. Intermediate Algebra. We take intermediate algebra to mean a semester course containing a rather systematic and extensive review of topics normally encountered in some form in elementary algebra, followed by new material on such topics as exponents and radicals, functions and graphs, quadratic equations, systems of equations, and inequalities, with selected topics from among complex numbers, logarithms, permutations and combinations, and progressions. The emphasis is on technical algebraic proficiency with occasional digressions into the applications of specific techniques.

Many of the students for whom the Panel would prescribe the year course in Mathematics E now take a year sequence composed of elementary and intermediate algebra. Mathematics E contains much nonalgebraic material and touches certain broader issues which the corresponding algebra sequence does not cover; Mathematics E, however, cannot be expected to provide as great a degree of technical algebraic proficiency as the conventional algebra sequence.

We suggest that what is appropriate here is a serious and realistic appraisal by the mathematics department at a given institution as to which sequence is more directly related to the real reasons why students take or are required to take courses at this level.

Intermediate algebra is not a more advanced course than Mathematics E, but rather one with different goals. The Panel feels that Mathematics A rather than intermediate algebra is the more natural continuation of Mathematics E. Thus, a school offering Mathematics E and Mathematics A might have no further need for intermediate algebra courses unless it has a significant number of students who (a) have an adequate competence in elementary algebra and (b) need a mastery of specialized algebraic techniques as opposed to a more generalized mathematical competence and (c) are not prepared for Mathematics A.

Finally, the introduction of Mathematics E, depending upon local circumstances, will make possible a considerable economy and simplification in the basic mathematics curriculum, an economy which should have special appeal to small colleges or to colleges having only a small number of poorly prepared students.

III. OUTLINE OF MATHEMATICS E

In this section we present an outline of one sequence of topics which the Panel feels is appropriate for use in implementing the purposes of the course. It should be re-emphasized that coverage of these topics is in itself neither necessary nor sufficient for the course to fulfill the spirit of the Panel's recommendations. One purpose in the presentation of the outline in detail is to demonstrate the existence of at least one sequence of topics which have obvious relevance to the interests and needs of present-day students and yet can be mastered by the students for whom the course is designed.

Although we feel we are recommending something more than coverage of a list of topics, the Panel has given extensive consideration to the question of which topics would best serve our purposes. We feel that others who give equally serious consideration to these questions will arrive at a sequence of topics in substantial agreement with ours.

Although the details of how much time should be allotted to each specific topic can only be determined by actually teaching the course, the Panel has in mind that approximately 50 per cent of the year's work would be devoted to Parts 1 through 4, approximately 25 per cent to Parts 5, 6, and 7, and the remaining 25 per cent to Parts 8 and 9.

The page references in the Outline are to the Commentary which follows.

Part 1 Flowcharts and Elementary Operations

There are two reasons for beginning this course with an introduction to computing. First, there is the rather obvious matter of initial motivation of the students. Second, we wish to introduce early the idea of a flowchart, which will permeate the entire course. (See pages 271-277.)

1-a Brief introduction to the nature and structure of digital computers. Specimens of computer programs and computer output, but no real programming until Part 5. Flowcharting as a preliminary device for communicating with the computer.

1-b Flowcharts. Further illustration of flowcharts by nonmathematical examples including loops and branches. Sequencing everyday processes.

1-c Addition and multiplication of whole numbers. Addition and multiplication as binary operations. The commutativity and associativity properties illustrated by everyday examples. Multiplication as repeated addition, illustrated by examples. Drill in these operations. Flowcharts for these operations, notion of variable, equality and order symbols. Introduction of the number line as an aid in illustrating the above and to provide for the introduction of the coordinate plane.

1-d The distributive property and base 10 enumeration. Distributive law done very intuitively and informally by examples on 2- or 3-digit numbers in expanded form. (See page 276.) Illustrate these two topics by means of simple multiplication.

1-e Orders of magnitude and very simple approximations. Relate order of magnitude to powers of 10. Motivate approximations to sums and products by means of simple examples. Lower and upper bound for approximations, no percentage errors. Introduction of the symbol $\approx$. (See pages 281-283.)

1-f Subtraction of whole numbers. Three equivalent statements:

$$a + b = c; \quad a = c - b; \quad b = c - a$$

Commutativity and associativity fail for subtraction, operation not always possible. Multiplication distributes over subtraction. Approximations as in 1-e. Drill.

1-g Exact division of whole numbers. Three equivalent statements for a and b not zero:

$$ab = c; \quad b = c/a; \quad a = c/b$$

Division is not always possible, division is noncommutative and nonassociative. Flowcharting. Computational practice.

1-h Division with remainder. Informal discussion of division with remainder. Flowchart process as handled by a computer. Approximations as in 1-e.

1-i English to mathematics. Translations of English sentences taken from real-life situations into algebraic symbolism. (See pages 277-281.)

Part 2 Rational Numbers

It is intended that this part shall have an informal and pragmatic flavor. Even though some references are made to certain of the field axioms, we do not wish to approach the number system from a structural point of view.

2-a Extending the number line to the negatives. Absolute value and distance.

2-b Rational operations on the integers. To be derived from plausibility arguments as novel as possible but not from the field axioms. Drill in these operations.

2-c Fractions with the four rational operations. Special case of the denominator 100 as percentage. Simple ratio and proportion. Drill in manipulations with fractions.

2-d Decimals. Use base 10 notation with negative exponents. Relation between fractions and decimals via division. Many practical applications and practice.

2-e Roundoff and truncation errors. (Most computers truncate rather than round off.) Significant digits and scientific notation.

2-f More on English to mathematics. Use the new ideas developed in this Part. More flowcharting with examples drawn from interest computations and financial problems, including the use of the computer. (See page 280.)

Part 3 Geometry I (See also Part 7)

The purpose of this material is to refresh the student's acquaintance with basic geometric vocabulary and then to present that minimal geometric background sufficient for the introduction of coordinate systems. (See pages 285-287.)

3-a Introduction to geometric ideas. Informal discussion of points, planes, segments, lines, angles, parallel and perpendicular lines.

3-b <u>Geometric figures</u>. Circles, triangles, special quadrilaterals, notion of congruence.

3-c <u>Use of basic instruments</u>. Ruler, protractor, compasses, T-square. Error in measurements.

3-d <u>Conversion of units</u>.

3-e <u>General introduction to linearity and proportion</u>. Many examples. Notion of similarity.

3-f <u>The coordinate plane</u>. Points and ordered pairs, road maps, etc.

3-g <u>The graph of $y = mx$</u>. Slope.

Part 4 Linear Polynomials and Equations

In this part there is to be a treatment of algebraic ideas at a level sufficient for the applications that follow, but which stops somewhat short of the technical algebraic competence usually sought in conventional courses in algebra. Students who wish a higher degree of technical competence may obtain it through appropriate work in the laboratory. (See Section V.)

4-a <u>English to mathematics</u>. A few word problems leading to one linear equation in one unknown as motivation for algebraic manipulation. Solve some equations by trial and error. Devise flowcharts for trial-and-error solutions.

4-b <u>Transformations of one equation in one variable</u>. Both identities such as $2x + 3x = 5x$ and $3(x + 2) = 3x + 6$ as well as transformations such as: if $4x + 5 = 11$, then $4x + 3 = 9$.

4-c <u>Flowchart for solving $ax + b = c$</u>. Include a variety of other forms.

4-d <u>Applications</u>. Word problems drawn from many different areas.

4-e <u>Situations leading to one equation in two variables</u>. (Motivation for next section.)

4-f <u>Transformations of one equation in two variables</u>. Leading, for example, to the form $y = mx + b$, being careful not to restrict the names of the variables to x and y.

4-g <u>Graphs of linear equations in two variables</u>. Slope of $y = mx + b$. Relation of $y = mx + b$ to $y = mx$.

4-h Solutions of two linear equations in two variables. Graphical and analytical methods, applications.

Part 5 The Computer

The amount of time devoted to this Part will turn out to be quite short or quite long depending on whether actual use is to be made of a computer. (See pages 16-17.)

5-a General discussion of the computer. Ability of a computer to respond to well-defined instructions. Illustrate with simple programs. Brief discussion of error due to truncation. Memory, operations, speed, with reference to the available equipment.

5-b Uses of the computer in modern society. Many different applications, with limitations of the computer stressed.

5-c Elementary instruction in programming. Language appropriate to the institution; writing programs from flowcharts.

5-d Varied applications. Drawing from material already presented, including more sophisticated financial problems. Run programs on computers when available.

Part 6 Nonlinear Relationships

We have included this material primarily to display the power of a mathematical model and to provide for development of the themes of flowcharting, approximation, and graphing which have been introduced earlier.

6-a Some examples of nonlinear relationships. Repeated doubling, and exponential growth of populations. Compound interest.

6-b The graph of $y = x^2$. Concept of square root and graphical evaluation of square root. Use of tables and approximation of square roots by averaging.

6-c Pythagorean theorem and distance formula. Very brief discussion of irrational numbers and the fact that lines and curves have no gaps.

6-d The graph of $y = ax^2$. Applications.

6-e The graph of $y = ax^2 + bx = x(ax + b)$. Roots and intercepts, maximum and minimum, applications.

6-f <u>Graphing of $y = ax^2 + bx + c$</u>. Use vertical translation from $y = ax^2 + bx$. Note that there may be 0, 1, or 2 roots of the corresponding quadratic equation.

6-g <u>Approximation of roots</u>. Use of the computer. (See page 276.)

6-h <u>Inverse, joint, and combined variation</u>. Applications.

6-i <u>Suitable bounds for accuracy and estimates</u>. Products and quotients, relative and percentage error, graphical illustrations. (See pages 281-284.)

Part 7 Geometry II

Our hope here is that the geometrical material presented will be made relevant and that there will be suitable links with the mathematical ideas introduced earlier. (See pages 285-287.)

7-a <u>Areas and perimeters of plane figures</u>. Rectangles, triangles, parallelograms, and circles. No extensive involvement with theorems and proofs. Perhaps compute area of irregular regions by use of rectangles and Monte Carlo methods.

7-b <u>Surface areas and volumes</u>. Use of formulas for areas and volumes of spheres, cylinders, parallelepipeds.

7-c <u>Applications</u>. Consumer problems, pollution problems, conversion of units.

7-d <u>Elementary constructions</u>. Use of straightedge and compasses. Include special triangles like isosceles right triangles, 30-60 right triangles, etc.

7-e <u>Further extension of work on similar figures</u>.

Part 8 Statistics

Besides the obvious interest and relevance of this material, it offers opportunities for use of virtually all of the ideas previously introduced in the course. We have in mind the use of statistical ideas in making practical decisions among realistic alternatives. (See pages 287-288.)

8-a <u>The role of statistics in society</u>. Problems of interpretation of charts, graphs, percentages.

8-b Descriptive statistics. Various kinds of graphs; mean, median, and mode; range and standard deviation; quartiles and percentiles.

8-c The normal distribution. Informal discussion.

8-d Statistics and the consumer. Informal discussion of bias; choosing samples. Flowcharts and computing should be used whenever appropriate.

Part 9 Probability

This Part represents a rather minimal introduction to the subject, avoiding any heavy involvement with combinatorics, but including one or more applications of complexity sufficient so that the methods of earlier chapters can be displayed to good advantage. (See pages 289-293.)

9-a Empirical probability. Mortality tables, long-run relative frequencies.

9-b A priori probability. Tossing coins, rolling dice, selecting discs from box. Experiments in which relative frequencies are compared with theoretical probabilities.

9-c Elementary counting principles. Emphasis on devising a procedure for listing of outcomes of an experiment, the procedure suggesting a principle or formula for obtaining the count. (See pages 291-292.)

9-d Further a priori probability. Independent trials of an experiment. Examples selected from everyday experiences such as athletics.

9-e Informal decision theory with examples. (See Exercise 15 in Appendix I.)

IV. COMMENTARY ON THE OUTLINE

An outline for a course is a good device for specifying the order of topics but a poor device for emphasizing the ideas which are to receive continuous attention. This section is devoted to making clear the intentions of the Panel concerning certain special topics and certain recurrent themes. Other topics of equal importance have been omitted from this discussion because the kind of treatment they should receive is relatively clear from the outline.

1. Computers and Computing

Mathematics E is to begin with a very brief description of how a digital computer receives its instructions and the operations it is capable of performing, preferably made concrete by exposure to actual computing machinery. The course returns to the subject in somewhat greater depth in Part 5 of the outline. At other places in the course opportunities to discuss and illuminate special topics from a computer point of view are to be exploited as these opportunities arise. The concept of a flowchart as a device for analysis is to be a central theme.

The purpose of introducing and utilizing computer-related ideas is basically the psychological one of getting the students involved with and thinking about mathematics. In a course of the kind we describe, this point is likely to be of crucial importance. The Panel has more than adequate anecdotal evidence that the injection of computing will provide very strong motivation for the student. A secondary reason is the ubiquity of the computer in our present society and the need for the students to understand its potentialities and limitations in order to function as citizens and employees.

The Panel feels very strongly that the basic validity of the course we propose would not be compromised by the absence of computing equipment at the institution where the course is being taught, nor would the lack of computer access lead us to prescribe a significant reduction in the emphasis on computer-related ideas such as flowcharting.

However, the Panel feels equally strongly that even a small amount of direct experience with computers will greatly enhance the motivational powers of these ideas and will result in a more effective course.

At the very least, the following should be done:

a. The students should have one or more guided field trips to a computer center to see how a problem is actually handled by a computer, e.g., they should understand what a computer program is and how it is related to a computer.

b. Students should be involved in team efforts to write quite simple programs.

c. The instructor should demonstrate how these programs are read by the computer and how the computer imparts its results.

It is clear that such a minimal degree of exposure would require only very limited access to computing equipment and would not imply actual possession of a computer.

In Part 5 we suggest further experience in programming. In various places that follow we mention some of the opportunities which exist for making some topics both more realistic and more challenging by actual computer usage.

The question of how extensively such opportunities can be utilized is clearly dependent on costs, even at colleges with extensive computer installations. The Panel feels that computer usage should certainly be carried beyond the minimum outlined above if at all possible. However, neither our experience nor the experience of others seems adequate at present to make any accurate estimates of the educational benefits to be expected as a function of costs.

2. Flowcharts

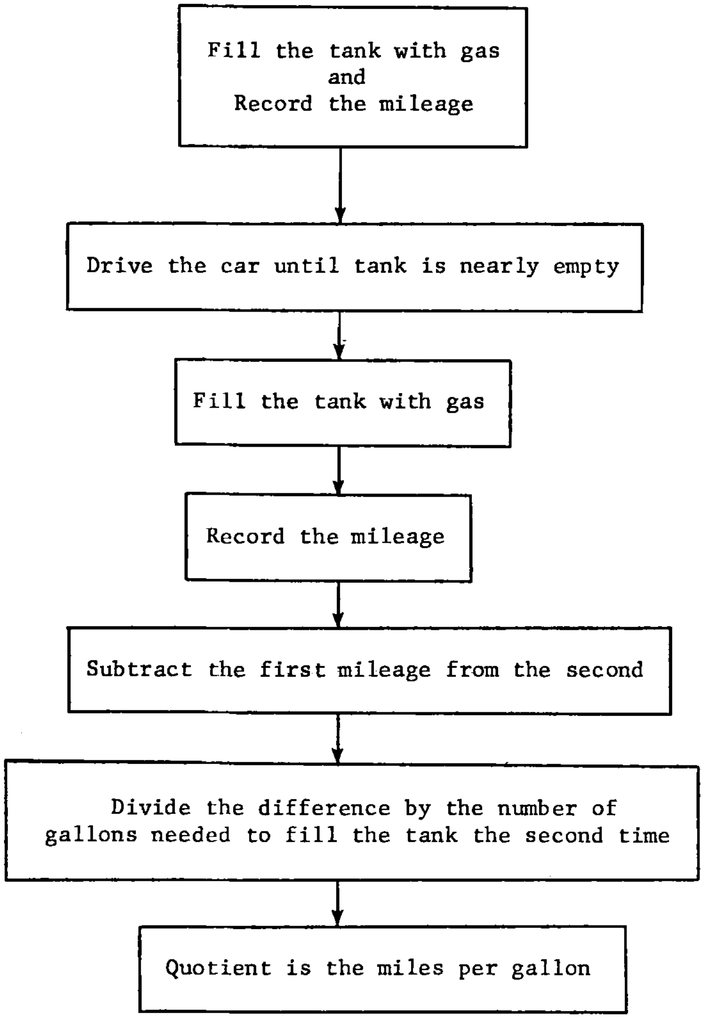
We have chosen to discuss flowcharting in the first part of the course both in order to arouse immediate student interest and because this technique is used extensively throughout the course. It must be made clear to the student that the computer operates only through very specific instructions presented to it in a special language. The idea of flowcharting can be first presented as an essential step in the process leading to the production of such detailed instructions.

The utilization of the construction of flowcharts as a technique in the analysis of problems recurs throughout the course being described. A flowchart is an extremely valuable way of describing a procedure to be followed either by another person or a computer. Its object is to break a problem up into easily managed steps whose interrelationships are clear.

It is not necessary to have a detailed familiarity with the actual capabilities of the computer to begin describing algorithmic processes with flowcharts. Perhaps the best examples to use in introducing flowcharting ideas are those of a nonmathematical nature such as the ones which follow. These first four examples illustrate the four kinds of elementary structure which flowcharts can have.

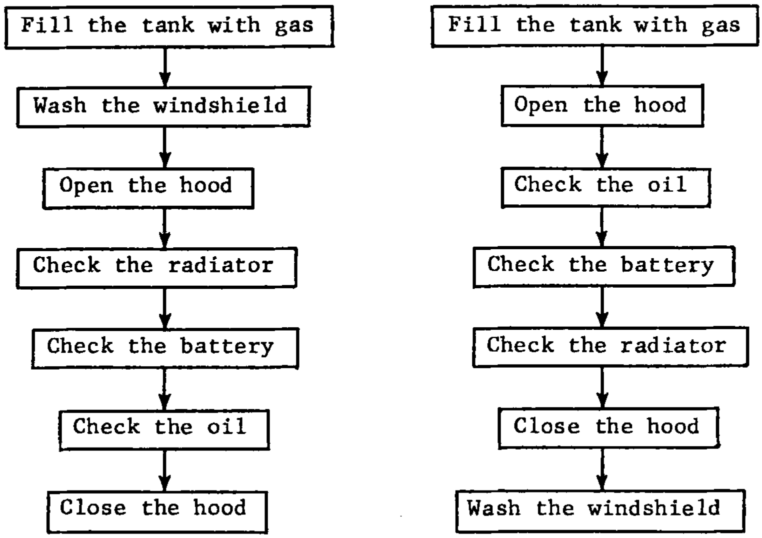
A. Sequential operations with no choice.

To determine the gas mileage for your car.



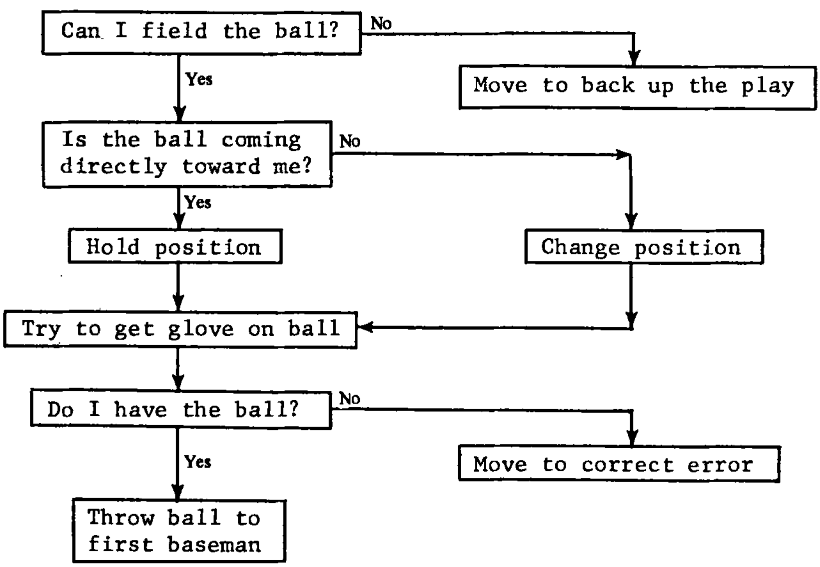
B. Sequential operations with a choice of order.

To service a car at a filling station.



C. Branches.

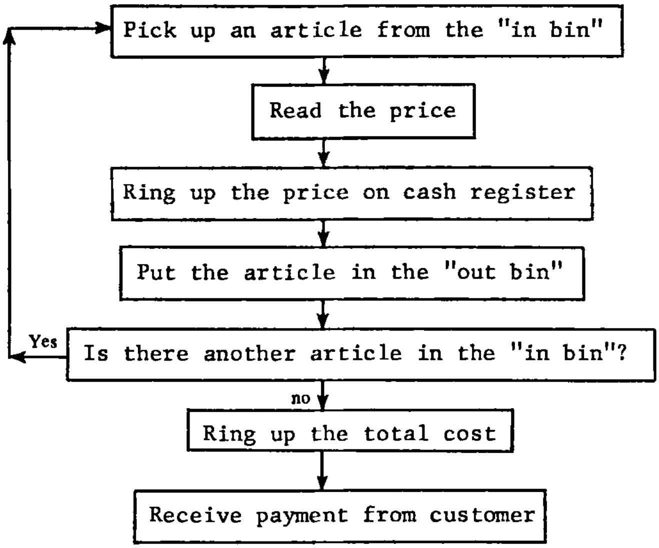
Reaction of a baseball shortstop to a groundball hit toward his position with no runner on base.



D. Loops

An algorithm may call for a process to be repeated any number of times to achieve a desired result. Thus, instead of repeating the same instructions for a set of objects, one may simply call for the same instructions to be applied to the set of objects until the set is exhausted. Representing such an algorithm by a flowchart gives rise to the so-called loop concept.

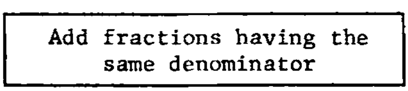
Action of cashier at a grocery store.



These flowcharts are all incomplete in the sense that the operations described within a given box are susceptible to further analysis into smaller steps. If a flowchart is to result in a computer program, the capabilities of the computer will determine completeness.

The student should be encouraged to construct flowcharts based on his own experience.

