Calculus: Catalyzing a National Community for Reform

Awards 1987-1995

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Contents

Foreword

Calculus: A Pump, not a Filter sums up the desire of mathematicians that calculus be valuable to students in pursuing their educational and career goals, not a barrier to success. Courses and materials developed under the aegis of calculus reform emphasize direct experience with methods and processes of inquiry. Students learn mathematics by doing mathematics, by applying mathematical tools to significant and engaging problems, by working with other students, and by writing about their work. The use of modern technology adds visual and numeric perspectives to the usual algebraic perspective.

The effect of calculus reform illustrates the profound impact possible through the orchestration of wellfocused efforts, the selection of the most promising models, the involvement of individuals involved in mathematical research and education, and wide support for replication, adoption, and adaptation across a variety of settings and institutional cultures. Model individual classroom practices have been successfully "scaled up" to affect large numbers of students at many institutions across the nation. This enterprise required a stronger partnership between the K-12 and undergraduate education communities and leadership from professional societies. Partnerships forged by this common venture are now being extended, strengthened, and sustained by interactions among key sectors-precollege and college; the community college and the four-year college and university; and the mathematics, scientific, and engineering communities.

Several difficult questions have been brought to the forefront, including-What is the proper mix of rigor and intuition in introductory mathematics courses? How should technology change what is learned and how it is learned? What should be the relationship between mathematics departments and other science and engineering departments? How can effective teaching and learning be assessed? How can the reward system encourage and recognize faculty contributions to mathematics education? How can research in mathematics education inform practice?

In summary, calculus reform has produced and continues to produce a large number of successful models and exemplary practices that are being shared among disciplines and institutions, while forging alliances and collaborations. It has stimulated a new community and has had implications well beyond calculus, resulting in an enterprise far greater than the sum of the individual projects that appear in this publication: *Calculus: Catalyzing a National Community for Reform.*

Preface

Calculus: Catalyzing a National Community for Reform is an internal report of the Directorate for Education and Human Resources, Division of Undergraduate Education of the National Science Foundation.

From 1987-1995, NSF invested more than $44 million dollars in calculus reform efforts. Over 350 awards were made during that period to a very broad spectrum of institutions. There is little doubt that NSF has been central to calculus reform. The MAA Report, Assessing Calculus Reform Efforts (ACRE) edited by James R. Leitzel, flatly states "there has been universal agreement that the calculus reform movement, as we now know it, would have been impossible without the NSF Calculus Initiative." The last sentence ofthe ACRE report further underscores the impact of the NSF Calculus Program: "Through the efforts of the NSF Calculus Initiative, particularly its emphasis on dissemination and implementation, calculus reform is truly a property of the entire mathematics community."

An important aspect of the NSF Calculus Program has been wide dissemination of project-based curriculum materials and reports on the projects. This report is in many ways a capstone volume. Thus sharing it broadly within the mathematics community seems especially appropriate.

After several years of calculus reform activities, discussions about the value and impact of reform programs continue. **In** publishing this report, the MAA is not endorsing anyone program. We do take this opportunity, however, to thank NSF for its strong interest and support of initiatives directed at improving the learning of calculus.

Don Albers
Director of Publications

NATIONAL SCIENCE FOUNDATION CALCULUS PROGRAM

William E. Haver *

The NSF Calculus Program has had widespread impact on Calculus courses, on introductory collegiate mathematics instruction, and indeed on collegiate mathematics at all levels. In this introduction, we will explore the nature of the NSF Calculus Program in an attempt to understand why it resulted in an impact of this magnitude.

Nationwide, the content of the Calculus course has been modified to include: (a) substantially more applications of mathematics, (b) the use of technology to improve the understanding of concepts, to encourage the formulation of conjectures, and to perform calculations that are normally too difficult to do by hand, and (c) a deeper understanding of Calculus from a geometric and numerical as well as analytic point of view. Calculus students today are making extensive use of modem technology; regularly completing long-term assignments; and frequently participating actively as members of study groups and activity teams. Ten years ago these activities were virtually unheard of in college mathematics classes.

Additional evidence of the substantial impact of the NSF Calculus Program as part of the national reform effort in Calculus is provided by the many faculty who are expending large portions of their teaching/scholarly efforts toward improving Calculus instruction nationally, in their own regions and institutions, and most significantly in their own classrooms. The major changes in the Advanced Placement Calculus courses, in the entirety of the overall undergraduate curriculum, and in the Graduate Record Exams also attest to the scope of the changes that have occurred.