We propose the study of flowcharting not only as an end in itself, but also for its usefulness throughout the course to explicate concrete details in situations which may not be purely computational--for example, the standard situation faced by a student unable to solve a problem in which he does not know where to begin. Thus the source of a student's difficulty in adding two fractions may be his inability to break the problem down into simpler parts. Such a student would probably be able to add fractions with the same denominator. Let him then write



It is then apparent that if this could be preceded by

$\boxed{\text{Get fractions to have the same denominator}}$ ———

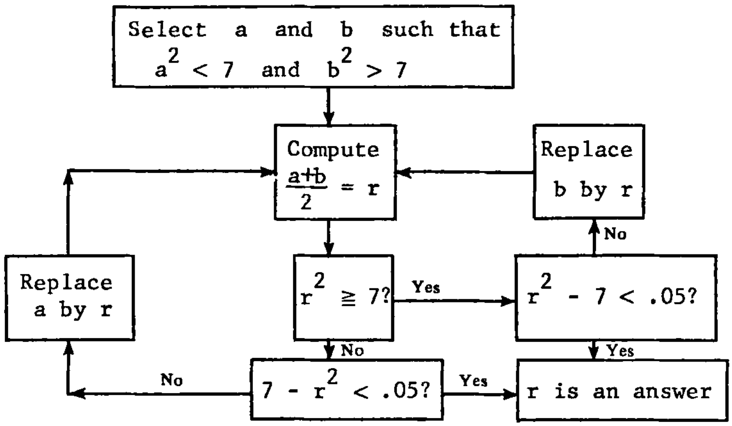
an important first step toward the solution of his problem would have been taken. A further analysis of the contents of this box via flowcharting should result in the solution.

Flowcharts can illuminate mathematical assertions that fail to have the desired impact when presented in the usual manner. For example, the two sequences of operations indicated in the equation for the distributive law can be dramatized by the exhibition of the two flowcharts that describe the sequences:

Compute

$x = a(b + c)$	$y = ab + ac$
Enter a, b, c	Enter a, b, c
↓	↓
Compute $d = b + c$	Compute $e = ab$
↓	↓
Compute $x = ad$	Compute $f = ac$
↓	↓
Print x	Compute $y = e + f$
	↓
	Print y

Once the student has been introduced to the graph of the quadratic equation, he should have no difficulty in understanding, for example, that if the graph of $y = 7 - x^2$ is above the x-axis at $x = a$ and below the x-axis at $x = b$, then the square root of 7 lies between a and b. This idea is exploited in the following flowchart of a simple algorithm for approximating the square root of 7. We have chosen this algorithm for its simplicity in spite of the fact that it is not the most efficient one for hand computations. The relative speed with which a computer would perform the work would be impressive to the students.



This flowchart illustrates graphically the idea of an iterative process and suggests the economy of presentation which loops in such charts provide. The example itself affords drill for the student who performs the successive approximations. Of course, access to a computer so that a program based on this chart could actually be run would provide added motivation for the student.

3. The Role of Applications and Model Construction

Since the development of mathematical literacy relevant to participation in human affairs is a major objective of the course, a strong relationship between the topics and their applicability must be established. The purpose of this section is to offer suggestions on the kind of applications that should be emphasized and the points at which they should be included.

Although it would be expected that separate sections devoted to "word problems" might be a part of the course, the mere inclusion of such sections is not sufficient to accomplish the desired goals. To aid in establishing the relevance of the topics, examples of practical problems should be used to motivate the topics. Additional examples need to be interspersed with sufficient frequency to insure that interest is maintained.

In order to help in overcoming one of the major difficulties associated with practical problems, separate sections entitled "English to Mathematics" are listed in the suggested outline. The purpose of those sections is to assure that attention is actually given to the process of translating English statements into mathematical equations and other mathematical models.

Care must be taken in the selection of examples to insure that the majority of the problems are actually practical. Too often in existing courses and texts, problems are "cooked up" to lead to a particular kind of equation. Consequently, some of the motivational value of the examples is lost because of their lack of relation to reality. Problems with sentences such as "John is three times as old as Mary was five years ago," or "Jack has twice as many quarters as dimes and seven more nickels than dimes," may lead to nice equations, but they are uninteresting simply because the situations are never encountered.

It is not being suggested that examples taking advantage of the playfulness of the human mind should be ignored. However, problems in which variables must represent quantities that are likely to be known in the situation being described should occur infrequently and only when a note of whimsy is desired.

Perhaps the most important problem that the instructor will face in teaching the typical Mathematics E student is to arouse his interest and to induce him to participate with some degree of enthusiasm in the course. It is hoped that the computer-related aspects of the course will help to bring this about. However, an appeal to the playful and puzzle-solving impulses of the student should also be used whenever possible. Most of the student audience under consideration is willing to think quite hard, and, indeed, in a mathematical fashion, when it comes to certain pursuits that it regards as pleasurable--for example, when it comes to playing card games. Moreover, a tricky puzzle will capture the attention of many of those students while a straight mathematical question will rarely do so. It is therefore very likely that if these elements are used whenever appropriate in Mathematics E they will jolt the students into awareness. For example, the old puzzle about the bellboy and the five dollars or the three warring species crossing the river two at a time[1] can be used to illustrate logical analysis and step-by-step consideration of problems. The elements of these puzzles should be brought out and used throughout the course in order to point out the intellectual similarity between dealing with them and with solving ordinary mathematical problems. Of course, in Part 9 many references should be given to the games that our students actually play, but even the seemingly more arid parts of the course can surely benefit from this kind of material. The Thirteen Colleges Curriculum Program[2] has made very successful use of this technique and has found it a very useful and efficient way to lead students into a study of mathematics that the students rejected originally. This material contains stimulating problems that have been used successfully by participants in the Thirteen Colleges Curriculum Program to engage

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1. Many such problems can be found in Mathematical Recreations by Maurice Kraitchik (New York, Dover Publications, Inc., 1942).

2. Thirteen Colleges Curriculum Program Annual Report. Analytical and Quantitative Thinking (Mathematics). Florida A and M University, Tallahassee, Florida, 1969.

the attention and reasoning power of students taking courses at approximately the level of Mathematics E.

The following points should be considered in the selection and presentation of examples and exercises: (1) The majority of problems should describe familiar situations in which variables represent quantities that could reasonably be expected to be unknown. (2) To develop the ability to translate English into mathematics, it may be necessary to begin with problems that students are able to solve without the aid of an equation or other model. However, at several points in the course, students will have the background necessary to construct models for problems that they will not be able to solve completely. If full advantage is taken of this situation, then applications can be used to motivate the need for being able to solve equations and otherwise manipulate models to produce solutions. (3) Students should frequently be asked to estimate an answer to an exercise prior to writing the equation or formulating a model, and to explain the process by which the estimation was made. Such explanations are often surprisingly close to being a correct verbal equation which can be more easily described by symbols than could the original problem. (4) Students should be encouraged to describe problems which they have actually encountered and which they would like to be able to solve. The experiences of students with charge accounts, savings accounts, tax problems, other college courses, vocational experiences, and situations that occur in playing cards and other games can be excellent sources of problems at various times within the course.

As an illustration, a sequence of problems that might be used near the beginning of the course to develop the ability to translate English into linear equations is given below. The problems begin with one that could be solved without a formal statement of the equation and progress to the point that the majority of students would need an equation to complete the solution.

1. A student has grades of 65 and 76 on two exams. In order to maintain the average that he desires, he must have accumulated 210 points after the third exam. Write an equation that can be used to find the grade which he must make on the next exam.

2. Suppose that the student in the previous problem wishes his average for the three exams to be 72. Write an equation that can be used to find the grade which he must make on the next exam.

3. Suppose that the third exam in problem 2 is a final exam and will count as two regular exams in computing his average. Write an equation that can be used to find the grade which he must make on the final exam in order that his average will be 72.

4. Suppose that in problem 3 the student may choose to use his textbook on the final exam, but if he so chooses, his final exam score is lowered by 10 points. Write an equation that can be used to find the grade which he must score on the final exam (before deduction) in order that his average will be 72.

Note that the equation for the first problem may be given by a sentence as simple as $P = 210 - 65 - 76$, which the student could certainly solve. In fact he could probably solve this problem without the aid of an equation. An equation for problem 4 is given by

$$\frac{65 + 76 + (X - 10) + (X - 10)}{4} = 72,$$

which the student probably could not solve at this point.

There is a wealth of possibilities for motivating and illustrating various topics throughout the course with consumer problems. Such problems seem to be especially adaptable to the notion of flowcharting. Two examples of such problems are given below. By adding such complicating factors as additional purchases, variable payments, and minimum service charges, one can expand the problems to the point where a flowchart is almost essential for describing the payment process. (See Appendix II.)

1. Suppose that a customer makes purchases on credit totaling \$560.00. Interest and all other charges on the debt make the total balance \$610.00. The customer must pay \$25.00 per month until he owes less than \$25.00. He will pay the remainder as a final payment. Construct a flowchart that contains boxes giving the amount owed and the number of payments remaining after n payments.

2. Suppose that a customer makes purchases on credit totaling \$560.00. On the last day of each month $1\frac{1}{2}\%$ of the unpaid balance is added to his account. The customer plans to pay \$25.00 each month until he owes an amount less than or equal to \$25.00. He will pay the remainder as the final payment. Payments are paid on the first of the month. Describe the payment process with a flowchart that contains boxes giving the amount paid and the amount owed after n payments.

The problems should include some that are more open-ended than is usually the case with textbook problems. Some problems should request a reasoned choice between alternative courses of action, and some should ask what additional information is needed in order for an ill-posed problem to become solvable.

It should be stressed that the Panel intends that "word" problems should be emphasized much more in this course than is usually the case. We do not have in mind teaching artificially neat procedures for solving quite special classes of problems (as, for example, in the rote methods frequently used to drill students in the solution of time-rate-distance problems). We believe that students in a course of this level can learn to create and analyze simple mathematical models. Many students are inhibited by their attitudes toward mathematics and beset by a fear of failure. If the problems are realistic enough to seem significant to the students and initially are simple enough to insure a ready solution by most students, we believe that the students can be helped to develop a regular use of mathematical ways of thinking in the analysis of practical situations.

Appendix I contains a list of problems and examples that illustrate the kind of applications that should be a part of the course. The list is not meant to convey any sort of desirable achievement level for the course. Neither is it meant to be comprehensive with respect to the areas from which applications should be selected. It does represent the spirit of the course relative to meaningful applications.

4. Estimation and Approximation

In a wide variety of practical problems an approximate solution will serve as well as the result of an exact calculation. In other situations (such as that of a shopper in a supermarket) estimation is the only practical course of action available. Even if an answer has been calculated exactly, it is useful as a check to obtain a quick approximate solution.

We hope that a graduate of this course will be an habitual estimator. The topic of estimation, like that of flowcharting, should appear throughout the course and should be presented in as many places as possible, taking advantage of opportunities as they arise. The course should make the student moderately proficient in estimating products, reciprocals and quotients, as well as powers and roots, of 2- and 3-digit numbers. Although the underlying theory is elementary, the approach must be through trial and error leading to the formulation of certain principles which in turn lead to the basic question of tolerance and control.

Because the audience we have in mind is not generally well prepared in arithmetic, one should start slowly, perhaps with relative errors discussed as percentages. These should be taken from everyday life: population figures, sports, betting, etc. Then one needs to present a crude technique of converting relative errors expressed as percentages. Thus, we discuss "48 parts in 99" and point out that it is roughly "50 parts in 100." Also, a relative error of 16/53 is approximately a 30% relative error, since 16/53 is approximately 15/50.

The product 13×27 can be estimated in several useful ways. For example,

(a) We could make each number smaller; replace 13 by 10 and 27 by 25 and obtain

$10 \times 25 = 250$, which we regard as roughly correct. We write

$13 \times 27 \approx 250$,

where the symbol $\approx$ means "is approximately equal to."

(b) We could make only one of the numbers smaller, using 10 and 27 for example, and write

$13 \times 27 \approx 10 \times 27 = 270$.

Clearly (a) and (b) are "underestimates," i.e.,

$13 \times 27 > 250$ and

$13 \times 27 > 270$.

(c) We could make each number larger; replace 13 by 15 and 27 by 30 and obtain

$15 \times 30 = 450$, writing

$13 \times 27 \approx 450$.

(d) We could make only one number larger, using 13 and 30, for example, and write

$13 \times 27 \approx 13 \times 30 = 390$.

Of course (c) and (d) are "overestimates." Thus, we can write

$270 < 13 \times 27 < 390$,

which the student must learn to read as two statements: 270 is less than 13×27, and 13×27 is less than 390. At this stage, we might suggest that an average be used. Thus, the average of 270 and 390 is 330; hence,

$13 \times 27 \approx 330$.

(e) In estimating the product we can make one factor larger and one smaller; this frequently leads to a better estimate. Thus, we could replace 13 by 10 and 27 by 30 and write

$13 \times 27 \approx 10 \times 30 = 300$.

The student will see that this estimate is perhaps better, but that we have lost control in the sense that we no longer know whether the estimate is larger or smaller than the correct answer.

Some students might wish to pursue these ideas further in the laboratory. They might be encouraged to consider questions such as: if a, b, and x are positive, x is small compared to a and b, and $a < b$, then is it the case that

$$\frac{a - x}{b - x} < \frac{a}{b} < \frac{a + x}{b + x} ?$$

Such a formulation might be conjectured empirically, and then an algebraic analysis attempted.

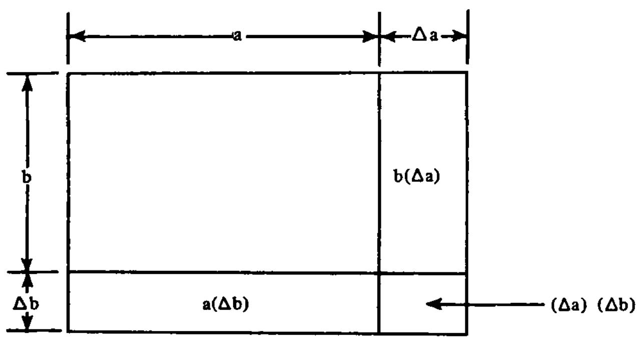
Somewhere in the course it might be well to introduce the student to the use of the slide rule, although the theory should be postponed until the student himself wants it and is ready for it. At the beginning it is just an instrument which serves to estimate products and quotients. Scientific notation would be introduced in discussing the position of the decimal point.

Before any formula about estimation is presented, the students should have been exposed to problems in which a quick rough estimate is useful.

<u>Example</u>: Steve wishes to mail 28 books to his new residence. The prices of the books range from \$6.95 to \$12.50. For how much should he insure the books?

The theory of approximation must, of course, be presented in stages, following the algebraic readiness of the student. Once the student has the necessary algebraic readiness, he might be introduced to the concept of relative error. Thus, if Δx denotes the error in x, then by the relative error $R(x)$ we mean $\Delta x/x$.

After doing a sufficient number of problems involving this concept, a student might be enticed to consider $R(ab)$. This can be motivated by considering the following diagram:



$$\text{Thus,}\quad \Delta(ab) = b\cdot\Delta a + a\cdot\Delta b + \Delta a\cdot\Delta b.$$

Now, dividing by $a\cdot b$, we obtain

$$R(ab) = R(a) + R(b) + R(a)\cdot R(b).$$

From this picture, a student can perceive that if $R(a)$ and $R(b)$ are "small enough" (depending on the situation), then an estimate for $R(ab)$ is

$$R(ab) \approx R(a) + R(b).$$

The student should, of course, have had the opportunity to conjecture this by considering several examples.

Having arrived at this formula, the student might be ready to understand, for example, that if $|R(a)| < 30\%$ and $|R(b)| < 20\%$, then the estimate for $R(ab)$, namely $R(a) + R(b)$, is correct to within 6%.

At the point where the student has learned to graph, we can use the graph of $y = x^2$ to get estimates of squares and of square roots (see also flowcharting). It should be interesting to the student that a smooth graph is a good "estimator." Of course, from the relation

$$R(ab) \approx R(a) + R(b),$$

we obtain

$$R(a^2) \approx 2R(a)$$

or

$$R(\sqrt{a}) \approx R(a)/2,$$

from which we conclude that the relative error in the square root is about half the relative error in the number.

Following this, the interested student could be shown (this should probably be done in the laboratory) that if $R(b)$ is "small enough," then

$$R\left(\frac{1}{b}\right) \approx -R(b),$$

and that if $R(a)$ is also "small enough," then

$$R\left(\frac{a}{b}\right) \approx R(a) - R(b).$$

At this stage, the student will have accepted the usefulness of these relations and will enjoy working examples and seeing how well he can quickly estimate products, quotients, powers, and roots.

5. Comments on Geometry

Geometric ideas should be presented in two stages. At the beginning (Part 3 in the outline) only basic geometric vocabulary and those ideas necessary for introducing the coordinate plane are presented. Once the student has gained some mathematical experience (Part 7 in the outline), he can be exposed to slightly more complicated geometric notions.

As early as in Part 3 it is fairly safe to assume that most of the students of whom we speak will have some notions of the basic geometric concepts of points, planes, segments, lines, angles, parallel lines, and perpendicular lines. It is hoped that the course can reinforce some of the correct notions the student already has, while at the same time pointing out a number of concrete uses of geometry in his daily life. Abstract definitions and an axiomatic approach must be avoided. It is appropriate to give the student experience with specific geometrical figures by performing constructions in the laboratory with the use of the ruler, compass, protractor, and draftsman's triangle. With even rudimentary ideas of similarity, one could discuss problems of indirect measurement.

After the student has been introduced to the idea of a plane, the coordinate plane can be regarded as analogous to the familiar devices employed in designating a specific section of a road map (by letter and number) or some particular locale in an ocean (by latitude and longitude).

The coordinate plane can then be introduced in the usual way, i.e., by reproducing on each axis the number line discussed in Parts 1 and 2, choosing the intersection of the two axes as the origin, and by making use of the concepts of perpendicular and parallel lines discussed earlier.

The graph of $y = mx$ can now be introduced, perhaps with an example such as the following:

Example: The owner of an ornate gift shop in a high-rent district has determined that he can make a reasonable profit and cover all overhead expenses (stock costs, rent, shipping costs, employees' salaries, taxes, utilities, office supplies, advertising, insurance, breakage, etc.) by making the selling price of each item equal to twice its cost. Let x be the cost of an item and y its selling price; then $y = 2x$. A table of corresponding values of x and y should be constructed and these ordered pairs plotted, with appropriate emphasis on the fact that the points are arrayed in a straight line.

The concept of steepness or slope might first be discussed in an already familiar context, such as the "pitch" of a roof or the "grade" of a road, with emphasis on the fact that the pitch, grade, or slope is determined by "rise over horizontal run." The slope of a line can be introduced by graphing functions such as $y = 4x$,

$y = 2x$, and $y = \frac{1}{2}x$ in the same plane and inviting students to compare the three lines. Such comparisons should make it difficult for them to avoid the notion that the constants 4, 2, and $\frac{1}{2}$ have something to do with the relative steepness of the lines.

Taking specific points on each of the lines $y = 4x$, $y = 2x$, and $y = \frac{1}{2}x$, a series of right triangles should be constructed by "walking away" from each point a varying horizontal distance and then "walking up" the required vertical distance to the line. Hopefully, the student will have done enough laboratory work on similarity (in Parts 3-5) to understand that all right triangles with hypotenuse on the same line and one leg horizontal are similar. Thus, successive computations of corresponding ratios will provide a simple illustration of the fact that the ratio of rise to horizontal run remains constant for the same line, and that the slope of each line is equal to the coefficient of x in $y = 4x$, $y = 2x$, and $y = \frac{1}{2}x$. Similar demonstrations with different examples can be used to show that the slope is positive if the line slants up to the right, is negative if the line slants down to the right, is nonexistent if the line is vertical, and is equal to the constant m in the equation $y = mx$.

In Part 7 the student should be introduced to the basic formulas for areas and perimeters of rectangles, triangles, parallelograms, and circles. He should also learn the formulas for surface areas and volumes of parallelepipeds, cylinders, and spheres. This, of course, provides an opportunity to do more work on approximations. The student will find this work more interesting if it is presented through useful examples such as the following:

Example 1: A man wants to cover his front yard with top soil before he plants his lawn. The yard is 100 ft. by 30 ft. and he wants to have the top soil 6 inches deep. How many cubic yards of top soil should he order?

Example 2: An oil discharge of fixed volume may spread over a very large area as the thickness of the slick decreases. Find an algebraic expression for the area covered by a given volume of oil in terms of the thickness of the slick.

Example 3: A certain noxious substance is discharged into a river as a by-product of a factory. Suppose the rate of flow of the stream in winter is at least 1,500,000 gallons per day and the rate of discharge of the pollutant is variable but may be as great as 2,000 gallons per day. If a concentration of 150 parts per million of the pollutant is the maximum that is considered "safe," what conclusion can be drawn as to the safety of the stream near the site of the factory? If the stream is safe at the factory site, how much additional pollutant can be discharged without exceeding a safe concentration? If the stream is unsafe at the factory, consider the situation downstream where additional inflow from tributaries may be thought of as diluting the concentration. How much inflow from tributaries will be needed before the augmented stream will be "safe"?

How does the situation change when the spring rains double the flow of the stream?

Some work on conversion of units, making use of mensuration formulas, should also be given--for example, the following:

<u>Exercise</u>: The base of a rectangular container of water is 2 ft. by 5 ft. Suppose that a metal ball 13 inches in diameter is dropped into the water and is completely submerged. How much will the level of the water rise?

The usefulness of similarity can be exemplified by pointing out that when a house is built, the contractors use plans to help them. These plans may be drawings or blueprints. In these the length of a line segment is much smaller than the actual length it represents. The ratio of the length of a line segment to the actual length it represents is called a scale. Maps are other scale drawings that students are familiar with. Many examples can be given of proportion using the idea of such scale drawings.

6. Remarks on the Material on Probability and Statistics

These two Parts can be considered as an opportunity to synthesize and apply many of the ideas of the preceding Parts. In addition, a case can be made for these topics as being among the most important mathematical concepts a citizen can acquire. He needs some knowledge of statistics in order to understand the news he hears on radio or television. Many of the problems facing the various governmental agencies in regard to taxes, welfare, education, and other public matters are comprehensible only to one who has some knowledge of statistics. Without an understanding of these problems, it is difficult, if not impossible, for a citizen to vote intelligently. The advertising of consumer goods is frequently couched in statistical or pseudo-statistical terms, and it has a direct bearing upon how one allocates one's income. In short, some knowledge of statistics seems important for everyone. Since the real payoff from a knowledge of statistics lies in using statistics to make decisions, and because most decisions of the sort considered daily are made in the presence of uncertainty, a basic grasp of elementary probability seems equally important.

What is proposed in Part 8 is a treatment, at a level accessible to the students to which the course is addressed, of the most fundamental notions of statistics. The main thrust of the Part should be the preparation of students to become intelligent consumers of statistical information rather than to be statisticians. Accordingly, they should be alerted to the pitfalls of interpretations based on some rather commonly encountered misuses of statistical information. Distortion of the scales of a statistical chart, faulty use of percentages, and poor sampling techniques are examples of causes of misinterpretations. An excellent and very readable book (for students, too) on this topic is How to Lie with Statistics by Darrell Huff and Irving Geis (New York, W. W. Norton and Company, Inc., 1954).

For example, one might pose the following question:

> Statistics show that in 1954 among fatal accidents due to automobiles, 25,930 occurred in clear weather, 370 in fog, 3,640 in rain, and 860 in snow. Do these statistics show that it is safest to drive in fog?

Students should become accustomed to analyzing and interpreting sets of numerical data. As a first step in this process, they need to be familiar with the construction and use of histograms, bar charts, line graphs, and pie diagrams. The World Almanac is a good source of material, e.g., census figures on U. S. population over a period of years, current distribution of population by age groups, personal income per capita, etc. Of course, daily newspapers and magazines contain other examples. The computation and use of measures of central tendency (mean, median, mode) and the use of percentile or other similar ranking indices should be presented with examples for actual hand-computation chosen so as to avoid an excessive amount of tedious labor and restricted to a list of numbers rather than grouped data. If a computer or a desk calculator is available, it would be possible in the laboratory to have interested students compute some of these measures using more meaningful data which they have collected or might be expected to encounter in vocational areas of particular interest to them.

Examples relevant to these matters might include discussions raised by questions such as:

> Is it safe for an adult who doesn't swim to step into a pool whose average depth is 4 feet?
>
> A newspaper reports that the average American family consists of 3.6 persons. What, if anything, does this mean? Is your family average?
>
> City A has average daily temperature of 75 degrees, as does city B. In city A, temperatures range from 10 to 99 degrees during the year. In city B, temperatures range from 60 to 80 degrees. For someone preferring moderate climate and hence considering moving to city A or city B, is there any reason to prefer one over the other and why?

An informal discussion of the normal distribution can be based upon the study of approximately bell-shaped histograms such as might be obtained, for example, from recording heights of a large number of college men. It may be observed that for such data the mean, median, and mode are approximately equal, so that phrases like "the average height is 69 inches" are not ambiguous. That this is not generally the case should also be noted. For instance, a distribution of family incomes would probably not have this property. The notion of standard deviation as a measure of dispersion and its relationship to the shape of distributions can be briefly touched upon--e.g., approximately 2/3 of the cases lie within one standard deviation of the mean, 95% within two standard deviations of the mean, and practically all are included within three standard deviations of the mean. In Part 9, after the notion of probability has been studied, it is possible to return to these properties in discussing problems of statistical inference.

An informal discussion of bias in sampling and the idea of a random sample might be enlivened by an experiment involving use of random number tables to select a random sample. A flowchart could be constructed to describe the steps in such a procedure.

The probability concepts proposed for Part 9 are simply those that bear upon the students' ability to make judgments in the presence of uncertainty. Little is proposed beyond introducing the general notions of the meaning of <u>a priori</u> and empirical probability, and developing an ability to understand the application of these notions in common-sense ways. These concepts are detailed in the outline, but their development should focus on the establishment of confidence in general impressions and interpretations rather than on the manipulation of formulas.

Empirical probability might be introduced by an example such as the American Experience Mortality Table. This table is based on birth and death records of 100,000 people alive at age 10. Entries give the number of these people living at various ages; for age 40 the entry 78,106 means that of the original group this many were still alive at age 40. The ratio 78,106/100,000 or .78 is called the empirical probability that a child of 10 will live to be 40. Students should be given some practice on the use of this table and caution on its misuse (e.g., probability .8 of living from age 10 to age 40 does not mean that in every group of ten individuals two will die before age 40).

The notion of <u>a priori</u> probability should be introduced from the point of view of an experiment performed with a set of objects in which certain outcomes are of interest. Some simple examples are: (1) A penny is tossed to see which of the outcomes, heads or tails, results. (2) One of several discs, each painted red or white, is drawn from a box and its color noted. (3) An individual is selected from a group of people and his opinion asked concerning a specific proposal of current interest to students.

It should, of course, be pointed out that experiments (1) and (2) are idealizations of certain processes having an intrinsic interest, such as the interaction of genes or the selection of a tax return for audit by the Internal Revenue Service.

It would be natural for these experiments to be performed in the laboratory with the results used to motivate the fundamental definitions. For example, in experiment (2), if it is known that the box has 50 white and 50 red discs, students will probably agree to assigning probability 1/2 for outcome "white disc." What if, instead, it is known that the box has 10 white and 90 red discs? What if the number of red and the number of white discs is unknown to the student? Experiments in the laboratory using boxes of discs with different proportions of the two colors might be undertaken with each student performing the experiment of drawing out a disc, then replacing it, say, 20 times, and recording the color of disc on each draw. A histogram of the relative frequencies collected by the class for outcome "white disc" illustrates the idea of variability and can lead to a discussion of how much confidence can be placed in information from a single, small sample. Combining the relative frequencies of groups of two students and then of five students, one can study the effect of sample size on variability. From an analysis of several histograms the class might be led to agree on a sample size when the composition of the box is unknown.

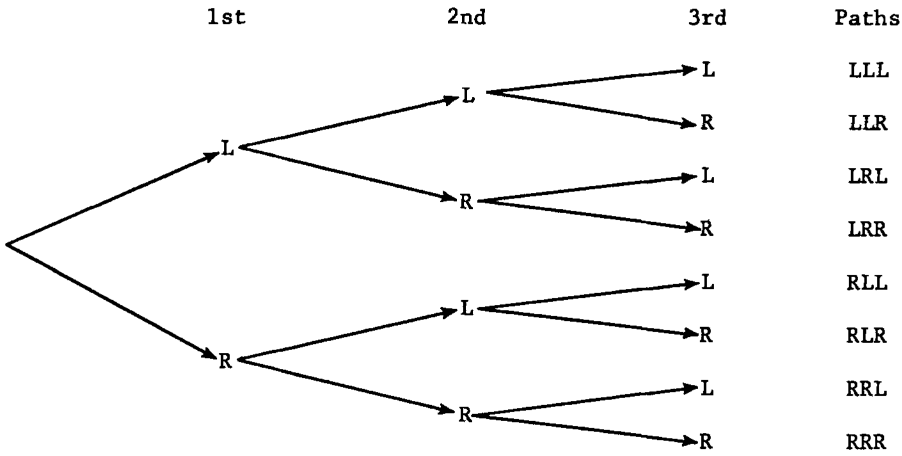
In experiment (3), if the percentage of the population in favor of the proposal is unknown and the problem is to estimate the unknown percentage, one should be careful to obtain a random selection from the population. For example, the percentage of the population in favor of building a new sports arena should not be estimated from those people who are leaving the old arena after a game. Estimation of the percentage of a population who favor a proposal is an example of statistical inference in which probability models are used to evaluate the likelihood with which assertions based on simple data are valid.

After some preliminary excursions into the computation of probabilities, it soon becomes apparent that a few guidelines to counting are needed. These should be based on the notion of first attempting a listing of (equally likely) outcomes. For problems involving a sequence of actions in succession--first do this, second that, etc.--the tree diagram provides a natural vehicle for presenting one of the so-called counting principles, the multiplicative one.

For example, in a psychology experiment a rat is placed at the entrance of a T-maze from which he runs either to the left arm, L, or the right arm, R.

(a) Suppose that the experiment is performed 3 times. List the possible paths of the rat on the three trials.

(b) If the food is always placed at L, how many of these paths could the rat take to receive the food at least two of the three times?



(a) See last column above; (b) 4: LLL, LLR, LRL, RLL.

With this basis for counting, the notion of the set of permutations of r objects chosen from n distinguishable objects, $n \geqq r$, can be pictured as an r-stage tree. Starting with many numerical examples, one arrives at the formula for the number $P(n,r)$. Simple discussion of factorials arising from the case of $P(n,r)$ is needed to facilitate arriving at the formula for the number of combinations (subsets) of r objects selected from a set of n objects. Students should compute a few factorials to see how fast they grow. A flow diagram for computing $n!$ might be developed. The formula for the number of combinations or subsets of r objects from a set of n should be derived after considering lists of permutations and of combinations in cases such as $n = 4$, $r = 2$; $n = 4$, $r = 3$. It should be stressed that this formula and the one for permutations are treacherous and should not be used until one has first ascertained (by an attempt at listing) that one of them is appropriate. Special combinatorial problems (e.g., circular permutations, permutations with repetition of symbols, etc.) should be avoided.

The formal treatment of probability theory should culminate in a discussion of the binomial distribution patterned after and illustrated by coin-tossing, die-throwing, etc. It is here that the convenience of the counting principles and formulas is noticeable. Some simple binomial tables should be available for student use in sampling (e.g., for $n = 10$, with various cases for p).

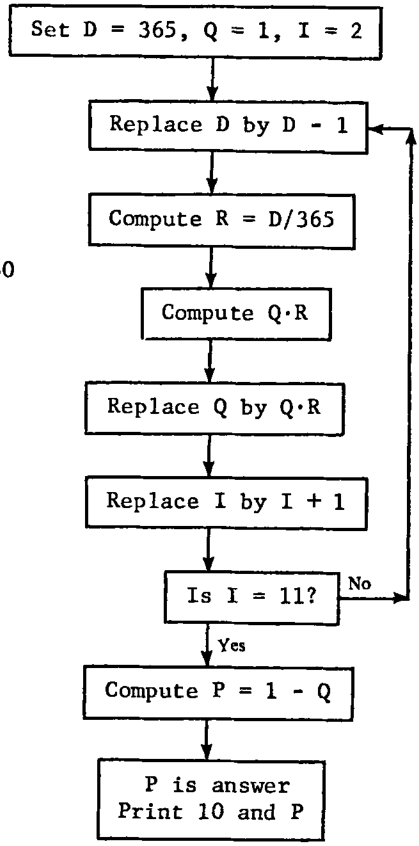
At this point let us consider an example to show the uses of the material of earlier chapters. (See Exercise 15 in Appendix I.)

Example: A famous probability example which interests beginning students is the so-called "birthday problem." The question is, "In a room containing N people, how large a number should N be so that the odds are 50-50 that at least two persons have the same birthday (month and day, not year)?" The answer, $N = 23$, defies the intuition of most people. The problem might be introduced as stated, various guesses given, and obvious extreme answers discarded. For example, for birthdays occurring in an ordinary year, if there are 366 people in the room, then it is certain that at least two have the same birthday. No doubt some will suggest that one half of 366 or, say, 180 is a good guess for the answer to the question, but the probability of coincidence of two or more birthdays for 180 people is still indistinguishable from 1! At this point the class can be encouraged to find the probability of coincidence in the case of small numbers N. For example, if $N = 2$, the number of possible (ordered) birthday pairs is $365 \cdot 365$, while the number of differing ordered pairs among these is $365 \cdot 364$, so that the required probability is $1 - \frac{365 \cdot 364}{365 \cdot 365}$ or $1 - 364/365 = 1/365$, indicating a small probability of two birthdays on the same day. Similarly, for $N = 3$ we find the probability of at least two birthdays falling on the same day to be $1 - \frac{365 \cdot 364 \cdot 363}{365 \cdot 365 \cdot 365}$, etc. It will be clear that the computation becomes prohibitive in a short time as one increases the size of N. Also, some remarks on approximating the fractions arising in such computations should tie in with previous discussions. Actual multiplication of numerator factors followed by division by the product of denominator factors would certainly lead to overflow on most computers. Writing, for example, the fractional part in the case $N = 3$ as $1 \cdot \frac{364}{365} \cdot \frac{363}{365}$ suggests a more hopeful procedure. A computer program (in BASIC language), with a corresponding flowchart, to compute and print out the values of this probability in the cases N = 10, 22, 23, 24, 25, 30, 40, 50, 60 is given below. Even with 60 people in the room, it is practically certain (probability .99) that at least two will have matching birthdays!

Birthday Problem BASIC Program

```
10  Read N
15  Let D = 365
20  Let Q = 1
25  For I = 2 to N
30  Let D = D - 1
35  Let R = D/365
40  Let Q = Q*R
45  Next I
50  Let P = 1 - Q
55  Print "N = "N; "P = "P
60  Go to 10
65  Data 10,22,23,24,25,30,40,50,60
70  End
```

Flowchart for case N = 10



The printout is given below:

```
N = 10      P = .116948
N = 22      P = .475695
N = 23      P = .507297
N = 24      P = .538344
N = 25      P = .5687
N = 30      P = .706316
N = 40      P = .891232
N = 50      P = .970374
N = 60      P = .994123
```

Now the student could be asked to consider just how significant the above 6-digit numbers are. What is the underlying assumption in the mathematical model? Is it true that each day of the year is equally likely for a birthday?

V. THE MATHEMATICS LABORATORY

The term "laboratory" will be used to indicate arrangements for teaching other than classroom instruction or undirected individual study. The word "laboratory" is also to mean a place (or places) in which would be located programmed materials, various audio-visual devices, books, and perhaps computer terminals. It might also include facilities for individual or group conferences. Laboratory facilities and organization may differ widely from school to school, and no prescriptive suggestions seem warranted.

Although we have not studied the role of mathematics laboratories in general, some form of a laboratory is an integral part of Mathematics E. Without a laboratory it is difficult to see how the flexibility necessary to deal with individual differences in preparation, ability, and goals can be achieved. Whatever form the laboratory may take, or whatever kinds of materials or devices are used, the total laboratory program must have three distinct goals:

(1) to correct deficiencies in preparation
(2) to make provision for individual goals
(3) to reinforce and extend the classroom instruction

We will discuss each of these three goals in turn.

1. The Remedial Goal

It is false to assume that any student knows all about mathematics up to a certain point in the standard curriculum, after which he knows nothing. Both the knowledge and the deficiencies of the students in the course will be scattered throughout their previous mathematical work, probably in small pieces. Small gaps in prerequisite knowledge and techniques can be diagnosed and treated in the laboratory.

The approach we have in mind is to determine as specifically as possible the individual deficiencies of a student and to prescribe as specifically as possible materials which the student can use individually to correct these deficiencies in advance of the time when these ideas and techniques are needed in the course.

From the outline one can see that Part 1 is essentially without prerequisites and that the prerequisites for Part 2 are minimal. Thus, while Part 1 is being taught in class the student can use the laboratory to master, by as many cycles of diagnosis and treatment as are necessary, the skills and concepts which are needed for Part 2. This leads in general to the idea that while one Part is being taught in class the student will be using the laboratory program to identify and correct any deficiencies he may have in the prerequisites for the next Part.

The problem of determining student needs for the remedial work of the laboratory is not trivial. Clearly, one requirement is the use of existing or teacher-constructed diagnostic tests. These tests must be highly discriminating and yet easily administered. They must pinpoint the student's deficiencies and indicate appropriate remedial activity.

Persons concerned with the design of the tests should also plan to minimize the degree of professional skill necessary to evaluate the results. Hopefully, the tests will be such that persons other than professional teachers can administer them and use the outcomes to assign appropriate remedial work.

Diagnostic tests should not constitute the only avenue for referring students to the laboratory for remedial work. If, on the basis of routine homework performance or class test results, it seems necessary for some students to repeat instruction in certain areas, instructors should have a means of referring these students to the laboratory for such reinforcement.

2. Provision for Individual Goals

Students will take (or be assigned to) this course for quite varied reasons. Beyond the mathematical literacy which is our fundamental purpose, some students who intend to continue in further mathematics courses will require more thorough grounding in or a broader coverage of fundamental algebraic skills. Others enrolled in technical or occupational curricula may need a further development of certain particular aspects of mathematics relevant to their curricula as well as applied problems directed specifically to their proposed major fields.

We imagine that corresponding to a given Part in the classroom presentation there might be several special-interest blocks composed of applications or illustrations of the same basic principles in different fields. For example, in connection with Part 8 on statistics, the prospective nurse might be studying statistics relative to public health, the prospective businessman the statistics of income and employment, and the prospective police administrator the statistics of crimes. Similarly, even such a straightforward subject as linear equations has applications to many subjects.

In addition to variant versions of an application, there may be need for optional blocks which endeavor to teach mathematical techniques not embodied in the main sequence of the course. For example, it is likely that a student interested in drafting might profit from some work in numerical trigonometry. We can also imagine that a student intending to take a course in chemistry might well profit from specific instruction in certain special techniques commonly used in chemistry courses. Finally, a student who does not possess any specific vocational objectives might even elect special-interest blocks in such subjects as number theory or more algebra.

We believe it is important that the student be allowed to choose freely from among the special-interest blocks that may appeal to him. Not only would this enhance the appeal of the laboratory material, but it might even be important to the student that he has been allowed to make some choice in order to adapt the course to his own goals as he sees them.

The availability of adaptation of the course to individual needs might well go far toward eliminating the demand for separate courses for special groups of students. In many local situations this flexibility might be good for the diplomatic relations between the mathematics faculty and the rest of the institution. In this connection, faculty members in other disciplines could well be invited to give advice or even aid in the construction of special-interest blocks for students interested in specializing in their fields.

An important class of students will be those who initially desire to use Mathematics E as preparation for Mathematics A or science or vocational courses having algebra as a prerequisite. We have previously remarked that the majority of students will not, in fact, go on. However, it would be desirable for this to be possible for those students with the requisite ability. Therefore, we suggest that at the beginning of the second semester a student who has such a goal and who is not heavily involved in the remedial aspect of this course could elect to devote the major portion of his laboratory time to studying programmed material on the techniques of algebra. In this way a course which will be terminal for the majority of students can become a nonterminal course for those willing and able to continue.

3. Reinforcement of Classroom Instruction

The laboratory may also be used to provide common extensions of the classroom work. As examples of such use of the laboratory we mention (a) the performance of certain geometric constructions by means of more effective devices than straightedge and compass, (b) computer programming and the use of computer terminals, (c) experiments in probability and statistics.

4. Management of a Mathematics Laboratory

The complexity of a mathematics laboratory may vary greatly, depending on the number of students at the level of Mathematics E which a given institution must serve. If only one or two sections of students are involved, procedures of diagnosis and prescription of suitable materials may reduce to conversation between the professor and the student. On the other hand, if several thousands of students and a large number of professors are involved in the basic mathematics program, the management of a laboratory may be a quite complex matter.

It may even be the case that some assistance from a computer is needed in order to keep records, to schedule conferences, and to give some feedback to the professor from the laboratory. If a laboratory reaches this degree of complexity, it would be natural for its services to be offered also to students whose mathematical deficiencies reveal themselves in other courses.

In some institutions the laboratory may be combined with similar endeavors in other disciplines and housed in a central learning resources center, thus relieving the mathematics faculty of the details of such an operation, much as a library relieves the mathematics faculty of the details of caring for collections of books. The difficulty with the latter arrangement is that the personnel of the laboratory would normally be expected to serve a tutorial function as well as performing the functions of clerks and librarians. Some colleges with well-developed laboratories have found students to be effective tutors and librarians provided they are properly directed and supervised. It is very important that professors have control over what the students are doing in the laboratory and that there is a method for professors to receive feedback from the laboratory.

There is naturally a question regarding the relative proportion of laboratory and classwork within the course. The answer would depend on local circumstances, and we have no desire to be prescriptive. The more strongly one believes in the importance of providing for individual differences and in the importance of insuring student activity, the greater the proportion of time one would tend to assign to the laboratory.

VI. QUALIFICATIONS FOR TEACHERS OF MATHEMATICS E

CUPM has considered in other reports the mathematical qualifications which it feels are necessary for the teaching of other mathematics courses in two- and four-year colleges. The present Panel feels that it would be unfortunate if teachers of Mathematics E were to constitute a subfaculty separate from those teaching other freshman and sophomore mathematics courses. Compared with other courses such as Mathematics A or Calculus, the teaching of the course would place somewhat smaller demands on depth in graduate mathematics training and somewhat greater demands on breadth in mathematics and related subjects. Thus, we do not see the need for a training program distinct from the usual one as preparation for the teaching of this course. However, teachers having little or no acquaintance with computing or statistics might require some small amount of additional training such as might be obtained at a summer institute or through some form of departmental or inservice training.

The teacher of Mathematics E must have certain attributes. Above all, he must be convinced of the value of the objectives of the course. In addition, he must have the ability to relate to a heterogeneous group of students. He must be willing to acquire an understanding of the applications of mathematics to the varied occupations to which these students may aspire; he must have sufficient understanding of computers, flowcharting, and estimation to weave these threads through the fabric of the course; he must have the versatility and experience in teaching needed to experiment with the laboratory to insure its success.

APPENDIX I

Sample Exercises

The following list of exercises was selected to demonstrate the existence of realistic problems that are likely to seem significant to the majority of students taking this course. The information given in the exercises is of the kind that would normally be encountered in the situation being described, and the questions asked in the problems are pertinent to the situation.

As indicated earlier in this recommendation, the exercises are not meant to convey any sort of desirable achievement level. In fact, the exercises given are generally more difficult than what might be termed an average exercise for the course. Most of the exercises can be reduced to a sequence of shorter and simpler problems. Some are appropriate for classroom discussion and perhaps would culminate in only a partial solution.

1. A builder quotes a prospective customer a price of $18 per square foot to build a certain style of house. The lot on which the house is to be built will cost an additional $4000. The customer knows that he can plan to spend at least $30,000, but no more than $40,000, on land and construction costs. Write an inequality whose solution will yield the range in which the size of the house must fall.

2. A grocery store advertises a peanut butter sale at a price of 2 jars for 85 cents. You notice that the net weight of peanut butter in each jar is 9 ounces. The same brand can be bought at 55 cents a jar containing 12 ounces of peanut butter. Should one buy the jars which are on sale?

3. Suppose you were required to take a 20 per cent cut in wages. If you are then given a 5 per cent increase in wages, does this mean that 1/4 of your cut has been restored?

4. What is wrong with the following procedure? To find the probability that an American citizen, chosen at random from a list of citizens, was born in a specified state, divide the number of favorable cases, 1, by the total number of states, 50, to obtain 1/50.

5. Two teams are playing a basketball game. A supporter of team A is willing to give you 3 to 1 odds and a supporter of team B will give you 2 to 1 odds, each betting on his favorite team. It is possible for you to bet x dollars with the first man and y dollars with the second and be \$10 ahead no matter which team wins. Write two equations involving x and y which express that fact.

6. A car salesman receives \$75 commission for each sale of one model of car and \$100 for each sale of another model. In a certain period of time he would like to receive a commission of \$3300. If he sold only cars of the first model, how many would he have to sell in order to earn the desired commission? Answer the same question for sales of only the second model of car. Set up an expression for the commission when cars of both models are sold. List additional items of information we would need in order to determine exactly how many cars of each model he must sell. Can he reach his goal exactly if he sells an odd number of the cars for which the commission is \$75? Explain. When three or four pairs of solutions have been found, what do you notice about the number of sales of the \$75 commission model? How many possible answers does the problem have?