While the evidence of the change that has occurred is well known within a large portion of the mathematics community, it may not be as well understood in the broader educational community. A summary ofthis evidence is provided in Part B: "Evidence of Impact ofNSF Calculus Program".

Large numbers of college and university mathematics faculty, including this author, are convinced of the extremely valuable nature of this change. We are convinced that the increased faculty attention to student learning is indeed resuiting in increased student learning; that the inclusion of applications and long-term projects as a portion of instruction is preparing students who are being better able to apply mathematics in more advanced situations; and that requirements of increased student communication are paying off in graduates who are better prepared employees.

It should be acknowledged, however, that some college and university mathematicians believe that the increased use of technology, the introduction of more applications, and the increased emphasis on student communication is a change in the wrong direction. In addition, there are others who believe that more evidence of improved student learning is necessary before a final decision can be made concerning the ultimate value of the change. *What is virtually unanimously agreed, however, is that the NSF Program is having a major impact on collegiate mathematics instruction.* In the following, evidence will be given to explain why the effect within the mathematics community has been so large.

The National Science Foundation's Calculus Program was initiated and sustained in response to a well enunciated national need recognized by leaders of the scientific, and particularly mathematics, community. From the outset, the goal of the Program was to make a large-scale impact on the quality of mathematics instruction and student learning. Based upon recommendations of mathematicians and other scientists, NSF identified problems and opportunities and described them in program announcements. However, at no time did the NSF prescribe particular solutions.

Beginning in 1987 and continuing through 1995, NSF invested a total of approximately 43 million dollars on the Calculus Reform effort. These funds include amounts allocated (a) through a variety of different undergraduate programs including Calculus, Faculty Enhancement (UFE), and Instrumentation and Laboratory Improvement (ILl), (b) through other Education and Human Resource programs such as Teacher Enhancement (TE), Evaluation, Research, and Communication (REC), and Human Resource Development (HRD), and (c) through special programs of the Division of Mathematical Sciences (DMS).

Forty-three million dollars is a sizable amount of money, but as the major vehicle for research and development of a large educational enterprise, it is quite modest. Approximately 680,000 students enroll in a Calculus course each semester (this includes first, second and third semester Calculus in colleges and universities and 250,000 students studying AP Calculus in high schools). Thus the $43 million represents approximately $3.34 for research and development per student semester course enrollment during the period of the program. Considering these expenditures as an investment in the future, the cost per student is negligible.

PART A:
Characteristics of the NSF Calculus Program

Following is a description of the particular positive characteristics of the NSF Calculus Program which were, in my opinion, most responsible for the program's impact.

> The NSF Calculus Program was initiated in response to a clear national need enunciated by influential leaders within the mathematics and broader scientific and engineering communities.

The mathematics community dates the Calculus reform effort to the 25 participant Tulane Conference held in January 1986. The Conference was chaired by Ronald Douglas and supported by the Sloan Foundation. It was consciously designed to ignite a national effort to reform the content of the Calculus sequence and the way that instruction was provided. The proceedings of the conference, *Toward a Lean and Lively Calculus* [2], were published by the Mathematical Association of America and provided not only a vision of what change could occur, but suggestions for implementation. At this time (1986) NSF did not provide much support for undergraduate education. However, it was clearly understood that significant financial support would be needed to bring about the radical changes envisioned.

The 22 months following the workshop were very active ones with the publication of the Neal Report to the National Science Board (NSF 86-100) [15] calling for renewed NSF support for undergraduate education, the mathematics discipline workshop report [10] highlighting the opportunity for change in Calculus, support for change from the engineering community, and the development of a relatively small but enthusiastic constituency for change among mathematicians in all components of higher education. This period culminated in the simultaneous formal announcement of the NSF Calculus Program and a 700 participant *Calculus for a New Century* [12] conference hosted by the National Research Council. Thus a strong base of support for improving Calculus existed before any NSF investment occurred, and this commitment to making a difference and to having an impact, existed independently of the Foundation.