7. A machinist is using a boring mill to rough-cut a collar for a steel shaft. In this process the speed of the lathe must be set carefully. Cutting too fast could burn the steel, while cutting too slowly is inefficient and produces a ragged edge. The speed in number of revolutions per minute (R) is given by $C = (\pi RD)/12$, where C is the best cutting speed for the materials being used in feet per minute and D is the diameter of the drill in inches. The value of π is approximately 3.14. If the drill is tool steel and the shaft is machine steel, D should be between 50 and 70 feet per minute. If the drill is 1 inch in diameter, what are the slowest and fastest rates of rotation at which the lathe should be set?

8. The current yearly gross salary of a state employee is \$7500. Each year he is given a raise equal to the rise in the cost of living. Each month 10% of his salary is withheld for federal income tax, 5% for his state income tax, 4.8% for social security tax, and \$9.50 for insurance. The rise in the cost of living during the current year is 6%. Write an equation whose solution will give his monthly take-home pay for the next year.

9. A housewife has a recipe for making brownies which calls for an 8" x 11" pan. By experience she knows that this will make two dozen servings. She is giving a big party and feels she needs 60 servings. She has two 8" x 11" pans and two 4" x 5" pans in the house. How many pans of which size should she use and how should she adjust her recipe? Note: This kind of problem gives an opportunity for some use of estimation. If the pans available have an area, say, 3.1 times that of the pan for which the recipe was written, the students should be made aware of the fact that tripling the recipe is good enough and that they shouldn't worry about the .1.

10. A risky operation used for patients with no other hope of survival has a survival rate of 80 per cent. What is the probability that at least four of the next five patients operated on will survive.

11. Assume that you are considering the purchase of a piece of land which costs $40,000. If a highway being planned passes through the land, the land will be worth $100,000. If, instead, the highway goes through nearby, the land will be worth $20,000. The probability that the highway will pass through the land is estimated to be .30. Evaluate the investment in the light of the probabilities given.

12. The following (with only trivial changes) is taken from the 1958 Boston and Maine Time Table:

Miles			A.M.
0	Dole Junction	Lv.	8:15
3	Hinsdale	Lv.	8:30
6	Ashuelot	Lv.	8:45
8	Winchester	Lv.	9:10
14	Wesport	Lv.	9:25
16	West Swanzey	Lv.	9:35
18	Swanzey	Lv.	9:45
21	Keene	Ar.	10:00

Observe that this trip of 21 miles requires an hour and three quarters and then check that this means that the average speed is 12 miles per hour. However, prove that an engine whose maximum speed is 20 miles an hour could not have made this trip on schedule.

13. Use data from the latest <u>World Almanac</u> giving U. S. population (official census) at 10-year intervals from 1880 to 1970 to make a line graph showing the population growth in the U. S. over this period. On the vertical scale start at 0 even though the population figures start at 50 million. In plotting points round off each population figure to the nearest million and choose scales of years on the horizontal axis in such a way as to make your picture approximately square.

Now make another line graph of the same data starting your vertical scale at 50 million and making the extent of your vertical scale about 4 inches while your horizontal scale is such that it extends across the width of your paper.

Compare the effect of the two graphs in portraying population growth.

14. A typical credit agreement reads:

Within 30 days after the billing date shown on each such monthly statement, Holder agrees to pay (1) the outstanding indebtedness ("New Balance") for "Purchases"; or (2) an installment of not less than 1/20th of such New Balance or $10, whichever is greater, and in addition a Finance Charge on the previous month's New Balance less "Payments and Credits" at the following rates: 1½% per month on so much of such amount as does not exceed $500; 1% per month on the excess of such amount over $500, or if the Finance Charge so computed is less than 50¢, a minimum Finance Charge of 50¢.

(Construct a flowchart for computing monthly payments. Do this for a $750 and a $100 purchase.)

15. Suppose that a person observing a carnival man in a game involving two tosses of a coin suspects the manner of tossing these coins favors both coins landing the same way, i.e., both heads or both tails. Such an outcome is unfavorable to the player. His decision on whether or not to play the game is based on the following rule: on the next throw of the two coins, if both show the same face he will not play; otherwise he will. Express the probability P that this person will play in terms of p, the probability with which the carnival man tosses a head. Determine P for $p = 0, .1, .25, .5, .75, 1$ and interpret the results.

16. One section of the Tax Reform Act of 1969 reads as follows:

"Low Income Allowance--

(1) The low income allowance is an amount equal to the sum of--

(A) the basic allowance, and
(B) the additional allowance.

(2) Basic Allowance--the basic allowance is an amount equal to the sum of--

(A) $200, plus
(B) $100, multiplied by the number of exemptions.

The basic allowance shall not exceed $1,000.

(3) Additional Allowance--

(A) the additional allowance is an amount equal to the excess (if any) of \$900 over the sum of--

(i) \$100, multiplied by the number of exemptions, plus
(ii) the income phase-out.

(B) Income Phase-out--The income phase-out is an amount equal to one-half of the amount by which the adjusted gross income for the taxable year exceeds the sum of--

(i) \$1,100 plus
(ii) \$625, multiplied by the number of exemptions."

The preceding statement offers a wealth of possibilities for problem construction including several possibilities for flowcharting problems.

(a) Write an equation that gives the basic allowance for a taxpayer with four exemptions.

(b) Write an equation that gives the basic allowance for a taxpayer with n exemptions.

(c) The statement limits the basic allowance to \$1000. Write an inequality whose solution will yield the maximum number of exemptions allowable for computing the basic allowance.

(d) Write an equation that gives the additional allowance when the income phase-out is \$400.

(e) Suppose the adjusted gross income for the taxable year is \$4400. Write an equation that gives the income phase-out. (See Appendix II.)

17. The following is the procedure for computing social security benefits:

(1) Determine the "number of years" figure. If you were born before 1930, start with 1956. If born after 1929, start with the year you reached 27. Using your appropriate starting year, count that year and each one thereafter up to but not including the year in which you will be 65 if a man, or 62 if a woman.

(2) List the amount of taxed earnings for all years beginning with 1951. List no more per year than the amount subject to social security tax. These amounts have been: \$3600 for 1951 through 1954; \$4200 for 1955 through 1958; \$4800 for 1959 through 1965; \$6600 for 1966 and 1967; and \$7800 for 1968 and succeeding years.

(3) Cross off this list your lowest earnings until the number remaining is equal to your "number of years" figure.

(4) Using the reduced list, add up all the earnings that are left, and divide by your number of years. This gives "average earnings," a figure which can be used with the aid of a table to determine the social security monthly benefit.

Construct a flowchart that describes the above process. (See Appendix II.)

18. Suppose you wish to buy a new car and you have determined that you do not want to pay more than \$130 per month for 30 months. If the interest is 5% per year on the original amount borrowed, then what price car should you consider? Approximately how many monthly payments are used to pay the interest? If you now decide that you might pay up to \$150 per month for 30 months, can you quickly estimate the price of a car you can consider?

19. The federal tax on capital gains in the stock market is roughly determined as follows: If you hold the stock for more than six months before selling it (a long-term gain), only half your profit is taxed; if you hold your stock six months or less before you sell it (a short-term gain), then your entire profit is fully taxed. For example, suppose you are in the 40% tax bracket and you have a stock which cost you \$1000. If you sell this stock for \$2000 within six months after you bought it, then your tax is 40% x \$1000, or \$400, so that your profit is \$600. If you sell this stock for \$2000 after holding it for more than six months, then your tax is 20% x \$1000, or \$200, so that your profit is \$800. Thus, long-term capital gain is clearly better than an equal amount of short-term gain.

Now suppose that your stock has shown a gain of \$1000 (paper profit), but six months has not lapsed since you purchased it and you fear that the stock will decline in price. Can you afford to risk losing part of your gain while you wait for it to become a long-term gain? Can you determine how much of your profit you can afford to lose and still be as well off after tax because of the lower tax on long-term gain?

If you think about it, you can show that the answer is

$$\frac{40\% - 20\%}{100\% - 20\%} = \frac{1}{4};$$

i.e., you can afford to lose 1/4 of your paper profit to wait for six months to pass and still have the same profit after tax. So a short-term gain of \$1000 is equivalent to a long-term gain of \$750 after tax. If you are in the 30% bracket, then what fraction of the paper profit can you afford to lose while waiting for six months to pass and still have the same profit after tax.

Examples of Artificial Word Problems

The exercises given as examples in this section are listed to demonstrate the kind of problem that is not in keeping with the spirit of the course. These problems are not necessarily considered to be innately "bad." In fact, some of the problems could justifiably be included in the course. However, the problems are obviously artificial and, hence, may fail to stimulate the pragmatically oriented students.

1. John is three times as old as Mary was five years ago. In twelve years he will be exactly twice as old as Mary. How old is Mary?

2. George and Frank can paint a house in 5 days by working together. Frank could paint the house in 8 days by working alone. How long would it take George to paint the house by working alone?

3. The tens digit of a 2-digit number is twice the units digit. The number is 20 more than the units digit. Find the number.

4. John has twice as many quarters as dimes and seven more nickels than dimes. If he has $2.30 in all, how many nickels does he have?

5. A certain room is 6 feet longer than it is wide. If the perimeter of the room is 82 feet, what is its width?

6. Compute three consecutive integers whose sum is 54.

7. Bob leaves city A at noon and drives 60 mph toward city B. Bill leaves city A at 1:30 p.m. and drives at 65 mph toward city B. At what time will Bill overtake Bob?

8. Frank bought thirteen pounds of meat for $10.55. He paid $1.10 per pound for beef and $.55 per pound for pork. How many pounds of each kind of meat did he buy?

APPENDIX II

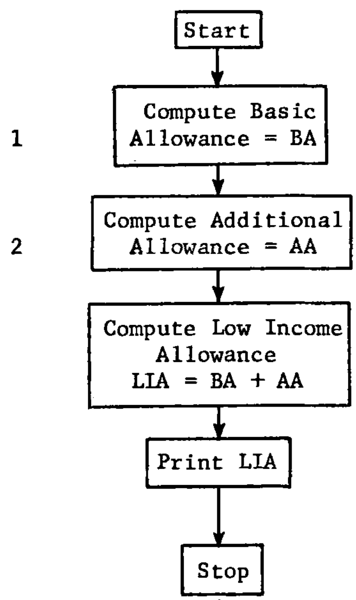
This Appendix contains analyses for exercises 15, 16, and 17.

15. This example, or one like it, might be used to introduce the notion of the uses of probability in decision making. Suppose the man can toss each coin so that for each, independently, p is the probability of heads. (If he tosses each fairly, $p = \frac{1}{2}$.) Then the probability of a head-tail combination is $P = 2p(1-p)$; this is

the probability that the observer will decide to play the game. Of course, p is unknown and can take on any value between 0 and 1. A graph of this quadratic, which students have made before (in the form $y = 2x(1 - x)$, probably), reveals that the maximum P is $\frac{1}{2}$, occurring when $p = \frac{1}{2}$ and the carnival man is tossing the pennies fairly. In the extreme cases of $p = 0$ or 1 (meaning what on the part of the carnival man?), we have $P = 0$, i.e., the observer will never play. Some inbetween values of p are of interest; should the carnival man have the finesse to insure always a $\frac{1}{4}$ chance of heads for each coin, $P = 3/8$--i.e., about $37\frac{1}{2}$ per cent of the time the observer engages in the game. The value $p = .1$ would result in $P = .18$, or less than 1 chance in 5 of the observer's succumbing. The main point is that the computation of P in terms of p, or the graph of P as a function of p, has provided the observer with some (probability) basis of evaluating the consequences of his rule of action. In this case, the decision rule, although based on a minimum amount of data, is a reasonable one.

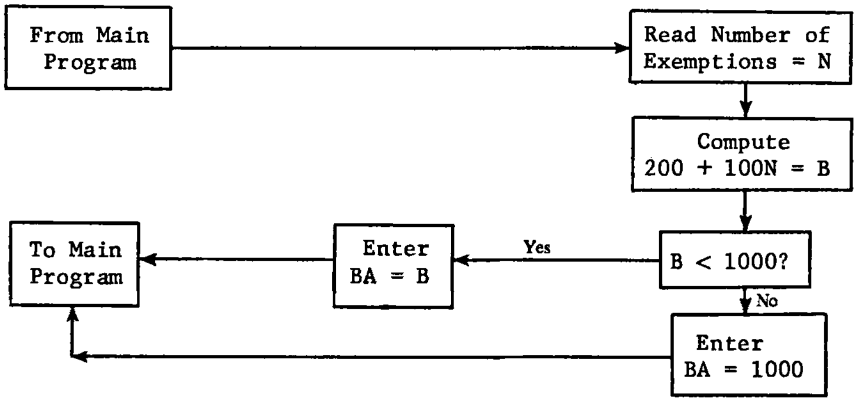
The flowcharts for Exercises 16 and 17 are especially illustrative of how flowcharting may be used to break large problems into sequences of smaller ones. Flowcharts for those two problems are given below:

(A) Flowchart for Exercise 16:

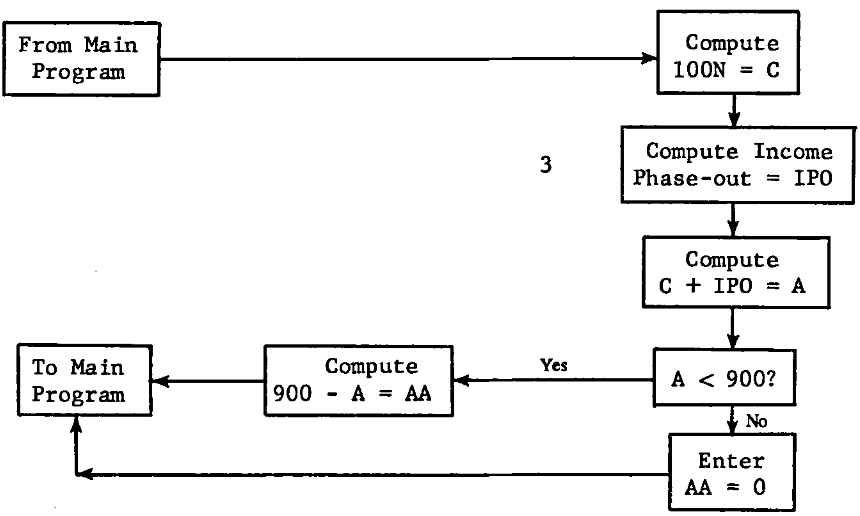


See next pages for expanded flowcharts for boxes 1 and 2.

Expansion of Box 1: Computing Basic Allowance

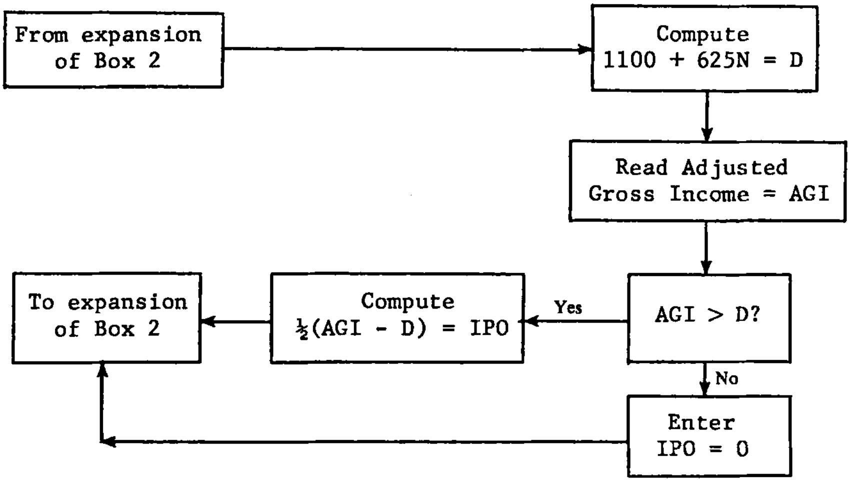


Expansion of Box 2: Computing Additional Allowance



See next page for expanded flowchart for Box 3.

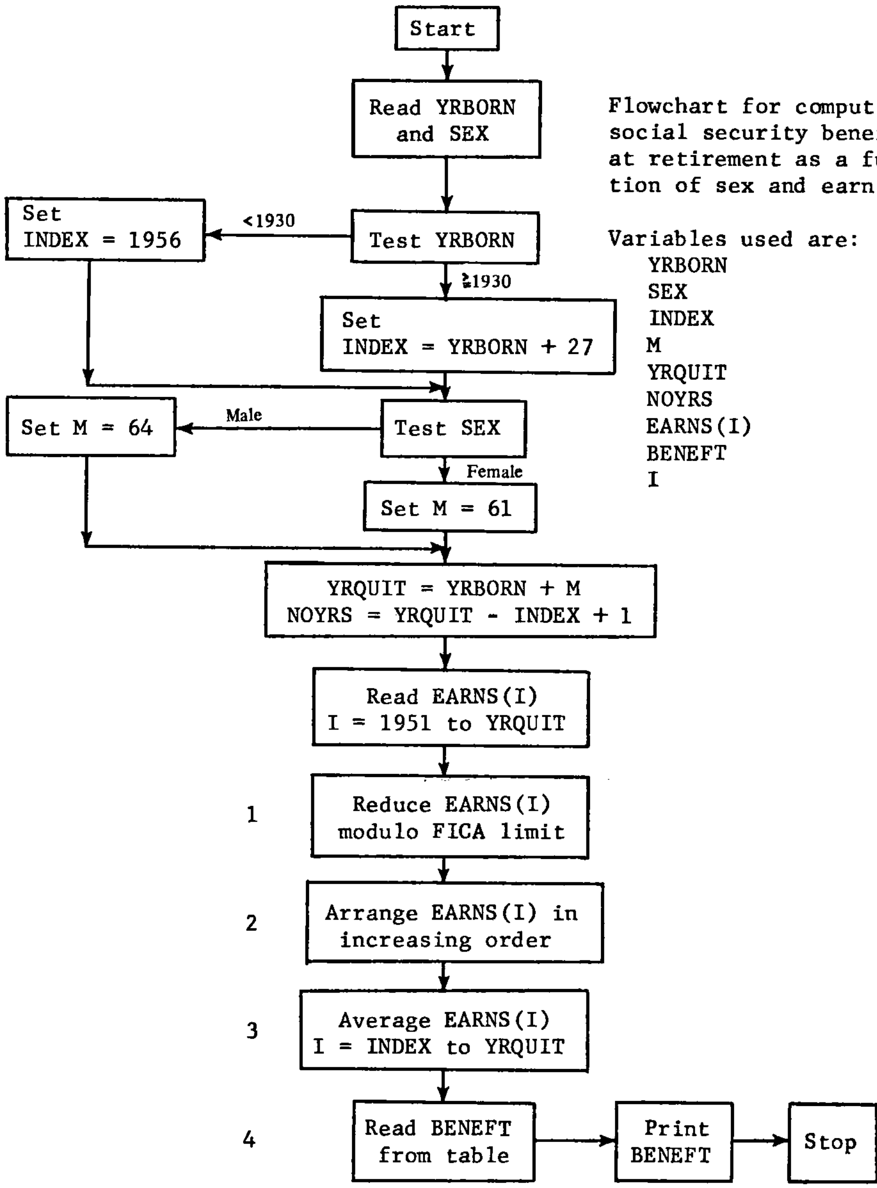
Expansion of Box 3: Computing Income Phase-out



(B) Flowchart for Exercise 17:

The table referred to in Part 4 of Exercise 17 is quite long. A portion of the table for a retired worker who has reached 65 is given below:

Average Earnings	Benefit (Per Month)
\$899 or less	\$55.00
\$900	\$70.00
\$1800	\$88.40
\$3000	\$115.00
\$4200	\$140.40
\$5400	\$165.00
\$6600	\$189.90
\$7800 or more	\$218.00

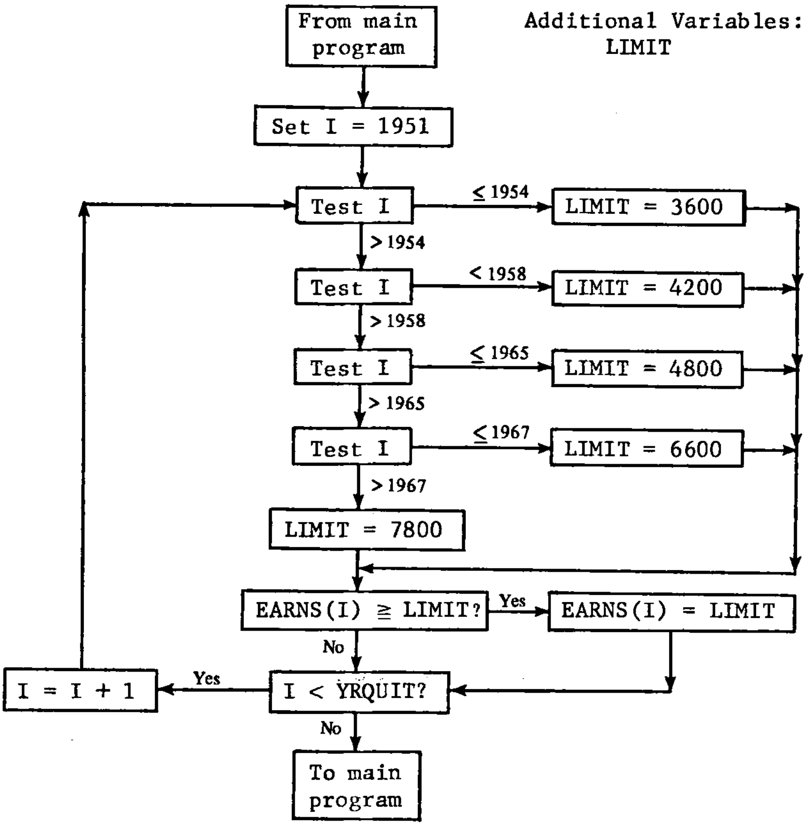


Flowchart for computing social security benefits at retirement as a function of sex and earnings

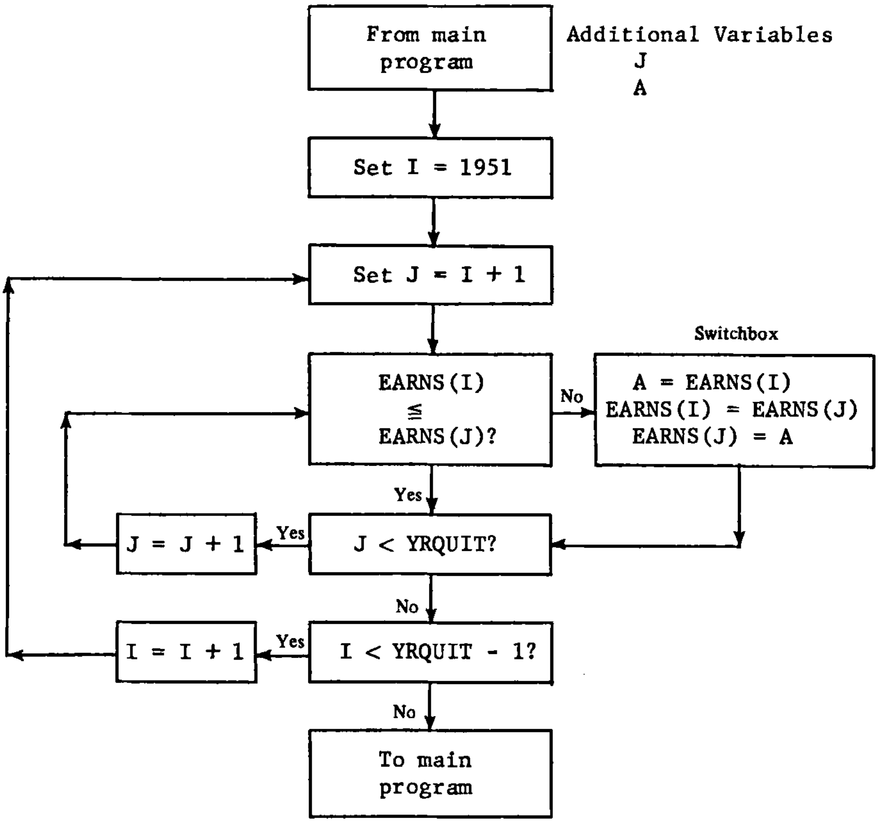
Variables used are:
YRBORN
SEX
INDEX
M
YRQUIT
NOYRS
EARNS(I)
BENEFT
I

See other pages for expanded flowcharts for Boxes 1, 2, 3 and a discussion of Box 4.

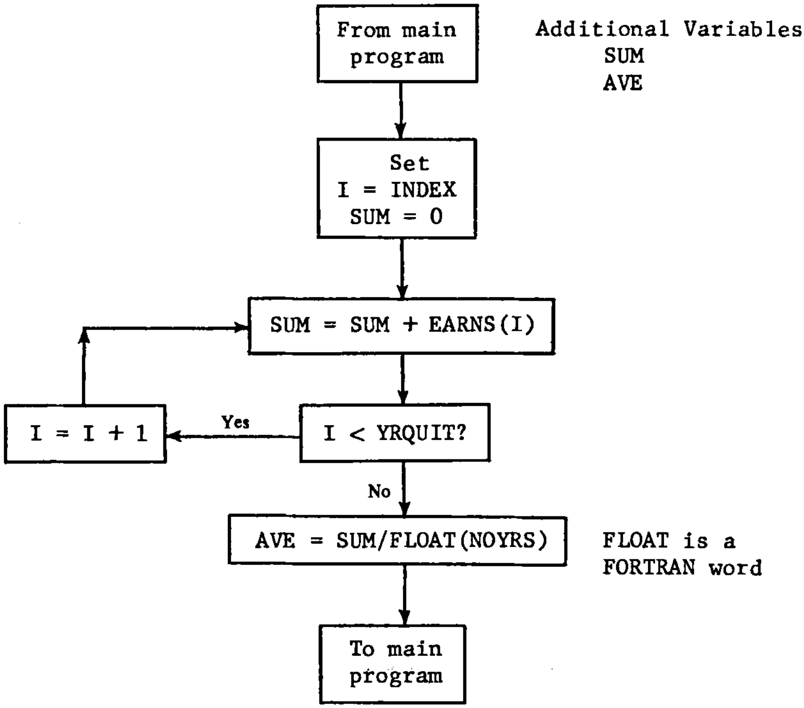
Expanded flowchart for Box 1--reducing actual earnings modulo the FICA limit for each year from 1951 to retirement



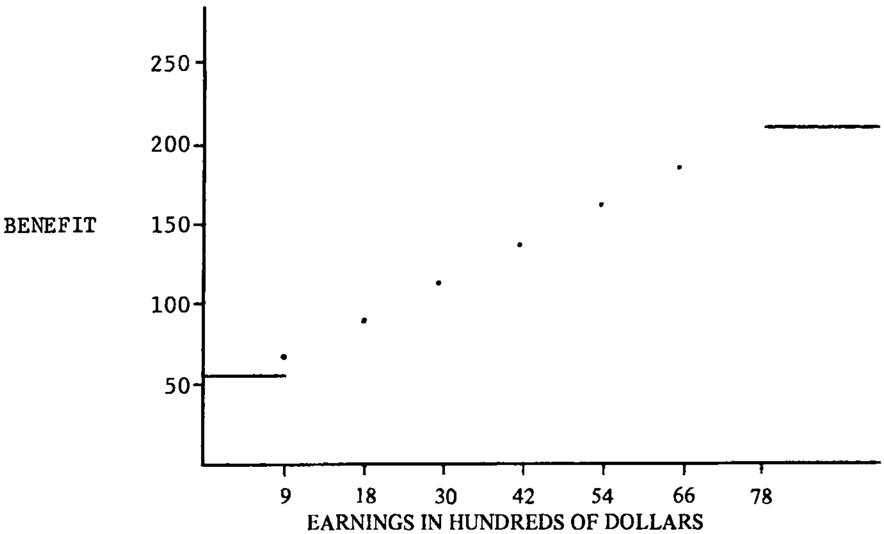
Expanded flowchart for Box 2--arranging FICA earnings in increasing order of magnitude from 1951 to retirement



Expanded flowchart for Box 3--computing average FICA earnings from index year to retirement



Box 4 of the flowchart offers several pedagogical opportunities. First, the student should plot a graph of benefit vs. earnings from the table on page 307. The graph might look like this:



Note that there is a jump of \$15 in benefit between earnings of \$899 and \$900.

Now, several approaches may be used for approximating the benefit associated with earnings between \$900 and \$7800. We list three possibilities that are all related to the fact that the isolated points on the graph seem to lie on a line.

1. The table could be stored in the computer memory and a program provided for interpolating linearly between adjacent points.

2. The student could select a line which actually passes through any two of the isolated points of the graph and determine its equation. There are several ways of doing this. For instance, the line on the points (3000, 115) and (5400, 165) has equation

$$\text{BENEFIT} = 115 + \frac{1}{48}(\text{EARNINGS} - 3000).$$

   This is a "good" line in the sense that it nearly contains the other points of the table, except for the first, the approximation of benefit being within \$2 of the table value in every case.

3. One could use a least-squares fit on some points of the graph. For instance, the equation

$$\text{BENEFIT} = \frac{253}{12000} \times \text{EARNINGS} + \frac{46071}{900}$$

fits the five points with abscissas 1800, 3000, 4200, 5400, and 6600 in this sense. This is a "good" line too. It yields benefits corresponding to tabular values of earnings that are within $1 of tabular benefits in all cases.

The table given in this example was for a man retiring at age 65. A class project could be to find similar information for a woman retiring at age 62 and to complete the program.

A BASIC LIBRARY LIST

FOR

TWO-YEAR COLLEGES

January 1971

TABLE OF CONTENTS

INTRODUCTION

The Basic Library List, [page 1], published by CUPM in 1965, was intended to define a minimal college mathematics library. More recently an <u>ad hoc</u> committee assisted by two-year college, four-year college, and university teachers prepared the present basic library list for two-year colleges. The aims of this list are quite similar to those of the Basic Library List, namely:

1. To provide the student with introductory material in areas of mathematics new to him

2. To provide the interested student with material collateral to the material he is studying in courses

3. To provide the student with material somewhat more advanced than he is likely to encounter in his course work

4. To provide the faculty with reference material, but generally below graduate level

5. To provide the general reader with elementary material in the field of mathematics

6. To provide trainees in various occupations, such as nurses, farmers, technologists, etc., with material designed for their particular needs

A further word concerning item 4 is in order. It is recognized that many faculty members at two-year colleges are still engaged in graduate study; however, it is felt that it is not the responsibility of the two-year college library to provide them reference material for their graduate courses. The reason for this is two-fold: first, such material should be available to them at the institution where they are pursuing graduate work; second, inclusion of such material in a two-year college library might place too heavy a financial strain on the two-year college.

The list is intended as a basic list from which the library can expand according to the needs and interests of the faculty and the students. Needs at different schools will, of course, differ, and the library should reflect the local needs; in this regard see the comment under Sections 6 through 11. There has been a concerted effort to keep the list small; one means of doing this has been to combine under one heading books of a somewhat different character. Alternate choices are listed so that a library can utilize its present holdings to the full. In the interest of keeping the list small, many books of merit have had to be omitted; it is also possible that, despite assiduous searching, the committee has overlooked books which should have been included. Furthermore, books which have been included in the list have been included because of their value as <u>library</u> books; no judgment is made as to their utility as texts for courses.

Some books are mentioned at more than one place in the list. This is not accidental. Since some schools will want to purchase only those portions pertinent to their programs, the committee wanted to be sure that relevant books were covered in each section.

The matter of library books in various remedial areas, i.e., arithmetic, elementary algebra, and the like, has been discussed at length by the committee. It is clear that these subjects are taught at the two-year college level, and that the character of the texts used there varies considerably from those used at lower levels. Despite this, we feel that for reference use by students the two-year college is well advised to include among its books those texts used by the local high schools or texts covering comparable material.

The library committee worked on this list during a two-year period ending in 1970; therefore, books with first publication dates after 1969 are generally _not_ included in the list. Finally, it must be recognized that the list covers a considerable range of sophistication beginning at quite an elementary level. The exposition in some of the more elementary books differs from the sort of presentation one expects at more advanced levels in being more discursive and less axiomatic. The mathematics may occasionally appear not to be in the best tradition of formal practice; however, these books fill a very real need for the audience intended and any solecisms encountered are not so serious as to remove the books from consideration.

After preliminary versions of this list were written, the _ad hoc_ committee sought the advice and comments of some 30 reviewers. The reviewers were chosen so that specialists in each of the areas represented in the list would be able to comment. The list thus reflects not only the competencies of the committee but also the informed views of the reviewers.

These recommendations contain about 510 volumes, of which approximately 170 volumes are to be chosen; this does not include journals or series in Sections 22 and 23.

The symbol * indicates that the book has been listed more than once.

1. HISTORICAL, GENERAL, AND RECREATIONAL

History--Both of the following:

1.1 Bell, Eric T. _Men of Mathematics_. New York, Simon and Schuster, Inc., 1961.

1.2 Boyer, Carl B. A History of Mathematics. New York, John Wiley and Sons, Inc., 1968.

And at least one of the following:

1.3a Eves, Howard. Introduction to the History of Mathematics, 3rd ed. New York, Holt, Rinehart and Winston, Inc., 1969.

1.3b Smith, D. E. History of Mathematics, 2 vols. New York, Dover Publications, Inc. Vol. I, General Survey of the History of Elementary Mathematics; Vol.II, Special Topics of Elementary Mathematics.

1.3c Struik, D. J., ed. A Source Book in Mathematics: Twelve Hundred to Eighteen Hundred. Cambridge, Massachusetts, Harvard University Press, 1969.

1.3d van der Waerden, B. L. Science Awakening. New York, Oxford University Press, 1961; New York, John Wiley and Sons, Inc., paper.

General--All of the following:

1.4 Courant, Richard and Robbins, Herbert. What is Mathematics? New York, Oxford University Press, 1941.

1.5 Eves, Howard and Newsom, Carroll V. Introduction to the Foundations and Fundamental Concepts of Mathematics, rev. ed. New York, Holt, Rinehart and Winston, Inc., 1965.

1.6 Klein, Felix. Elementary Mathematics from an Advanced Standpoint. Vol. 1, Arithmetic, Algebra, Analysis. New York, Dover Publications, Inc., 1968.

1.7 National Council of Teachers of Mathematics. Enrichment Mathematics for the Grades (27th Yearbook) and Enrichment Mathematics for the High Schools (28th Yearbook). Washington, D. C., National Council of Teachers of Mathematics, 1963.

1.8 Rademacher, Hans and Toeplitz, Otto. The Enjoyment of Mathematics: Selections from Mathematics for the Amateur. Princeton, New Jersey, Princeton University Press, 1965.

1.9 Sawyer, Walter W. Mathematician's Delight. Baltimore, Maryland, Penguin Books, Inc., 1943.

1.10 Steinhaus, Hugo. Mathematical Snapshots, 2nd ed. New York, Oxford University Press, 1969.

And at least two of the following:

1.11a Cadwell, James H. Topics in Recreational Mathematics. New York, Cambridge University Press, 1966.

1.11b Court, Nathan A. Mathematics in Fun and in Earnest. New York, Mentor Press, 1961. Out of print.

1.11c Kac, Mark and Ulam, Stanislaw M. Mathematics and Logic: Retrospect and Prospects. New York, Frederick A. Praeger, Inc., 1968.

1.11d Kasner, Edward and Newman, James R. Mathematics and the Imagination. New York, Simon and Schuster, Inc., 1940.

1.11e Lockwood, Edward H. and Prag, A. A Book of Curves. New York, Cambridge University Press, 1961.

1.11f Ogilvy, C. Stanley. Tomorrow's Math: Unsolved Problems for the Amateur. New York, Oxford University Press, 1962.

1.11g Pedoe, Daniel. Gentle Art of Mathematics. New York, The Macmillan Company, 1963; Baltimore, Maryland, Penguin Books, Inc., 1969, paper.

1.11h Sawyer, Walter W. Prelude to Mathematics. Baltimore, Maryland, Penguin Books, Inc., 1955.

1.11i Stein, Sherman K. Mathematics: The Man-Made Universe, An Introduction to the Spirit of Mathematics, 2nd ed. San Francisco, California, W. H. Freeman and Company, 1969.

And one of the following:

1.12a Kline, Morris. Mathematics in Western Culture. New York, Oxford University Press, 1964.

1.12b Scientific American Editors. Mathematics in the Modern World. San Francisco, California, W. H. Freeman and Company, 1968.

Mathematical Recreations--At least one of the following:

1.13a Ball, W. W. R. and Coxeter, H. S. M. Mathematical Recreations and Essays, rev. ed. New York, The Macmillan Company, 1962.

1.13b Kraitchik, Maurice. Mathematical Recreations, 2nd ed. New York, Dover Publications, Inc., 1953.

And at least one of the following (problems and puzzles):

1.14a Bakst, Aaron. Mathematical Puzzles and Pastimes, 2nd ed. New York, Van Nostrand Reinhold Company, 1965.

1.14b Gamow, George and Stern, Marvin. Puzzle-Math. New York, Viking Press, Inc., 1958.

1.14c Gardner, Martin, ed. Scientific American Book of Mathematical Puzzles and Diversions. New York, Simon and Schuster, Inc., 1964.

1.14d Gardner, Martin, ed. Second Scientific American Book of Mathematical Puzzles and Diversions. New York, Simon and Schuster, Inc., 1965.

1.14e Gardner, Martin. Unexpected Hanging and Other Mathematical Diversions. New York, Simon and Schuster, Inc., 1968.

1.14f Graham, Lloyd A. Ingenious Mathematical Problems and Methods. New York, Dover Publications, Inc., 1959.

1.14g Graham, Lloyd A. Surprise Attack in Mathematical Problems. New York, Dover Publications, Inc., 1968.

1.14h Mott-Smith, Geoffrey. Mathematical Puzzles for Beginners and Enthusiasts, 2nd ed. New York, Dover Publications, Inc., 1954

1.14i Phillips, Hubert C. My Best Puzzles in Mathematics. New York, Dover Publications, Inc., 1961.

Various Topics (about mathematics and mathematicians)--All of the following:

1.15 Committee on Support of Research in the Mathematical Sciences. The Mathematical Sciences: A Collection of Essays. Cambridge, Massachusetts, MIT Press, 1969.

1.16 Cundy, Henry M. and Rollett, A. P. Mathematical Models, 2nd ed. New York, Oxford University Press, 1961.

1.17 Hadamard, Jacques. Psychology of Invention in the Mathematical Field. New York, Dover Publications, Inc., 1945.

1.18 Hardy, G. H. Mathematician's Apology, rev. ed. New York, Cambridge University Press, 1967.

1.19 Newman, James R. The World of Mathematics, 4 vols. New York, Simon and Schuster, 1962. Vol. I, Men and Numbers; Vol. II, World of Laws and the World of Chance; Vol. III, Mathematical Way of Thinking; Vol. IV, Machines, Music and Puzzles.

1.20 Pólya, Gyorgy. How to Solve It, 2nd ed. New York, Doubleday and Company, Inc., 1957.

1.21 Pólya, Gyorgy. Mathematical Discovery on Understanding, Learning and Teaching Problem Solving, 2 vols. New York, John Wiley and Sons, Inc., 1962.

Sets and Collections of Books

1.22 New Mathematical Library, 22 vols. New York, Random House/ Singer School Division.

Numbers: Rational and Irrational (NML 1). Ivan Niven

*What is Calculus About? (NML 2). W. W. Sawyer

An Introduction to Inequalities (NML 3). E. Beckenbach and R. Bellman

*Geometric Inequalities (NML 4). Nicholas D. Kazarinoff

The Lore of Large Numbers (NML 6). P. J. Davis

Uses of Infinity (NML 7). Leo Zippin

Geometric Transformations (NML 8). I. M. Yaglom, translated by Allen Shields

Continued Fractions (NML 9). Carl D. Olds

*Graphs and Their Uses (NML 10). Oystein Ore

Hungarian Problem Book I and II (NML 11 and 12). Translated by E. Rapaport

Episodes from the Early History of Mathematics (NML 13). A. Aaboe

Groups and Their Graphs (NML 14). I. Grossman, et al.

The Mathematics of Choice (NML 15). Ivan Niven

From Pythagoras to Einstein (NML 16). K. O. Friedrichs

The MAA Problem Book II (NML 17).

*First Concepts of Topology (NML 18). W. G. Chinn and N. E. Steenrod.

Geometry Revisited (NML 19). H. S. M. Coxeter and S. L. Greitzer.

Invitation to Number Theory (NML 20). Oystein Ore

Geometric Transformations II (NML 21). I. M. Yaglom, translated by Allen Shields

Elementary Cryptanalysis--A Mathematical Approach (NML 22). Abraham Sinkov

2. FINITE MATHEMATICS

Although Finite Mathematics is not well defined, it is generally understood to encompass modern problems in elementary set theory, logic, probability, linear programming, and theory of games solved by methods not involving the calculus. In the following list, all the books deal with these topics.

At least two of the following:

2.1a Crouch, Ralph B. Finite Mathematics and Statistics for Business. New York, McGraw-Hill Book Company, 1968.

*2.1b Kaye, Norman J. Elementary Quantitative Techniques for Business Problem Solving. Belmont, California, Dickenson Publishing Company, Inc., 1969.

2.1c Kemeny, John G., et al. Finite Mathematics with Business Applications. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1962.

*2.1d Kemeny, John G., et al. Introduction to Finite Mathematics, 2nd ed. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1967.

2.1e Marcus, Marvin. A Survey of Finite Mathematics. Boston, Massachusetts, Houghton Mifflin Company, 1969.

2.1f Richardson, William H. Finite Mathematics. New York, Harper and Row, Publishers, 1968.

2.1g Wheeler, Ruric E. and Peeples, W. D. Modern Mathematics for Business Students. Belmont, California, Brooks/Cole Publishing Company, 1969.

3. PREPARATION FOR CALCULUS

The following list is at the level of Mathematics 0 [page 75] or Mathematics A [page 216]. It is intended to provide reference material for courses leading to the calculus but does not include programmed materials or books for remedial work.

At least two of the following:

3.1a Dolciani, Mary P., et al. Modern Introductory Analysis. Boston, Massachusetts, Houghton Mifflin Company, 1970.

3.1b Golightly, Jacob F. Precalculus Mathematics--Algebra and Trigonometry. Philadelphia, Pennsylvania, W. B. Saunders Company, 1968.

3.1c Horner, Donald R. Precalculus: Elementary Functions and Relations. New York, Holt, Rinehart and Winston, Inc., 1969.

3.1d Hu, Sze-Tsen. Elementary Functions and Coordinate Geometry. Chicago, Illinois, Markham Publishing Company, 1969.

3.1e Knight, Ronald A. and Hoff, William E. Introduction to the Elementary Functions. Belmont, California, Dickenson Publishing Company, Inc., 1969.

3.1f Marcus, Marvin and Minc, Henryk. Elementary Functions and Coordinate Geometry. Boston, Massachusetts, Houghton Mifflin Company, 1969.

At least two of the following:

3.2a Allendoerfer, Carl B. and Oakley, Cletus O. Principles of Mathematics, 3rd ed. New York, McGraw-Hill Book Company, 1969.

3.2b Good, R. A. Introduction to Mathematics. New York, Harcourt Brace Jovanovitch, Inc., 1966.

3.2c Haag, Vincent H. and Western, Donald W. Introduction to College Mathematics, 2nd ed. New York, Harcourt Brace Jovanovitch, Inc., 1968.

3.2d Meserve, Bruce E., et al. Principles of Advanced Mathematics, rev. ed. New York, Random House/Singer School Division, 1970.

3.2e Pownall, Malcolm W. A Prelude to the Calculus. New York, McGraw-Hill Book Company, 1967.

3.2f Shanks, Merrill E., et al. Pre-Calculus Mathematics, 2nd ed. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1968.

3.2g Rosenbloom, Paul C. and Schuster, Seymour. Prelude to Analysis. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1966.

3.2h Zwier, Paul J. and Nyoff, Larry R. Essentials of College Mathematics. New York, Holt, Rinehart and Winston, Inc., 1969.

4. CALCULUS

General Calculus. There are many good calculus books available. Several of these should be in the library. The following represent some of the various possible approaches.

4.1a Bers, Lipman. Calculus. New York, Holt, Rinehart and Winston, Inc., 1969.

4.1b Crowell, Richard H. and Slesnick, William E. Calculus with Analytic Geometry. New York, W. W. Norton and Company, Inc., 1968.

4.1c de Leeuw, Karel. Calculus. New York, Harcourt Brace Jovanovitch, Inc., 1966.

4.1d Johnson, Richard E. and Kiokemeister, F. L. Calculus with Analytic Geometry, 4th ed. Boston, Massachusetts, Allyn and Bacon, Inc., 1964.

4.1e Protter, Murray H. and Morrey, Charles B., Jr. Calculus with Analytic Geometry: A First Course, 2nd ed. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1963.

4.1f Sherwood, George E. and Taylor, Angus E. Calculus, 3rd ed. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1954.

4.1g Thomas, George B., Jr. Calculus and Analytic Geometry, 4th ed. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1968.

Honors Calculus--One or more of the following:

4.2a Apostol, Tom M. Calculus, 2 vols. Waltham, Massachusetts, Blaisdell Publishing Company. Vol. I, One-Variable Calculus with an Introduction to Linear Algebra, 2nd ed., 1967; Vol. II, Multi-Variable Calculus and Linear Algebra, with Applications to Differential Equations and Probability, 2nd ed., 1969.