This deep commitment did not develop spontaneously. The leadership and political acumen of Ronald Douglas at SUNY Stony Brook, Bernard Madison at the National Research Council, and many others, including NSF staff, were critical. In addition, the commitment could not have developed if there had not been a real need and a vision of what might occur. Of course, the possibility of financial support to turn hopes into an opportunity to bring about change provided a big boost.

Before the Calculus Program began, there was a committed group of individuals ready to make the most of the opportunity the program would provide.

> From its inception the Calculus Program had a clear goal of bringing about large scale improvement in Calculus instruction for all students nationwide.

The goal from the beginning was to bring about what in the 1990s is described as *systemic change.* This meant dramatically changing the content and quality of Calculus instruction in all sections of Calculus at all types of institutions in all portions of the country. Because the *system* that determines what is taught and how it is taught at the college level includes the value structure of the faculty, a goal was to actively involve mathematics leaders and professional societies. Because faculty rely on textbooks and other instructional materials, another goal from the beginning was not only to develop, test, and refine new materials, but to involve commercial publishers to assure that these materials would be available to and considered by faculty.

The leaders of the Harvard Calculus Consortium report that they were recruited by Deborah Hughes-Hallett in telephone conversations that began with the sentence, "Do you want to change the World?" So, the effort clearly was mission oriented. It was not to teach a perfect Calculus course to a few students at a few locations. It was not to develop ideal materials that particularly talented teachers could use. It was not a demonstration project. The goal was to improve Calculus instruction for all students.

> The NSF Calculus Program, with input from the community, identified problems and opportunities, but did not prescribe solutions.

While there was a firm commitment to major improvement, the NSF did not have a blueprint for the nature of the change. The initial program announcement stated that the "focus will be on raising students' conceptual understanding, problem solving skills, analytical and transference skills, while implementing new methods to reduce tedious calculations." It also summarized the *Toward a Lean and Lively Calculus* report and referred readers to this report, the National Academy of Sciences October 1987 colloquium, and sessions on Calculus which had recently occurred at meetings of professional societies. The program announcement asserted that "Awareness of the need for revision and renewal in Calculus curriculum is evident throughout science and engineering." No guidelines for the nature of this revision and renewal were provided. However, the long term goal was clearly stated: "It is expected that revitalization and reorganization can be accomplished that will have long-lasting impact on undergraduate instruction in the United States."

> Rather than concentrating its resources on a small number of relatively large projects, NSF provided support for a large number of relatively small projects along with a limited number of larger projects.

This strategy was partially a result of the conscious decision not to predetermine the nature of the desired change and partially a response to the large number of high quality proposals received. The decision was based primarily on the view that content and instructional style were so entrenched that the creation, by itself, of an "ideal" text would bring about little change in common practice.

A major strength of American science and the National Science Foundation is the clear understanding that the best work should be funded. This view could have led to a decision to determine a limited number of best projects and to invest all of the limited resources in these. What happened, however, was that the decision was made to support the best proposals which made extensive use of computing technology, the best proposals that were based on graphing calculators, and the best that assumed that high powered technology would be extensively used but that were technology neutral. The decision was also made to fund the best proposals that built courses based on long-term student projects, the best proposals to integrate the teaching of Calculus and science, and the best proposals to fmd ways to address the special needs of minorities, women, and individuals with disabilities. Possibly as a result of these decisions, leadership for Calculus reform has come from a wide range of types of institutions.

Whatever the motivation of those who made the decision, the number of abstracts included in this volume provides clear proof that large numbers of relatively small projects were indeed supported through a variety of NSF programs.

> The NSF Calculus Program placed a premium on supporting the translation of promising ideas to different environments, the development of local laboratories and laboratory materials, and extensive "Beta Testing" of new materials.

Individual teachers who are especially talented and especially committed can make effective use of almost any set of curricular materials that they develop themselves. Materials and approaches however will bring about systemic change only if they can be used by all faculty. This view resulted in a high premium being placed on projects to adapt and adopt promising materials and approaches. In some cases, curriculum developers submitted new proposals to support beta testing of their materials. In other cases, a large university or a group of institutions was funded to test, refine, and adopt or adapt materials and approaches developed elsewhere. This had two major consequences:

It provided a mechanism to provide partial additional support to the most successful projects. The survival of the most successful happened naturally through the external review process.

It kept the projects from being idiosyncratic. Those that depended upon the skills or idiosyncrasies of an individual entrepreneur either did not attract interest from potential beta testers or were refined and honed by skilled adapters.

Since most of the approaches made use of computer technology and/or calculators, the Instrumentation and Laboratory Improvement (ILl) program provided excellent support for this process.

> The problems and difficulties associated with large-scale implementation of change were recognized, and projects that showed promise in attacking these problems were strongly encouraged.

Within the NSF, the issue of large-scale implementation was referred to as "the hard part." The obstacles to large-scale change are tremendous. Faculty lead busy professional lives with multiple priorities, and although the amount of out-of-class time for a faculty member whose major interest is *not* Calculus instruction could be increased to some degree, it was not possible (or desirable) to increase this time unrealistically. In addition, by necessity, graduate students and part-time faculty provide a large portion of Calculus instruction nationwide. Most of these individuals had little experience with the type of instruction or even the mathematics that was forming the basis of the reformed Calculus courses.

So the issue of large-scale change was very challenging, even in departments that had a commitment to bringing about change. The Calculus Program took the view that this was just as intellectual, academic, mathematical, and educational an issue as was the initial development of new ideas and approaches. The funded projects and the availability of high quality materials is evidence that in this regard the Calculus Program followed through on its intentions.

> Faculty development was recognized from very early on as critical to effective use of the newly developed educational materials, emerging technologies, and the teaching of new mathematics and science.

Toward a Lean and Lively Calculus [2] and *Calculus for a New Century* [12] called for ambitious curricular reform. The initial proposals for planning grants and subsequently large-scale curriculum development grants called for the development of radically different courses. The proposed change was large enough that faculty could not be expected to teach the new courses without first gaining an appreciation of the revised goals and approaches and then learning from the experiences of those who had tried the new approaches.

Through the Undergraduate Faculty Enhancement Program and other dissemination efforts, the Foundation was able to support a large array of workshops and training sessions. Many of these were funded as a part of the regular Calculus Program with financial support from other NSF programs. The commitment of the full mathematics community was of particular importance to this part of the endeavor. Individuals who received support for curricular development projects devoted much more time than provided for in the grants, but they did have the added incentive of receiving recognition (and sometimes even reward) for improving instruction at their own schools, and in some cases the potential for income from the sale of commercial materials. Of great importance to the Calculus reform, however, were the individuals who organized, developed, and taught in the many workshops and short courses which provided vital support for those who wished to use these new materials and methods, often with little local support.

The value of and necessity for the training is described in the MAA report *Assessing Calculus Reform Efforts [13]:*

"The in-depth survey [conducted by the MAA as the reform movement was gathering momentum] indicated that at virtually every one of the mathematics departments where some level of reform, however modest, was occurring, the faculty collectively had attended many presentations, often a dozen or more, about Calculus reform at conferences, professional meetings, or workshops."

Conscious and continual efforts were undertaken to develop and then to nurture a national community of mathematicians and mathematics educators committed to bringing about improvement in the student learning of Calculus.

Beginning in 1988, NSF Program Directors hosted annual meetings of project directors of all projects that were receiving NSF support at the time of the meeting. The meetings accomplished their goals to (a) develop a community of Calculus curricular developers, (b) promote sharing ideas and approaches that work, and (c) advise NSF staff of issues and opportunities. Because the approaches taken by the various large-scale projects were quite different, participants vigorously and emphatically championed their own approaches. Discussions often turned on which materials and approaches constituted 'real' change and which were merely mild reform. On other occasions, questions were raised to which approaches could indeed bring about broad national change and which would only work for the developer. Other sessions focused on the goals of Calculus reform, assessment, appropriate preparation for Calculus, and the impact of Calculus reform on other disciplines and other mathematics courses.

As in any successful intellectual community, participants attempted to understand what others had learned and then to make use of this knowledge in their own work. Admittedly, some of the project directors kept track of how much financial support others were receiving, and some kept track of how many institutions adopted each approach. But the project directors universally developed and displayed mutual respect. Subsequently, meetings of project directors were held in other mathematics programs and later for many DUE projects [9].

The Foundation also took active steps to nurture a broader community committed to Calculus reform and lower level mathematics education. Each year new proposals were reviewed by 6-8 person panels. Then after evaluations were completed, reviewers met together to discuss the ideas and proposals that each panel thought were most exciting and promising. This practice of meeting as an at-large panel to discuss the most exciting ideas soon was adopted for all NSF undergraduate mathematics programs, and then for all programs in undergraduate education.