4.2b Courant, Richard. Differential and Integral Calculus, 2 vols. (translated by E. J. McShane) New York, John Wiley and Sons, Inc. Vol. I, 2nd ed., 1937; Vol. II, 1936.

4.2c Hardy, G. H. A Course of Pure Mathematics. New York, Cambridge University Press, 1959.

4.2d Spivak, Michael. Calculus. New York, The Benjamin Company, Inc., 1967.

Background--At least one of the following:

4.3a Boyer, Carl B. History of the Calculus and Its Conceptual Development. New York, Dover Publications, Inc., 1959.

*4.3b Khinchin, Alexander Y. Eight Lectures on Mathematical Analysis. Lexington, Massachusetts, D. C. Heath and Company, 1965.

*4.3c Sawyer, W. W. What is Calculus About? New York, Random House, Inc., 1961.

4.3d Selected Papers on Calculus, Tom Apostol, editor. Belmont, California, Dickenson Publishing Company, Inc., 1969.

4.3e Toeplitz, Otto. Calculus: A Genetic Approach. (edited by G. Köthe, translated by L. Lange) Chicago, Illinois, University of Chicago Press, 1963.

See also 1.22.

Calculus of Several Variables--At least one of the following:

4.4a Fadell, Albert G. Vector Calculus and Differential Equations, vol. III. New York, Van Nostrand Reinhold Company, 1968.

4.4b Osserman, Robert. Two-Dimensional Calculus. New York, Harcourt Brace Jovanovitch, Inc., 1968.

4.4c Williamson, Richard, et al. Calculus of Vector Functions. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1968.

Advanced Calculus--At least one of the following:

4.5a Apostol, Tom M. Mathematical Analysis: A Modern Approach to Advanced Calculus. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1957.

4.5b Buck, R. Creighton. Advanced Calculus, 2nd ed. New York, McGraw-Hill Book Company, 1965.

4.5c Kaplan, Wilfred. Advanced Calculus. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1952.

4.5d Kreider, Donald L., et al. Introduction to Linear Analysis. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1966.

4.5e Taylor, Angus E. Advanced Calculus. Waltham, Massachusetts, Blaisdell Publishing Company, 1955.

5. STATISTICS AND PROBABILITY

General--At least one of the following:

5.1a Huff, Darrell and Geis, Irving. How to Lie with Statistics. New York, W. W. Norton and Company, Inc., 1954.

5.1b Levinson, Horace C. Chance, Luck and Statistics, 2nd ed. New York, Dover Publications, Inc., 1963.

5.1c Moroney, M. J. Facts From Figures. Baltimore, Maryland, Penguin Books, Inc., 1956.

Elementary Statistics--At least one of the following:

5.2a Blackwell, David. Basic Statistics. New York, McGraw-Hill Book Company, 1969.

5.2b Dixon, Wilfrid J. and Massey, F. J., Jr. Introduction to Statistical Analysis, 3rd ed. New York, McGraw-Hill Book Company, 1969.

5.2c Freund, John E. Modern Elementary Statistics, 3rd ed. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1967.

5.2d Hodges, Joseph L. and Lehmann, E. L. Basic Concepts of Probability and Statistics, 2nd ed. San Francisco, California, Holden-Day, Inc., 1970.

5.2e Hoel, Paul G. Elementary Statistics, 2nd ed. New York, John Wiley and Sons, Inc., 1966.

5.2f Mode, Elmer B. Elements of Probability and Statistics, 3rd ed. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1966.

5.2g Mosteller, Frederick, et al. Probability with Statistical Applications, 2nd ed. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1970.

5.2h Wallis, Wilson A. and Roberts, Harry V. Statistics: A New Approach. New York, The Macmillan Company, 1956.

5.2i Wolf, Frank L. Elements of Probability and Statistics. New York, McGraw-Hill Book Company, 1962.

Mathematical Statistics--At least one of the following:

5.3a Brunk, H. D. Introduction to Mathematical Statistics, 2nd ed. Waltham, Massachusetts, Blaisdell Publishing Company, 1965.

5.3b Hoel, Paul G. Introduction to Mathematical Statistics, 3rd ed. New York, John Wiley and Sons, Inc., 1970.

5.3c Hogg, Robert V. and Craig, A. T. Introduction to Mathematical Statistics, 3rd ed. New York, The Macmillan Company, 1970.

5.3d Meyer, Paul L. Introductory Probability and Statistical Applications, 2nd ed. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1970.

5.3e Mood, Alexander M. and Graybill, F. A. Introduction to the Theory of Statistics, 2nd ed. New York, McGraw-Hill Book Company, 1963.

Elementary Probability--At least one of the following:

5.4a Berman, Simeon M. Elements of Probability. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1969.

5.4b Gangolli, R. A. and Ylvisaker, Donald. Discrete Probability. New York, Harcourt Brace Jovanovitch, Inc., 1967.

5.4c Gnedenko, Boris V. and Khinchin, Alexander Y. Elementary Introduction to the Theory of Probability, 5th ed. (translated by Leo F. Boron) New York, Dover Publications, Inc., 1961.

5.4d Goldberg, Samuel. Probability: An Introduction. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1960.

5.4e Scheerer, Anne C. Probability on Discrete Sample Spaces with Applications. Scranton, Pennsylvania, Intext Educational Publishers, 1969.

5.4f Thompson, W. A., Jr. Applied Probability. New York, Holt, Rinehart and Winston, Inc., 1969.

Intermediate Probability

5.5 Feller, William. Introduction to Probability Theory and Its Applications, vol. I, 3rd ed. New York, John Wiley and Sons, Inc., 1968.

And at least one of the following:

5.6a Breiman, Leo. Probability and Stochastic Processes, with a View Towards Applications. Boston, Massachusetts, Houghton Mifflin Company, 1969.

5.6b Parzen, Emanuel. Modern Probability Theory and Its Applications. New York, John Wiley and Sons, Inc., 1960.

5.6c Rozanov, Y. A. Introductory Probability Theory. (translated by M. Silverman) Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1969.

Other Approaches--At least one of the following:

5.7a Chernoff, Herman and Moses, L. E. Elementary Decision Theory. New York, John Wiley and Sons, Inc., 1959.

5.7b Kraft, Charles H. and van Eeden, Constance. Nonparametric Introduction to Statistics. New York, The Macmillan Company, 1968.

5.7c Savage, I. Richard. Statistics: Uncertainty and Behavior. Boston, Massachusetts, Houghton Mifflin Company, 1968.

Applied Statistics--At least one of the following:

5.8a Chorafas, Dimitris N. Statistical Processes and Reliability Engineering. New York, Van Nostrand Reinhold Company. Out of print.

5.8b Cochran, W. G. and Cox, G. M. Experimental Designs, 2nd ed. New York, John Wiley and Sons, Inc., 1957.

5.8c Grant, Eugene L. Statistical Quality Control, 3rd ed. New York, McGraw-Hill Book Company, 1964.

5.8d Hays, William L. Statistics for Psychologists. New York, Holt, Rinehart and Winston, Inc., 1963.

5.8e Mainland, Donald. Elementary Medical Statistics, 2nd ed. Philadelphia, Pennsylvania, W. B. Saunders Company, 1963. Out of print.

5.8f Schefler, William C. Statistics for the Biological Sciences. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1969.

5.8g Schlaifer, Robert. Introduction to Statistics for Business Decisions. New York, McGraw-Hill Book Company, 1961.

5.8h Wasserman, W. and Neter, J. Fundamental Statistics for Business and Economics. Boston, Massachusetts, Allyn and Bacon, Inc., 1966.

5.8i Wine, Russell L. Statistics for Scientists and Engineers. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1964.

5.8j Yates, Frank. Sampling Methods for Censuses and Surveys, 3rd ed. New York, Hafner Publishing Company, Inc., 1960.

Tables--At least one of the following:

*5.9a Burington, Richard S. and May, Donald C., Jr. Handbook of Probability and Statistics with Tables, 2nd ed. New York, McGraw-Hill Book Company, 1969.

*5.9b Chemical Rubber Company. Handbook of Tables for Probability and Statistics, 2nd ed. Cleveland, Ohio, The Chemical Rubber Company, 1968.

5.9c Owen, Donald B. Handbook of Statistical Tables. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1962.

6. VOCATIONAL MATHEMATICS

The books under this heading are specific to a particular program offered in various two-year colleges. If that program is not offered, there may be no need for the library to purchase these books.

The listings are reference materials for general shop courses. The books in 6.2 are somewhat more general than the ones in 6.3. For further books that may be useful under this heading, see also those listed in Section 8--Technology.

6.1 Grazda, Edward E., _et al_. _Handbook of Applied Mathematics_, 4th ed. New York, Van Nostrand Reinhold Company, 1966.

At least one of the following:

6.2a Levine, Samuel. _Vocational and Technical Mathematics in Action_. New York, Hayden Book Company, 1969.

6.2b Slade, Samuel and Margolis, L. _Mathematics for Technical and Vocational Schools_, 5th ed. New York, John Wiley and Sons, Inc., 1968.

And at least one of the following:

6.3a McMackin, Frank J. and Shaver, John H. _Mathematics of the Shops_, 3rd ed. New York, Van Nostrand Reinhold Company, 1968.

6.3b Wolfe, J. H. and Phelps, E. R. _Practical Shop Mathematics_, 2 vols., 4th ed. New York, McGraw-Hill Book Company. Vol. 1, Elementary, 1959; Vol. 2, Advanced, 1960.

7. BUSINESS

The books under this heading are specific to a particular program offered in various two-year colleges. If that program is not offered, there may be no need for the library to purchase these books.

At least two of the following:

7.1a Bush, Grace A. and Young, John E. _Foundations of Mathematics with Applications to the Social and Management Sciences_. New York, McGraw-Hill Book Company, 1968.

*7.1b Kaye, Norman J. _Elementary Quantitative Techniques for Business Problem Solving_. Belmont, California, Dickenson Publishing Company, Inc., 1969.

7.1c Locke, Flora M. and Dehr, D. _College Mathematics for Business_. New York, John Wiley and Sons, Inc., 1969.

7.1d Roueche, Nelda W. Business Mathematics: A Collegiate Approach. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1969.

7.1e Snyder, Llewellyn R. Essential Business Mathematics, 5th ed. New York, McGraw-Hill Book Company, 1967.

Mathematics of Finance--At least two of the following:

7.2a Cissell, Robert and Cissell, Helen. Mathematics of Finance, 3rd ed. Boston, Massachusetts, Houghton Mifflin Company, 1969.

7.2b Curtis, Arthur B. and Cooper, J. (Revised by W. McCallion) Mathematics of Accounting, 4th ed. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1961.

7.2c Freund, John E. College Mathematics with Business Applications. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1969.

7.2d Hart, William L. Mathematics of Investment, 4th ed. Lexington, Massachusetts, D. C. Heath and Company, 1958.

7.2e Rider, P. R. and Fisher, C. H. Mathematics of Investment. Ann Arbor, Michigan, Ulrich's Books, Inc., 1951.

7.2f Rosenberg, R. Robert. College Mathematics, 4th ed. New York, McGraw-Hill Book Company, 1967.

Mathematics of Management--At least one of the following:

7.3a Corcoran, A. Wayne. Mathematical Applications in Accounting. New York, Harcourt Brace Jovanovitch, Inc., 1968.

7.3b Dean, Burton V., et al. Mathematics for Modern Management. New York, John Wiley and Sons, Inc., 1963.

7.3c Goetz, Billy E. Quantitative Methods: A Survey and Guide for Managers. New York, McGraw-Hill Book Company, 1965.

7.3d Springer, Clifford H., et al. Mathematics for Management Sciences. Homewood, Illinois, Richard D. Irwin, Inc. Vol. I, Basic Mathematics, 1965; Vol. II, Advanced Methods and Models, 1965; Vol. III, Statistical Inference, 1966; Vol. IV, Probabilistic Models, 1968.

7.3e Stern, Mark E. Mathematics for Management. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1963.

Every library should have the following:

7.4 Minrath, William R. Handbook of Business Mathematics, 2nd ed. New York, Van Nostrand Reinhold Company, 1967.

8. TECHNOLOGY

The books under this heading are specific to a particular program offered in various two-year colleges. If that program is not offered, there may be no need for the library to purchase these books.

Engineering Technology--At least one of the following:

8.1a Blakeley, Walter R. Calculus for Engineering Technology. New York, John Wiley and Sons, Inc., 1968.

8.1b Placek, Ronald J. Technical Mathematics with Calculus. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1968.

8.1c Rice, Harold S. and Knight, Raymond M. Technical Mathematics with Calculus, 2nd ed. New York, McGraw-Hill Book Company, 1966.

8.1d Washington, Allyn J. Basic Technical Mathematics with Calculus, 2nd ed. Menlo Park, California, The Cummings Publishing Company, 1970.

Electronics and Electricity--At least two of the following:

8.2a Adams, Lovincy J. and Journigan, R. P. Applied Mathematics for Electronics. New York, Holt, Rinehart and Winston, Inc., 1967.

8.2b Barker, Forrest I. and Wheeler, Gershon J. Mathematics for Electronics. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1968.

8.2c Herrick, Clyde N. Mathematics for Electronics. Columbus, Ohio, Charles E. Merrill Publishing Company, 1967.

8.2d Korneff, Theodore. Introduction to Electronics. New York, Academic Press, Inc., 1966.

8.2e National Radio Institute Staff. Mathematics for Electronics and Electricity. New York, Holt, Rinehart and Winston, Inc. Out of print.

8.2f Nunz, Gregory J. and Shaw, William L. Electronics Mathematics, 2 vols. New York, McGraw-Hill Book Company, 1967. Vol. 1, Arithmetic and Algebra; Vol. 2, Algebra, Trigonometry and Calculus.

8.2g Singer, Bertrand B. Basic Mathematics for Electricity and Electronics, 2nd ed. New York, McGraw-Hill Book Company, 1965.

8.2h Westlake, John H. and Noden, Gordon E. Applied Mathematics for Electronics. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1968.

8.2i Zelinger, G. Basic Matrix Analysis and Synthesis. Elmsford, New York, Pergamon Press, Inc., 1966.

Chemical Technology--One of the following:

8.3a Bard, Allen J. Chemical Equilibrium. New York, Harper and Row, Publishers, 1966.

8.3b Freiser, Henry and Fernando, Quintus. Ionic Equilibria in Analytic Chemistry. New York, John Wiley and Sons, Inc., 1963.

8.3c Margolis, Emil J. Chemical Principles in Calculations of Ionic Equilibria. New York, The Macmillan Company, 1966.

8.3d Robbins, Omer, Jr. Ionic Reactions and Equilibria. New York, The Macmillan Company, 1967.

One of the following:

8.4a Andersen, Laird B. and Wenzel, L. A. Introduction to Chemical Engineering. New York, McGraw-Hill Book Company, 1961.

8.4b Anderson, H. V. Chemical Calculations. New York, McGraw-Hill Book Company, 1955.

8.4c Hamilton, L. F., et al. Calculations of Analytic Chemistry, 7th ed. New York, McGraw-Hill Book Company, 1969.

8.4d Nyman, Carl J. and King, George B. Problems for General Chemistry and Qualitative Analysis. New York, John Wiley and Sons, Inc., 1966.

8.4e Peters, M. S. Elementary Chemical Engineering. New York, McGraw-Hill Book Company, 1954.

Health Sciences--At least one of the following:

8.5a Asperheim, Mary K. Pharmacology for Practical Nurses, 2nd ed. Philadelphia, Pennsylvania, W. B. Saunders Company, 1967.

8.5b Lipsey, Sally Irene. Mathematics for Nursing Science. New York, John Wiley and Sons, Inc., 1965.

8.5c Sackheim, George I. Programmed Mathematics for Nurses, 2nd ed. New York, The Macmillan Company, 1961.

8.5d Sisson, Harriet E. Applied Pharmaceutical Calculations. Minneapolis, Minnesota, Burgess Publishing Company, 1966.

Other Technologies

Standard reference books and handbooks in other specialized technologies and vocations, e.g., mechanical engineering, agricultural engineering, etc., should be in the library. Since most of these books deal more with the specific field than with the mathematics involved in that field, it is felt that the choice of such books should be left to those intimately involved in the field, rather than to members of the mathematics staff.

9. DATA PROCESSING

The books under this heading are specific to a particular program offered in various two-year colleges. If that program is not offered, there may be no need for the library to purchase these books.

Overview--At least two of the following:

9.1a Allen, Paul. Exploring the Computer. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1967.

9.1b Boore, William F. and Murphy, G. R. Computer Sampler: Management Perspectives on the Computer. New York, McGraw-Hill Book Company, 1968.

9.1c Davis, Gordon B. Computer Data Processing. New York, McGraw-Hill Book Company, 1969.

9.1d Moursund, David G. How Computers Do It. Belmont, California, Wadsworth Publishing Company, Inc., 1969.

9.1e Sanders, Donald H. Computers in Business: An Introduction. New York, McGraw-Hill Book Company, 1968.

9.1f Swanson, Robert W. Introduction to Business Data Processing and Computer Programming. Belmont, California, Dickenson Publishing Company, Inc., 1967.

9.1g Wheeler, Gershon J. and Jones, Donlan F. Business Data Processing: An Introduction. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1966.

9.1h Withington, Frederick G. Use of Computers in Business Organizations. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1966.

Unit Record Operations--At least one of the following:

9.2a Claffey, William J. Principles of Data Processing. Belmont, California, Dickenson Publishing Company, Inc., 1967.

9.2b Levy, Joseph. Punched Card Data Processing. New York, McGraw-Hill Book Company, 1967.

9.2c Micallef, Benjamin A. Electronic Accounting Machine Fundamentals. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1968.

9.2d Salmon, Lawrence J. IBM Machine Operation and Wiring, 2nd ed. Belmont, California, Wadsworth Publishing Company, Inc., 1966.

Assembly Language--At least one of the following:

9.3a Cashman, Thomas J. and Shelly, Gary B. IBM System/360 Assembler Language. Fullerton, California, Anaheim Publishing Company, 1969.

9.3b Chapin, Ned. 360 Programming in Assembly Language. New York, McGraw-Hill Book Company, 1968.

9.3c Computer Usage Company. Programming the IBM System-360. New York, John Wiley and Sons, Inc., 1966.

9.3d Golden, James T. and Leichus, Richard M. IBM 360 Programming and Computing. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1967.

9.3e Struble, George L. Assembler Language Programming: The IBM System-360. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1969.

COBOL--At least one of the following:

9.4a Raun, Donald L. Introduction to COBOL Computer Programming for Accounting and Business Analysis. Belmont, California, Dickenson Publishing Company, Inc., 1966.

9.4b Wendel, Thomas M. and Williams, William H. Introduction to Data Processing and COBOL. New York, McGraw-Hill Book Company, 1969.

10. COMPUTING--PROGRAMMING LANGUAGES

The books under this heading are specific to a particular program offered in various two-year colleges. If that program is not offered, there may be no need for the library to purchase these books.

It is assumed that one or more books will be obtained concerning the particular computing systems which are available to the institution. Consequently, no books have been listed which apply to a particular system. This is not meant to indicate that no machines are referred to in the listed books, but rather that the book would not be listed if its applications were only to a particular system. While books are listed for the more widely used programming languages, it is presumed that primary attention would be given to books concerning those languages used within the institution. It should be noted that additional books on computing are listed in Section 9; in particular, books on assembler languages and COBOL are listed therein.

Introductory--At least one of the following:

10.1a Forsythe, A. I., et al. Computer Science: A First Course. New York, John Wiley and Sons, Inc., 1969.

10.1b Hull, Thomas E. and Day, D. D. F. Introduction to Computers and Problem Solving. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1969.

And at least one of the following:

10.2a Arden, Bruce W. Introduction to Digital Computing. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1963.

10.2b Galler, Bernard A. Language of Computers. New York, McGraw-Hill Book Company, 1962.

10.2c Rice, John K. and Rice, John R. Introduction to Computer Science: Problems, Algorithms, Languages, Information and Computers. New York, Holt, Rinehart and Winston, Inc., 1969.

Digital Computing--General References--At least one of the following:

10.3a Bartee, Thomas C. Digital Computer Fundamentals, 2nd ed. New York, McGraw-Hill Book Company, 1966.

10.3b Conway, Richard W., et al. Theory of Scheduling. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1967.

10.3c Desmonde, William H. Computers and Their Uses. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1964.

10.3d Klerer, Melvin and Korn, Granino A. Digital Computer User's Handbook. New York, McGraw-Hill Book Company, 1967.

10.3e Maisel, Herbert. Introduction to Electronic Digital Computers. New York, McGraw-Hill Book Company, 1969.

10.3f Schriber, Thomas J. Fundamentals of Flowcharting. New York, John Wiley and Sons, Inc., 1969.

Programming Languages--At least one for each language available in the institution.

FORTRAN IV

10.4a Dimitry, Donald L. and Mott, Thomas H., Jr. Introduction to FORTRAN IV Programming. New York, Holt, Rinehart and Winston, Inc., 1966.

10.4b McCammon, Mary. Understanding FORTRAN. New York, Thomas Y. Crowell Company, 1968.

10.4c McCracken, Daniel D. Guide to FORTRAN Programming. New York, John Wiley and Sons, Inc., 1961.

10.4d Organick, Elliot I. FORTRAN IV Primer. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1966.

10.4e Rule, Wilfred P. FORTRAN IV Programming. Boston, Massachusetts, Prindle, Weber and Schmidt, Inc., 1968.

PL/I

10.5a Bates, Frank and Douglas, Mary L. Programming Language: One. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1967.

10.5b Lecht, Charles Philip. The Programmer's PL/I: A Complete Reference. New York, McGraw-Hill Book Company, 1968.

10.5c Pollack, S. V. and Sterling, T. D. Guide to PL-I. New York, Holt, Rinehart and Winston, Inc., 1969.

10.5d Sprowls, R. Clay. Introduction to PL/I Programming. New York, Harper and Row, Publishers, 1969.

10.5e Weinberg, Gerald M. PL/I Programming Primer. New York, McGraw-Hill Book Company, 1966.

BASIC

10.6 Kemeny, John G. and Kurtz, T. E. Basic Programming. New York, John Wiley and Sons, Inc., 1967.

COBOL--See 9.4

Analog and Hybrid Computing--At least one of the following:

10.7a Johnson, Clarence L. Analog Computer Techniques, 2nd ed. New York, McGraw-Hill Book Company, 1963.

10.7b Korn, Granino A. and Korn, Theresa M. Electronic Analog and Hybrid Computers. New York, McGraw-Hill Book Company, 1964.