The Mathematical Association of America (MAA) was provided with funding to initiate a program of sharing results ofNSF education awards at regional meetings and startup funds were provided for *UME Trends* (Undergraduate Mathematics Education) to share information on efforts to improve undergraduate education. The MAA CRAFTY (Calculus Reform and the First Two- Years) Committee with NSF support initiated poster sessions at annual MAN AMS meetings for sharing among not only NSF funded projects but all those interested in Calculus reform. The Foundation encouraged publishers to develop newsletters and to host workshops, and NSF staff participated in these activities.

In summary, with support from the Sloan Foundation and the National Research Council, the beginning of a national Calculus community was developed. The NSF then actively pursued its goal of deepening and broadening this community.

Support was provided to assess a wide variety of different aspects of the Calculus reform effort.

By its very nature, systemic reform is difficult to assess. When "implementing new methods to reduce tedious calculations" is a major stated goal, it would not be appropriate to base the assessment of the Calculus Program on the ability of students to excel on the final exams which were prevalent in 1986 and gave significant weight to hand computations of derivatives and integrals.

Therefore, a wide variety of different approaches to assessment was employed. When the program was in its initial stages, William Browder at Princeton University and Lida Barrett of Mississippi State University, President of the American Mathematical Society (AMS) and the MAA respectively, reviewed the initial directions of the program. A multi-year grant was made in the Fall of 1991 to MAA to assess overall (not exclusively NSF supported) Calculus reform, resulting in the *Assessing Calculus Reform Efforts* [13] report published in 1995. When James Lightbourne became the Lead Program Director of the Calculus Program in 1991, he placed great emphasis on assessment activities. A working group on Assessment, chaired by Alan Schoenfeld, was formed in November 1992 to guide assessment activities in Calculus and issued its report in 1997.

In addition to these external characteristics, there were a number of internal (to NSF) characteristics which were also instrumental in bringing about the large change which did occur.

The Calculus Program continued over a long period of time. It is almost impossible to overstate the importance of this perseverance over time.

NSF announced the Calculus Program in October 1987; the Calculus Program made its last award as a special program in September 1994; the last project funded under the program will end as a funded project in March 1999; and NSF has made a commitment to take a special interest in continuing to support curricular development in Calculus into the future through its Course, Curriculum and Laboratory Improvement programs. THIS IS A LONG TIME.

For a program to succeed, it needs a commitment over time to effect change. This time is necessary to accomplish any fundamental, systemic change in higher education.

First, there is strong momentum against change. In mathematics, this includes both resistance to technology as well as to not teaching some beautiful and useful mathematics. If a call for change/reform/improvement does not persevere, it is easy for an individual or a department to simply wait it out.

Second, fundamental change results from a collaborative, iterative process. No one engaged in large-scale curricular development gets it right the first time. But if a number of people try different approaches and then share their experiences, many get it better the next time.

Third, it makes no sense for a huge national enterprise like Calculus instruction to change overnight. Materials must be developed, refined, and adapted. Most faculty should do other work (mathematics research, applications of mathematics, other instructional acti vities) and after others have refined new approaches, then these faculty should consider the desirability of change. But at this stage they are the ones who need support, encouragement and the opportunity to make their own contribution.

The NSF Calculus Program was a cooperative activity of a large number of different programs and, indeed divisions, within the Foundation.

Much of the original impetus for the program came from the Division of Mathematical Sciences (DMS) under the leadership of Judy Sunley. From near its inception, administration and guidance for the program was provided by the Division of Undergraduate Education (DUE) and Division Director Robert Watson. Louise Raphael who was then in DMS moved to DUE to initially manage the program. The location within DUE was critical in that it provided continual interaction with the engineering, physics, chemistry, biology, and the broader scientific community. This is especially important since within universities faculty are often isolated from those in other disciplines. Since the Calculus Program was centered in undergraduate education, it profited from daily involvement with other disciplines conceming selection of reviewers, applications of Calculus, site visits of on-going projects, impact of the revised Calculus on courses in other departments, and determination of program goals.