10.7c Stice, James E. and Swanson, Bernet S. Electronic Analog Computer Primer. Waltham, Massachusetts, Blaisdell Publishing Company, 1965.

11. TEACHING

The books under this heading are specific to a particular program offered in various two-year colleges. If that program is not offered, there may be no need for the library to purchase these books.

The chief function of the two-year college with reference to teacher training seems to be the providing of the subject matter foundations for future teachers. Library books for this purpose are found elsewhere in this publication. It is only at the elementary school teacher level that there appears to be a call for special courses and related library books in the junior college. The following list suggests a few books dealing with pedagogy at both the elementary and secondary levels for the use of teachers who might use the college's facilities or students who are interested in teaching careers.

Elementary School Teacher Preparation in Mathematics

At least one of the following:

11.1a Brumfiel, Charles F. and Krause, Eugene F. Elementary Mathematics for Teachers. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1969.

11.1b Fehr, Howard F. and Hill, Thomas J. Contemporary Mathematics for Elementary Teachers. Lexington, Massachusetts, D. C. Heath and Company, 1966.

11.1c Garstens, Helen L. and Jackson, Stanley B. Mathematics for Elementary School Teachers. New York, The Macmillan Company, 1967.

*11.1d School Mathematics Study Group. Studies in Mathematics. Vol. IX, A Brief Course in Mathematics for Elementary School Teachers. Pasadena, California, A. C. Vroman, Inc., 1963.

And at least one of the following:

11.2a Moise, Edwin E. The Number Systems of Elementary Mathematics: Counting, Measurements and Coordinates. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1965.

11.2b National Council of Teachers of Mathematics. Topics in Mathematics for Elementary School Teachers (29th Yearbook) and More Topics in Mathematics for Elementary School Teachers (30th Yearbook). Washington, D. C., National Council of Teachers of Mathematics. 29th Yearbook, 1964; 30th Yearbook, 1969.

11.2c Peterson, John A. and Hashisaki, Joseph. Theory of Arithmetic, 2nd ed. New York, John Wiley and Sons, Inc., 1967.

And at least one of the following:

11.3a Keedy, Mervin L. and Nelson, Charles W. Geometry, A Modern Introduction. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1965.

11.3b Ringenberg, Lawrence A. Informal Geometry. New York, John Wiley and Sons, Inc., 1967.

11.3c Smart, James R. Introductory Geometry: An Informal Approach. Belmont, California, Brooks/Cole Publishing Company, 1967.

Teaching of Mathematics--At least one of the following:

11.4a Fehr, Howard F. and Phillips, Jo M. Teaching Modern Mathematics in the Elementary School. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1967.

11.4b Riedesel, C. Alan. Guiding Discovery in Elementary School Mathematics. New York, Appleton-Century-Crofts, 1967.

And at least one of the following:

11.5a Johnson, Donovan A. and Rising, Gerald R. Guidelines for Teaching Mathematics. Belmont, California, Wadsworth Publishing Company, Inc., 1967.

11.5b Willoughby, Stephen S. Contemporary Teaching of Secondary School Mathematics. New York, John Wiley and Sons, Inc., 1967.

And at least one of the following:

11.6a Butler, D. H. and Wren, F. L. The Teaching of Secondary Mathematics, 5th ed. New York, McGraw-Hill Book Company, 1969.

11.6b Dubisch, Roy. Teaching of Mathematics, 2nd ed. New York, John Wiley and Sons, Inc., 1963.

12. NUMERICAL ANALYSIS

Introductory Texts--At least one of the following:

12.1a Dodes, Irving A. and Greitzer, S. L. Numerical Analysis with Scientific Applications. New York, Hayden Book Company, Inc., 1964.

12.1b Dorn, William S. and Greenberg, Herbert J. Mathematics and Computing: With FORTRAN Programming. New York, John Wiley and Sons, Inc., 1967.

Texts Combined with Introductory Programming--At least one of the following:

12.2a James, Merlin L., *et al*. *Applied Numerical Methods for Digital Computation with FORTRAN*. Scranton, Pennsylvania, Intext Educational Publishers, 1967.

12.2b McCormick, John M. and Salvadori, M. G. *Numerical Methods in FORTRAN*. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1964.

12.2c McCracken, Daniel D. and Dorn, William S. *Numerical Methods and FORTRAN Programming*. New York, John Wiley and Sons, Inc., 1964.

Intermediate Texts--At least one of the following:

12.3a Conte, Samuel D. *Elementary Numerical Analysis: An Algorithmic Approach*. New York, McGraw-Hill Book Company, 1965.

12.3b Fox, Leslie and Mayers, D. F. *Computing Methods for Scientists and Engineers*. New York, Oxford University Press, 1968.

12.3c Macon, Nathaniel. *Numerical Analysis*. New York, John Wiley and Sons, Inc., 1963.

12.3d Moursund, David G. and Duris, C. S. *Elementary Theory and Application of Numerical Analysis*. New York, McGraw-Hill Book Company, 1967.

12.3e Stiefel, E. L. *An Introduction to Numerical Mathematics*. (translated from the German by W. C. Rheinboldt) New York, Academic Press, Inc., 1963.

Intermediate Numerical Linear Algebra Texts--At least one of the following:

12.4a Forsythe, George E. and Moler, C. B. *Computer Solution of Linear Algebraic Systems*. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1967.

12.4b Fox, Leslie. *Introduction to Numerical Linear Algebra*. New York, Oxford University Press, 1965.

Advanced Texts--At least one of the following:

12.5a Fröberg, Carl E. Introduction to Numerical Analysis, 2nd ed. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1969.

12.5b Henrici, Peter K. Elements of Numerical Analysis. New York, John Wiley and Sons, Inc., 1964.

12.5c Householder, Alston S. Principles of Numerical Analysis. New York, McGraw-Hill Book Company, 1953.

12.5d Isaacson, Eugene and Keller, H. B. Analysis of Numerical Methods. New York, John Wiley and Sons, Inc., 1966.

12.5e Ralston, Anthony. First Course in Numerical Analysis. New York, McGraw-Hill Book Company, 1965.

Some books with a reference character--At least one of the following:

12.6a Carnahan, Brice, et al. Applied Numerical Methods. New York, John Wiley and Sons, Inc., 1969.

12.6b Handscomb, David C., ed. Methods of Numerical Approximation. Elmsford, New York, Pergamon Press, Inc., 1966.

12.6c Kelly, Louis G. Handbook of Numerical Methods and Applications. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1967.

12.6d Mathematical Methods for Digital Computers. Edited by A. Ralston and H. S. Wilf. New York, John Wiley and Sons, Inc. Vol. I, 1960; Vol. II, 1967.

13. MATHEMATICS FOR THE PHYSICAL SCIENCES

General Works--At least one of the following:

13.1a Boas, M. L. Mathematical Methods in Physical Sciences. New York, John Wiley and Sons, Inc., 1966.

13.1b Collins, R. E. Mathematical Methods for Physicists and Engineers. New York, Van Nostrand Reinhold Company, 1968.

13.1c Page, Chester H. Physical Mathematics. New York, Van Nostrand Reinhold Company, 1955. Out of print.

Engineering Case Studies

13.2 Noble, Ben. Applications of Undergraduate Mathematics in Engineering. New York, The Macmillan Company, 1967.

Applied Algebra--At least one of the following:

13.3a Hall, George G. Applied Group Theory. New York, American Elsevier Publishing Company, Inc., 1967.

13.3b Hohn, Franz E. Applied Boolean Algebra, 2nd ed. New York, The Macmillan Company, 1966.

Applied Analysis--At least one of the following:

13.4a Brouwer, Dirk and Clemence, Gerald M. Methods of Celestial Mechanics. New York, Academic Press, Inc., 1961.

13.4b Churchill, Ruel V. Operational Mathematics. New York, McGraw-Hill Book Company, 1958.

13.4c Lanczos, Cornelius. Applied Analysis. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1956.

13.4d Lawden, Derek F. Mathematics of Engineering Systems, 2nd ed. New York, Barnes and Noble, Inc., 1959.

13.4e Sokolnikoff, Ivan S. and Redheffer, R. M. Mathematics of Physics and Modern Engineering, 2nd ed. New York, McGraw-Hill Book Company, 1966.

13.4f Urwin, Kathleen M. Advanced Calculus and Vector Field Theory. Elmsford, New York, Pergamon Press, Inc., 1966.

13.4g Von Kármán, T. and Biot, M. A. Mathematical Methods in Engineering: An Introduction to the Mathematical Treatment of Engineering Problems. New York, McGraw-Hill Book Company, 1960.

13.4h Wylie, Clarence R. Advanced Engineering Mathematics, 3rd ed. New York, McGraw-Hill Book Company, 1966.

Operations Research

13.5 Kaufmann, Arnold. The Science of Decision Making. (translated by R. Audley) New York, McGraw-Hill Book Company, 1968.

And at least one of the following:

13.6a Hillier, Frederick S. and Lieberman, Gerald J. Introduction to Operations Research. San Francisco, California, Holden-Day, Inc., 1967.

13.6b Kaufmann, Arnold and Faure, R. Introduction to Operations Research. (translated by H. C. Sneyd) New York, Academic Press, Inc., 1968.

13.6c Sasieni, Maurice W., et al. Operations Research: Methods and Problems. New York, John Wiley and Sons, Inc., 1959.

See also 14.10

14. MATHEMATICS FOR THE SOCIAL AND LIFE SCIENCES

General Books--At least one of the following:

14.1a Kemeny, John G. and Snell, J. Laurie. Mathematical Models in the Social Sciences. Waltham, Massachusetts, Blaisdell Publishing Company, 1962.

14.1b Lazarfeld, Paul T. and Henry, Neil W., eds. Readings in Mathematical Social Science. Cambridge, Massachusetts, MIT Press, 1968.

14.1c Massarik, F. and Ratoosh, P., eds. Mathematical Explorations in Behavioral Sciences. Homewood, Illinois, Richard D. Irwin, Inc., 1965. Out of print.

Elementary Mathematics for Social and Biological Sciences--At least one of the following:

14.2a Gelbaum, Bernard R. and March, James G. Mathematics for the Social and Behavioral Sciences. Vol. 1, Probability, Calculus and Statistics. Philadelphia, Pennsylvania, W. B. Saunders Company, 1969.

*14.2b Kemeny, John G., et al. Introduction to Finite Mathematics, 2nd ed. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1957.

Economics--At least one of the following:

14.3a Archibald, George C. and Lipsey, Richard G. An Introduction to a Mathematical Treatment of Economics. London, Weidenfeld and Nicholson, 1967.

14.3b Beach, E. F. Economic Models. New York, John Wiley and Sons, Inc., 1957.

14.3c Boot, Johannes. Mathematical Reasoning in Economics and Management Sciences: Twelve Topics. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1967.

14.3d Bushaw, Donald W. and Clower, Robert W. Introduction to Mathematical Economics. Homewood, Illinois, Richard D. Irwin, Inc., 1957. Out of print.

Sociology--At least one of the following:

14.4a Bartos, Otomar J. Simple Models of Group Behavior. New York, Columbia University Press, 1967.

14.4b Berger, Joseph, et al. Types of Formalization in Small Group Research. Boston, Massachusetts, Houghton Mifflin Company, 1963.

14.4c Coleman, James S. Introduction to Mathematical Sociology. New York, Free Press, 1964.

Psychology--At least one of the following:

14.5a Atkinson, Richard C., et al. Introduction to Mathematical Learning Theory. New York, John Wiley and Sons, Inc., 1965.

14.5b Luce, Robert D., et al. Handbook of Mathematical Psychology, 3 vols. New York, John Wiley and Sons, Inc., 1963-1965.

14.5c Luce, Robert D., et al. Readings in Mathematical Psychology, 2 vols. New York, John Wiley and Sons, Inc., 1963.

14.5d Miller, George. Mathematics and Psychology. New York, John Wiley and Sons, Inc., 1964.

Political Science--At least one of the following:

14.6a Alker, Hayward R., Jr. Mathematics and Politics. New York, The Macmillan Company, 1965.

14.6b Riker, William H. Theory of Political Coalitions. New Haven, Connecticut, Yale University Press, 1962.

14.6c Saaty, Thomas L. Mathematical Models of Arms Control and Disarmament. New York, John Wiley and Sons, Inc., 1968.

14.6d Tullock, Gordon. Toward a Mathematics of Politics. Ann Arbor, Michigan, University of Michigan Press, 1967.

Biological Sciences--At least one of the following:

14.7a Keyfitz, Nathan. Introduction to the Mathematics of Population. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1968.

14.7b Lotka, Alfred J. Elements of Mathematical Biology. New York, Dover Publications, Inc., 1957.

14.7c Nahikian, Howard M. Modern Algebra for Biologists. Chicago, Illinois, University of Chicago Press, 1964.

Game Theory--At least one of the following:

14.8a Luce, Robert D. and Raiffa, H. Games and Decisions. New York, John Wiley and Sons, Inc., 1957.

14.8b Owen, Guillermo. Game Theory. Philadelphia, Pennsylvania, W. B. Saunders Company, 1968.

14.8c Rapoport, Anatol. Fights, Games, and Debates. Ann Arbor, Michigan, University of Michigan Press, 1960.

14.8d Rapoport, Anatol. Two-Person Game Theory: The Essential Ideas. Ann Arbor, Michigan, University of Michigan Press, 1966.

14.8e Williams, John D. Compleat Strategyst. New York, McGraw-Hill Book Company, 1965.

Programming--At least one of the following:

14.9a Dantzig, George B. Linear Programming and Extensions. Princeton, New Jersey, Princeton University Press, 1963.

14.9b Gass, Saul I. Linear Programming, 2nd ed. New York, McGraw-Hill Book Company, 1969.

14.9c Glicksman, Abraham M. Linear Programming and the Theory of Games. New York, John Wiley and Sons, Inc., 1963.

14.9d Haley, K. B. Mathematical Programming for Business and Industry. New York, St. Martin's Press, Inc., 1967.

14.9e Vajda, S. Mathematical Programming. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1961.

Mathematical Topics of Special Interest to the Social and Life Sciences--At least two of the following:

14.10a Allen, Roy G. Mathematical Economics, 2nd ed. New York, St. Martin's Press, Inc., 1959.

14.10b Ash, R. B. Information Theory. New York, John Wiley and Sons, Inc., 1965.

14.10c Beckenbach, Edwin F., ed. Applied Combinatorial Mathematics. New York, John Wiley and Sons, Inc., 1964.

14.10d Goldberg, Samuel. Introduction to Difference Equations. New York, John Wiley and Sons, Inc., 1958.

14.10e Harary, Frank, et al. Structural Models: An Introduction to the Theory of Directed Graphs. New York, John Wiley and Sons, Inc., 1965.

14.10f Pierce, J. R. Symbols, Signals and Noise: The Nature and Process of Communication. New York, Harper and Row, Publishers, 1962.

14.10g Saaty, Thomas L. Optimization in Integers and Related Extremal Problems. New York, McGraw-Hill Book Company, 1970.

See also 13.5 and 13.6

15. ANALYSIS AND DIFFERENTIAL EQUATIONS

Differential Equations--At least one of the following:

15.1a Agnew, Ralph P. Differential Equations, 2nd ed. New York, McGraw-Hill Book Company, 1960.

15.1b Boyce, William and DiPrima, R. C. Elementary Differential Equations and Boundary Value Problems, 2nd ed. New York, John Wiley and Sons, Inc., 1969.

15.1c Coddington, Earl A. Introduction to Ordinary Differential Equations. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1961.

15.1d Ford, Lester R. Differential Equations, 2nd ed. New York, McGraw-Hill Book Company, 1955.

15.1e Rainville, Earl D. and Bedient, Phillip E. Short Course in Differential Equations, 4th ed. New York, The Macmillan Company, 1969.

15.1f Ross, S. L. Differential Equations. Waltham, Massachusetts, Blaisdell Publishing Company, 1964.

15.1g Spiegel, Murray R. Applied Differential Equations, 2nd ed. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1967.

15.1h Tenenbaum, Morris and Pollard, Harry. Ordinary Differential Equations. New York, Harper and Row, Publishers, 1963.

Partial Differential Equations--At least one of the following:

15.2a Berg, Paul W. and McGregor, James L. Elementary Partial Differential Equations. San Francisco, California, Holden-Day, Inc., 1966.

15.2b Broman, Arne. Introduction to Partial Differential Equations: From Fourier Series to Boundary Value Problems. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1970.

15.2c Churchill, Ruel V. Fourier Series and Boundary Value Problems, 2nd ed. New York, McGraw-Hill Book Company, 1963.

Infinite Series--At least one of the following:

15.3a Green, James A. Sequences and Series. (edited by W. Ledermann) New York, Dover Publications, Inc., 1958.

15.3b Knopp, Konrad. Infinite Sequences and Series. (translated by F. Bagemihl) New York, Dover Publications, Inc., 1956.

Fourier Series--At least one of the following:

*15.4a Jackson, Dunham. Fourier Series and Orthogonal Polynomials. LaSalle, Illinois, Open Court Publishing Company, 1941.

15.4b Rogosinski, W. Fourier Series, 2nd ed. New York, Chelsea Publishing Company, Inc., 1959.

15.4c Seeley, Robert T. Introduction to Fourier Series and Integrals. New York, The Benjamin Company, Inc., 1966.

15.4d Tolstov, Georgy P. Fourier Series, 2nd ed. (translated by Richard A. Silverman) Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1962.

Real Variables

*15.5 Boas, Ralph P., Jr. A Primer of Real Functions. New York, John Wiley and Sons, Inc., 1960.

And at least one of the following:

15.6a Randol, Burton. Introduction to Real Analysis. New York, Harcourt Brace Jovanovitch, Inc., 1969.

15.6b Royden, H. L. Real Analysis, 2nd ed. New York, The Macmillan Company, 1968.

15.6c Rudin, Walter. Principles of Mathematical Analysis, 2nd ed. New York, McGraw-Hill Book Company, 1964.

Complex Variables--At least one of the following:

15.7a Ahlfors, Lars V. Complex Analysis, 2nd ed. New York, McGraw-Hill Book Company, 1966.

15.7b Churchill, Ruel V. Complex Variables and Applications, 2nd ed. New York, McGraw-Hill Book Company, 1960.

15.7c Knopp, Konrad. Theory of Functions, Parts I and II. Problem Books I and II. (translated by F. Bagemihl) New York, Dover Publications, Inc., 1947. Part I, Elements of the General Theory of Analytic Functions; Part II, Applications and Continuations of the General Theory.

General--At least one of the following:

15.8a Gelbaum, Bernard and Olmsted, John. Counterexamples in Analysis. San Francisco, California, Holden-Day, Inc., 1964.

15.8b Smirnov, Vladimir I. Course of Higher Mathematics, 5 vols. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1964. Vol. I, Elementary Calculus; Vol. II, Advanced Calculus; Vol. III, Part 1, Linear Algebra; Vol. III, Part 2, Complex Variables, Special Functions; Vol. IV, Boundary Value Problems, Integral Equations and Partial Differential Equations; Vol. V, Integration and Functional Analysis.

16. ALGEBRA

Theory of Equations--At least one of the following:

16.1a Conkwright, N. B. Introduction to the Theory of Equations. Waltham, Massachusetts, Blaisdell Publishing Company, 1957.

16.1b Dickson, Leonard E. A New First Course in the Theory of Equations. New York, John Wiley and Sons, Inc., 1939.

16.1c MacDuffee, Cyrus C. Theory of Equations. New York, John Wiley and Sons, Inc., 1954.

16.1d Uspensky, James V. Theory of Equations. New York, McGraw-Hill Book Company, 1948.

Elementary Linear Algebra--At least one of the following:

16.2a Beaumont, Ross A. Linear Algebra. New York, Harcourt Brace Jovanovitch, Inc., 1965.

16.2b Curtis, Charles W. Linear Algebra: An Introductory Approach, 2nd ed. Boston, Massachusetts, Allyn and Bacon, Inc., 1968.

16.2c Shields, Paul C. Elementary Linear Algebra. New York, Worth Publishers, Inc., 1968.

16.2d Zelinsky, Daniel. First Course in Linear Algebra. New York, Academic Press, Inc., 1968.

Intermediate Linear Algebra--At least one of the following:

16.3a Finkbeiner, Daniel T. Introduction to Matrices and Linear Transformations, 2nd ed. San Francisco, California, W. H. Freeman and Company, 1966.

16.3b Hohn, Franz E. Elementary Matrix Algebra, 2nd ed. New York, The Macmillan Company, 1964.

16.3c Schneider, Hans and Barker, George P. Matrices and Linear Algebra. New York, Holt, Rinehart and Winston, Inc., 1968.

16.3d Staib, John H. Introduction to Matrices and Linear Transformations. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1969.

Advanced Linear Algebra--At least one of the following:

16.4a Halmos, Paul R. Finite-Dimensional Vector Spaces, 2nd ed. New York, Van Nostrand Reinhold Company, 1958.

16.4b Hoffman, Kenneth and Kunze, Ray. Linear Algebra, 2nd ed. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1971.

16.4c Mostow, George D. and Sampson, Joseph H. Linear Algebra. New York, McGraw-Hill Book Company, 1969.

16.4d Nering, Evar D. Linear Algebra and Matrix Theory, 2nd ed. New York, John Wiley and Sons, Inc., 1970.

Applied Linear Algebra

16.5 Noble, Ben. Applied Linear Algebra. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1969.

Introductory Abstract Algebra--At least one of the following:

16.6a Andree, Richard V. Selections from Modern Abstract Algebra. New York, Holt, Rinehart and Winston, Inc., 1958.

16.6b Weiss, Marie J. and Dubisch, Roy. Higher Algebra for the Undergraduate, 2nd ed. New York, John Wiley and Sons, Inc., 1962.

Elementary Abstract Algebra

16.7 McCoy, Neal H. Introduction to Modern Algebra, rev. ed. Boston, Massachusetts, Allyn and Bacon, Inc., 1968.

And at least one of the following:

16.8a Dean, Richard A. Elements of Abstract Algebra. New York, John Wiley and Sons, Inc., 1966.

16.8b Fraleigh, John B. First Course in Abstract Algebra. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1967.

Intermediate Abstract Algebra--At least one of the following:

16.9a Birkhoff, Garrett and MacLane, Saunders. Survey of Modern Algebra, 3rd ed. New York, The Macmillan Company, 1965.

16.9b Herstein, I. N. Topics in Algebra. Waltham, Massachusetts, Blaisdell Publishing Company, 1964.

16.9c Lewis, D. J. Introduction to Algebra. New York, Harper and Row, Publishers, 1965.

16.9d Paley, Hiram and Weichsel, Paul M. First Course in Abstract Algebra. New York, Holt, Rinehart and Winston, Inc., 1966.

Advanced Abstract Algebra

16.10 van der Waerden, B. L. Modern Algebra. (translated by T. J. Benac) New York, Frederick Ungar Publishing Company, Inc. Vol. I, 1949; Vol. II, 1950.

17. NUMBER THEORY

General and Historical--At least one of the following:

17.1a Fraenkel, Abraham A. Integers and Theory of Numbers. New York, Academic Scripta Mathematica Studies (Yeshiva University: Scripta Mathematica Studies, No. 5), 1955.

17.1b Ogilvy, C. Stanley and Anderson, John T. Excursions in Number Theory. New York, Oxford University Press, 1966.

17.1c Ore, Oystein. Number Theory and Its History. New York, McGraw-Hill Book Company, 1948.

Elementary--At least one, preferably two, of the following:

17.2a Barnett, I. A. Elements of Number Theory. Boston, Massachusetts, Prindle, Weber and Schmidt, Inc., 1969.

17.2b Davenport, H. Higher Arithmetic: Introduction to the Theory of Numbers, 3rd ed. New York, Hillary House Publishers, 1968.

17.2c Dudley, Underwood. Elementary Number Theory. San Francisco, California, W. H. Freeman and Company, 1969.

17.2d Jones, Burton W. Theory of Numbers. New York, Holt, Rinehart and Winston, Inc., 1955.

17.2e McCoy, Neal H. Theory of Numbers. New York, The Macmillan Company, 1965.

17.2f Stewart, Bonnie M. Theory of Numbers, 2nd ed. New York, The Macmillan Company, 1964.

17.2g Uspensky, James V. and Heaslet, M. A. Elementary Number Theory. New York, McGraw-Hill Book Company, 1939.

Advanced--At least one of the following:

17.3a LeVeque, William J. Topics in Number Theory, vol. 1. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1956.

17.3b Nagell, Trygve. Introduction to Number Theory, 2nd ed. New York, Chelsea Publishing Company, Inc., 1964.

17.3c Niven, Ivan and Zuckerman, H. S. Introduction to the Theory of Numbers, 2nd ed. New York, John Wiley and Sons, Inc., 1966.

Larger Reference Works--At least one of the following:

17.4a Dickson, Leonard E. History of the Theory of Numbers, 3 vols. New York, Chelsea Publishing Company, Inc.

17.4b Hardy, Godfrey H. and Wright, E. M. Introduction to the Theory of Numbers, 4th ed. New York, Oxford University Press, 1960.

17.4c Shanks, Daniel. Solved and Unsolved Problems in Number Theory, vol. 1. New York, Spartan Books, Inc., 1962.

17.4d Sierpiński, Wacław. Elementary Theory of Numbers. (translated from the Polish by A. Hulanicki) Polska Academia Nauk Monografie Matematyczne, Tom. 42. New York, Hafner Publishing Company, 1964.

18. LOGIC, FOUNDATIONS, AND SET THEORY

Philosophy

18.1 Barker, Stephen F. Philosophy of Mathematics. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1964.

General

18.2 Wilder, Raymond L. Introduction to the Foundations of Mathematics, 2nd ed. New York, John Wiley and Sons, Inc., 1965.

And at least one of the following:

18.3a Fraenkel, Abraham A. Set Theory and Logic. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1966.

18.3b Meschkowski, Herbert. Evolution of Mathematical Thought. (translated by J. H. Gayl) San Francisco, California, Holden-Day, Inc., 1965.

18.3c Nagel, Ernest and Newman, James R. Gödel's Proof. New York, New York University Press, 1958.

18.3d Stoll, Robert R. Sets, Logic and Axiomatic Theories. San Francisco, California, W. H. Freeman and Company, 1961.

18.3e Vilenkin, N. Ya. Stories About Sets. New York, Academic Press, Inc., 1968.

Elementary Logic--At least one of the following:

18.4a Dinkines, Flora. Elementary Concepts of Modern Mathematics. Part 2, Introduction to Mathematical Logic. New York, Appleton-Century-Crofts, 1964.

18.4b Exner, Robert M. and Rosskopf, Myron S. Logic in Elementary Mathematics. New York, McGraw-Hill Book Company, 1959.

18.4c Kenelly, John W. Informal Logic. Boston, Massachusetts, Allyn and Bacon, Inc., 1967.

18.4d Suppes, P. and Hill, S. First Course in Mathematical Logic. Waltham, Massachusetts, Blaisdell Publishing Company, 1964.

Mathematical Logic--At least one of the following:

18.5a Copi, Irving M. Symbolic Logic, 3rd ed. New York, The Macmillan Company, 1967.

18.5b Kalish, Donald and Montague, Richard. Logic: Techniques of Formal Reasoning. New York, Harcourt Brace Jovanovitch, Inc., 1964.

18.5c Quine, Willard Van Orman. Mathematical Logic, rev. ed. New York, Harper and Row, Publishers, 1951.

18.5d Suppes, P. C. Introduction to Logic. New York, Van Nostrand Reinhold Company, 1957.

18.5e Tarski, Alfred. Introduction to Logic and to the Methodology of Deductive Sciences, 3rd ed. New York, Oxford University Press, 1965.

Elementary Set Theory--At least one of the following:

18.6a Breuer, Joseph. Introduction to Theory of Sets. (translated by H. Fehr) Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1958.

18.6b Dinkines, Flora. Elementary Concepts of Modern Mathematics. Part 1, Elementary Theory of Sets. New York, Appleton-Century-Crofts, 1964.

18.6c Kamke, E. Theory of Sets. (translated by F. Bagemihl) New York, Dover Publications, Inc., 1950.

*18.6d Lipschutz, Seymour. Set Theory and Related Topics. (Schaum's Outline Series) New York, McGraw-Hill Book Company, 1964.

Advanced Set Theory

18.7 Halmos, Paul R. Naive Set Theory. New York, Van Nostrand Reinhold Company, 1960.

Number Systems--At least one of the following:

18.8a Cohen, Leon W. and Ehrlich, Gertrude. Structure of the Real Number System. New York, Van Nostrand Reinhold Company, 1963.

18.8b Feferman, S. Number Systems: Foundations of Algebra and Analysis. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1964.

18.8c Hamilton, Norman T. and Landin, J. Set Theory: The Structure of Arithmetic. Boston, Massachusetts, Allyn and Bacon, Inc., 1961.

Foundations of Computer Science--At least one of the following:

18.9a Arbib, Michael A. Brains, Machines and Mathematics. New York, McGraw-Hill Book Company, 1964.

18.9b Knuth, Donald E. Art of Computer Programming. Vol. 1, Fundamental Algorithms. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1968.

18.9c Minsky, Marvin. Computation: Finite and Infinite Machines. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1967.

19. GEOMETRY

General--All of the following:

19.1 Coxeter, H. S. M. Introduction to Geometry. New York, John Wiley and Sons, Inc., 1961.

19.2 Eves, Howard. Survey of Geometry. Boston, Massachusetts, Allyn and Bacon, Inc. Vol. I, 1963; Vol. II, 1965.

19.3 Hilbert, David and Cohn-Vossen, Stephan. Geometry and the Imagination. (translated by P. Nemenyi) New York, Chelsea Publishing Company, Inc., 1952.

Elementary Geometry--At least one of the following:

19.4a Moise, Edwin E. Elementary Geometry from an Advanced Standpoint. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1963.

19.4b Prenowitz, Walter and Jordan, Meyer. Basic Concepts of Geometry. Waltham, Massachusetts, Blaisdell Publishing Company, 1965.

19.4c Wylie, Clarence R. *Foundations of Geometry*. New York, McGraw-Hill Book Company, 1964.

Vector Geometry--At least one of the following:

19.5a Hausner, Melvin. *Vector Space Approach to Geometry*. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1965.

19.5b Schuster, Seymour. *Elementary Vector Geometry*. New York, John Wiley and Sons, Inc., 1962.

Non-Euclidean Geometry--At least one of the following:

19.6a Kulczycki, Stefan. *Non-Euclidean Geometry*. Elmsford, New York, Pergamon Press, Inc., 1961.

19.6b Wolfe, Harold E. *Introduction to Non-Euclidean Geometry*. New York, Holt, Rinehart and Winston, Inc., 1945.

Projective and Affine Geometry--At least one of the following:

19.7a Blumenthal, Leonard M. *Modern View of Geometry*. San Francisco, California, W. H. Freeman and Company, 1961.

19.7b Coxeter, H. S. M. *Projective Geometry*. Waltham, Massachusetts, Blaisdell Publishing Company, 1964.

19.7c Fishback, William T. *Projective and Euclidean Geometry*. New York, John Wiley and Sons, Inc., 1969.

Differential Geometry--At least one of the following:

19.8a O'Neill, Barrett. *Elementary Differential Geometry*. New York, Academic Press, Inc., 1966.

19.8b Willmore, Thomas James. *Introduction to Differential Geometry*. New York, Oxford University Press, 1959.

Special Topics--Any of the following:

19.9a Albert, A. Adrian and Sandler, Reuben. *Introduction to Finite Projective Planes*. New York, Holt, Rinehart and Winston, Inc., 1968.

19.9b Dorwart, Harold L. Geometry of Incidence. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1965.

19.9c Jeger, Max. Transformation Geometry. (Mathematical Studies Series, Vol. I) New York, American Elsevier Publishing Company, 1966.

19.9d Kaplansky, Irving. Linear Algebra and Geometry: A Second Course. Boston, Massachusetts, Allyn and Bacon, Inc., 1969.

*19.9e Kazarinoff, Nicholas D. Geometric Inequalities. New York, Random House, Inc., 1961.

19.9f Weyl, Hermann. Symmetry. Princeton, New Jersey, Princeton University Press, 1952.

19.9g Yaglom, I. M. and Boltyanskii, V. G. Convex Figures. New York, Holt, Rinehart and Winston, Inc. Out of print.

20. TOPOLOGY

Intuitive Approaches to Topology--At least one of the following:

20.1a Arnold, Bradford Henry. Intuitive Concepts in Elementary Topology. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1962.

20.1b Bing, R. H. Elementary Point Set Topology. Slaught Memorial Paper No. 8. Washington, D. C., Mathematical Association of America, 1960.

20.1c Fréchet, Maurice and Fan, Ky. Initiation to Combinatorial Topology. (translated from the French by Howard Eves) Boston, Massachusetts, Prindle, Weber and Schmidt, Inc., 1967.

20.1d Lietzmann, Walter. Visual Topology. New York, American Elsevier Publishing Company, 1965.

A somewhat more rigorous approach with many of the classical theorems:

*20.2 Chinn, William G. and Steenrod, Norman E. First Concepts of Topology. New York, Random House/Singer School Division, 1966.

Algebraic Topology--At least one of the following:

20.3a Aleksandrov, P. S. Combinatorial Topology, 3 vols. Baltimore, Maryland, Graylock Press. Vol. I, Introduction, Complexes, Coverings, Dimensions, 1956; Vol. II, Betti Groups, 1957; Vol. III, Homological Manifolds, Duality, Classification, and Fixed Point Theorems, 1960.

20.3b Blackett, Donald W. Elementary Topology: Combinatorial and Algebraic Approach. New York, Academic Press, Inc., 1967.

20.3c Massey, William S. Algebraic Topology: An Introduction. New York, Harcourt Brace Jovanovitch, Inc., 1967.

20.3d Wallace, Andrew Hugh. Introduction to Algebraic Topology. Elmsford, New York, Pergamon Press, Inc., 1957.

General Topology--At least one of the following:

20.4a Baum, John D. Elements of Point Set Topology. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1964.

20.4b Bushaw, Donald. Elements of General Topology. New York, John Wiley and Sons, Inc., 1963. Out of print.

20.4c Kuratowski, Kazimierz. Introduction to Set Theory and Topology. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1962. Out of print.

20.4d Mendelson, Bert. Introduction to Topology, 2nd ed. Boston, Massachusetts, Allyn and Bacon, Inc., 1968.

20.4e Pervin, William J. Foundations of General Topology. New York, Academic Press, Inc., 1964.

Graph Theory

*20.5 Ore, Oystein. Graphs and Their Uses. New York, Random House, Inc., 1963.

21. TABLES AND DICTIONARIES

The library should contain at least one mathematical dictionary and one or more sets of tables, both numerical and functional.

Following is a list of several such dictionaries and tables; there are others equally good available.

Abramowitz, Milton and Stegun, Irene A., eds. Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables. New York, Dover Publications, Inc., 1964.

Burington, Richard S. Handbook of Mathematical Tables and Formulas, 4th ed. New York, McGraw-Hill Book Company, 1965.

*Burington, Richard S. and May, Donald C., Jr. Handbook of Probability and Statistics with Tables, 2nd ed. New York, McGraw-Hill Book Company, 1969.

*Chemical Rubber Company. Handbook of Tables for Probability and Statistics, 2nd ed. Cleveland, Ohio, Chemical Rubber Company, 1968.

Chemical Rubber Company. Standard Mathematical Tables, 19th ed. Cleveland, Ohio, Chemical Rubber Company, 1971.

Davis, Harold T. Tables of Mathematical Functions, 2 vols. San Antonio, Texas, Trinity University Press, 1963.

Davis, Harold T. and Fisher, Vera. Tables of Mathematical Functions, vol. 3. San Antonio, Texas, Trinity University Press, 1962.

Dwight, Herbert B. Mathematical Tables of Elementary and Some Higher Mathematical Functions, 3rd ed. New York, Dover Publications, Inc., 1961.

Dwight, Herbert B. Tables of Integrals and Other Mathematical Data, 4th ed. New York, The Macmillan Company, 1961.

James, Glenn and James, Robert C. Mathematics Dictionary, 3rd ed. New York, Van Nostrand Reinhold Company, 1968.

Karush, William. Crescent Dictionary of Mathematics. New York, The Macmillan Company, 1962.

Larsen, Harold. Rinehart Mathematical Tables, Formulas and Curves, enl. ed. New York, Holt, Rinehart and Winston, Inc., 1953.

Marks, Robert W. New Mathematics Dictionary and Handbook. New York, Grosset and Dunlap, Inc., 1964.

Newman, J. R. The Universal Encyclopedia of Mathematics. New York, New American Library, Inc., 1965.

Nielsen, Kaj L. Logarithmic and Trigonometric Tables to Five Places, rev. ed. New York, Barnes and Noble, Inc., 1961.

The Universal Encyclopedia of Mathematics. New York, Simon and Schuster, Inc., 1964.

Weintraub, S. Tables of Cumulative Binomial Probability Distribution for Small Values of p. New York, Free Press, 1963.

22. JOURNALS

The American Mathematical Monthly. Mathematical Association of America, Inc., 1225 Connecticut Avenue, N.W., Washington, D. C. 20036 Ten issues per year.

The Arithmetic Teacher. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington, D. C. 20036 Eight issues per year.

The Mathematical Gazette. G. Bell and Sons, Ltd., Portugal Street, London, W.C. 2, England. Five issues per year.

Mathematics Magazine. Mathematical Association of America, Inc., 1225 Connecticut Avenue, N.W., Washington, D. C. 20036 Five issues per year.

The Mathematics Teacher. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington, D. C. 20036 Eight issues per year.

The Two-Year College Mathematics Journal. Prindle, Weber and Schmidt, Inc., 53 State Street, Boston, Massachusetts 02109 Two issues per year.

23. SERIES AND COLLECTIONS

A number of excellent series of monographs on various topics in mathematics exist. Quality varies somewhat within each series. Listing of a series here by no means implies that every book in every series should be purchased, for some volumes cover topics not appropriate to the two-year college.

Blaisdell Scientific Paperbacks. Waltham, Massachusetts, Blaisdell Publishing Company, Inc. A series of six pamphlets that are translations of the Russian series "Popular Lectures in Mathematics." Out of print.

Korovkin, P. P. Inequalities. 1961

Kostovskii, A. N. Geometrical Constructions Using Compasses. 1961

Smogorzhevskii, A. S. The Ruler in Geometrical Constructions. 1961

*Sominskii, I. S. The Method of Mathematical Induction. 1961

Uspenskii, V. A. Some Applications of Mechanics to Mathematics. 1961

*Vorobev, N. N. Fibonacci Numbers. 1961

Carus Mathematical Monographs. Washington, D. C., Mathematical Association of America, Inc.

No. 1. Calculus of Variations. G. A. Bliss

No. 2. Analytic Functions of a Complex Variable. D. R. Curtiss

No. 3. Mathematical Statistics. H. L. Rietz

No. 4. Projective Geometry. J. W. Young

*No. 6. Fourier Series and Orthogonal Polynomials. Dunham Jackson

No. 7. Vectors and Matrices. C. C. MacDuffee

No. 8. Rings and Ideals. N. H. McCoy

No. 9. The Theory of Algebraic Numbers. Harry Pollard

No. 10. The Arithmetic Theory of Quadratic Forms. B. W. Jones

No. 11. Irrational Numbers. Ivan Niven

No. 12. Statistical Independence in Probability, Analysis and Number Theory. Mark Kac

*No. 13. A Primer of Real Functions. R. P. Boas, Jr.

No. 14. Combinatorial Mathematics. H. J. Ryser

No. 15. Non-Commutative Rings. I. N. Herstein

No. 16. Dedekind Sums. Hans Rademacher and Emil Grosswald

Mathematics: Its Content, Methods, and Meaning, 3 vols. Edited by A. D. Aleksandrov, et al. Translated by S. H. Gould. Cambridge, Massachusetts, MIT Press.

MAA Studies in Mathematics. Washington, D. C., Mathematical Association of America, Inc.

Vol. 1. Studies in Modern Analysis. R. C. Buck, editor

Vol. 2. Studies in Modern Algebra. A. A. Albert, editor

Vol. 3. Studies in Real and Complex Analysis. I. I. Hirschman, Jr., editor

Vol. 4. Studies in Global Geometry and Analysis. S. S. Chern, editor

Vol. 5. Studies in Modern Topology. P. J. Hilton, editor

Vol. 6. Studies in Number Theory. W. J. LeVeque, editor

Vol. 7. Studies in Applied Mathematics. A. H. Taub, editor

Schaum's Outline Series. New York, McGraw-Hill Book Company.

Advanced Calculus. Murray R. Spiegel

Analytic Geometry. Joseph H. Kindle

Calculus, 2nd ed. Frank Ayres, Jr.

College Algebra. Murray R. Spiegel

Complex Variables. Murray R. Spiegel

Descriptive Geometry. Minor C. Hawk

Differential Equations. Frank Ayres, Jr.

Elementary Algebra. Barnett Rich

Finite Mathematics. Seymour Lipschutz

First Year College Mathematics. Frank Ayres, Jr.

General Topology. Seymour Lipschutz

Group Theory. B. Baumslag and B. Chandler

Laplace Transforms. Murray R. Spiegel

Linear Algebra. Seymour Lipschutz

Mathematical Handbook of Formulas and Tables. Murray R. Spiegel

Mathematics of Finance. Frank Ayres, Jr.

Matrices. Frank Ayres, Jr.

Modern Algebra. Frank Ayres, Jr.

Numerical Analysis. Francis Scheid

Plane Geometry with Coordinate Geometry. Barnett Rich

Projective Geometry. Frank Ayres, Jr.

Real Variables. Murray R. Spiegel

**Set Theory and Related Topics*. Seymour Lipschutz

Statistics. Murray R. Spiegel

Theory and Problems of Probability. Seymour Lipschutz

Trigonometry. Frank Ayres, Jr.

Vector Analysis. Murray R. Spiegel

School Mathematics Study Group. *Studies in Mathematics*. Pasadena, California, A. C. Vroman, Inc.

Euclidean Geometry Based on Ruler and Protractor Axioms (SM-1)

Protractor Axioms (SM-2)

Structure of Elementary Algebra (SM-3)

Geometry (SM-4)

Concepts of Informal Geometry (SM-5)

Number Systems (SM-6)

Intuitive Geometry (SM-7)

Concepts of Algebra (SM-8)

*Brief Course in Mathematics for Elementary School Teachers (SM-9)

Applied Mathematics in the High School (SM-10)

Mathematical Methods in Science (SM-11)

A Brief Course in Mathematics for Junior High School Teachers (SM-12)

Inservice Course for Primary School Teachers (SM-13)

Introduction to Number Systems (SM-14)

Calculus and Science (SM-15)

Some Uses of Mathematics (SM-16)

Mathematical Concepts of Elementary Measurement (SM-17)

Puzzle Problems and Games Project (SM-18)

Reviews of Recent Research in Mathematics Education (SM-19)

Slaught Memorial Papers. (not all available) Washington, D. C., Mathematical Association of America, Inc.,

No. 3 Proceedings of the Symposium on Special Topics in Applied Mathematics.

No. 5 The Conjugate Coordinate System for Plane Euclidean Geometry. W. B. Carver

No. 6 To Lester R. Ford on His Seventieth Birthday. A collection of 14 articles.

No. 7 Introduction to Arithmetic Factorization and Congruences from the Standpoint of Abstract Algebra. H. S. Vandiver and M. W. Weaver

No. 11 Papers in Analysis. A collection of 23 articles.

No. 12 Differentiation of Integrals. A. M. Bruckner

Topics in Mathematics. Lexington, Massachusetts, D. C. Heath and Company.

Configuration Theorems. B. I. Argunov and L. A. Skornyakov, 1963

What is Linear Programming? A. S. Barsov, 1964

Equivalent and Equidecomposable Figures. V. G. Boltyanskii, 1963

Mistakes in Geometric Proofs. Ya. S. Dubnov, 1963

Proof in Geometry. A. I. Fetisov, 1963

Induction in Geometry. L. I. Golovina and I. M. Yaglom, 1963

Computation of Areas of Oriented Figures. A. M. Lopshits, 1963

Areas and Logarithms. A. I. Markushevich, 1963

Summation of Infinitely Small Quantities. I. P. Natanson, 1963

Hyperbolic Functions. V. G. Shervatov, 1963

How to Construct Graphs. G. E. Shilov, 1963

Simple Maxima and Minima Problems. I. P. Natanson, 1963
(The above two are bound as one volume.)

*_The Method of Mathematical Induction_. I. S. Sominskii, 1963

Algorithms and Automatic Computing Machines. B. A. Trakhtenbrot, 1963

*_Fibonacci Numbers_. N. N. Vorobev, 1963

An Introduction to the Theory of Games. E. S. Venttsel, 1963

Translations from the Russian: A Survey of Recent East European Mathematical Literature. Lexington, Massachusetts, D. C. Heath and Company.

Multicolor Problems. E. B. Dynkin and V. A. Uspenskii, 1963

Problems in the Theory of Numbers. E. B. Dynkin and V. A. Uspenskii, 1963

Random Walks. E. B. Dynkin and V. A. Uspenskii, 1963

*Eight Lectures on Mathematical Analysis. Alexander Y. Khinchin, 1965

Convex Figures and Polyhedra. L. A. Lyusternik, 1966

Infinite Series. A. I. Markushevich, 1967