The Division of Mathematical Sciences continued to participate fully in the program, providing a strong link with mathematicians whose current primary responsibilities lie in the area of research mathematics. Fundamental change in undergraduate instruction requires the full support of and full consultation with all components of higher education. Under the leadership of others such as Bernard McDonald, John Polking, Fred Wan, and Deborah Lockhart, DMS assured that the program maintained strong ties with the research community.

The Division of Elementary, Secondary, and Informal Education (ESIE) also participated thoroughly in the program. Many of the most talented students in the United States study Calculus in high school under the Advanced Placement program. If fundamental change in Calculus was to occur, newly developed materials needed to be tested at this level, appropriate training needed to be provided for AP teachers, and strong tiess needed to be formed among those who provided leadership in the AP program and those active in Calculus Reform. ESIE Division Directors Joan Leitzel and Margaret Cozzens assured that this happened. ESIE also participated fully in reconsidering appropriate preparation for Calculus whether it occurred in the schools or colleges.

Other EHR divisions also brought their expertise and resources to bear. Under the leadership of Kenneth Travers, the Division of Research, Evaluation, and Communications (originally Research, Evaluation, and Dissemination) provided the expertise of its staff in the areas of assessment and dissemination and resources to support this component of the Calculus Program.

On average, NSF devoted the time of approximately 1 1/2 Program Officers each year to the Calculus Program and associated projects; however, instead of two individuals devoting the bulk of their time to

Calculus effort, during the typical year six or seven different individuals devoted a portion of their time to the program.

Some of these individuals had long-term appointments to the Foundation and were able to provide stability and continuity. Also as a group the involved individuals had recent experience as faculty and leaders at a wide variety of different types of educational institutions (competitive liberal arts colleges, urban universities, community colleges, Historically Black Colleges and Universities (HBCUs), major research universities, etc.) and brought an understanding of the needs and the capabilities to contribute of each sector. In addition, they represented many academic disciplines including evaluation, engineering, chemistry, physics, biology, as well as mathematics. The large number of different individuals also made possible the involvement of the various NSF programs previously described.

NSF would not have been able to take the pro-active role that it did without the active, albeit part-time, involvement of so many different individuals. For example, it was natural and commonplace for a Program Officer to conduct a short site visit of a Calculus project as an add-on to other NSF travel. The national community of individuals committed to improving Calculus was strengthened by a group of individuals at NSF with this commitment as well as by individuals who returned to faculty status after working on Calculus at NSF. Many NSF program officers contributed to the Calculus Program by inviting reviewers, conducting site visits, and providing guidance on particular projects. The following list of individuals is restricted to those who actually formally recommended funding of Calculus projects. The individuals are listed by the year in which they were first at the Foundation. After each name the individual's Division within NSF, the individual's previous academic affiliation, and the year that he or she completed NSF service, if applicable. If their primary discipline was not mathematics, that has also been noted. The names of individuals who served as Lead Program Director for Calculus are indicated by *.

1987

Louise Raphael*, DUEIDMS; Howard Univ.; 1988 Duncan McBride, Physics, DUE, Swarthmore College William Lucas, DUE; Claremont Graduate School; 1987

1988

Joe Adney, ESIE; Michigan State Univ.; 1991
John Kenelly*, DUE; Clemson Univ.; 1988 Tom Berger, ESIE; Univ. of Minnesota; 1991
Deborah Lockhart, DMS, Michigan Technological Univ.

1989

John (Spud) Bradley*, DUE; Univ. of Tennessee; 1991. (returned to NSFIESIE in 1994 after service with American Mathematical Society)
William Haver, DUE; Virginia Commonwealth Univ., 1994
Jack Lohmann, Engineering; DUE; Univ. of Michigan, 1991
Joan Ferrini-Mundy, ESIE; Univ. of New Hampshire, 1992

1990

Glenda Lappan, ESIE; Michigan State Univ., 1992
Anne Steiner, DMS; Iowa State Univ., 1991
Herb Levitan, Life Sciences, DUE; Univ. of Maryland

1991

Elizabeth Teles, DUE; Montgomery Community College
James Lightbourne*, DUE; West Virginia Univ.
Ken Travers, REC; Univ. of Illinois, 1995
Ray Collings, ESIE; Tri-County Technical College, 1993
Marjorie Enneking, ESIEIDUE; Portland State Univ., 1993
William McHenry, Chemistry, HRD; Mississippi State Univ.

1992

Peter Braunfeld, ESIE; U ni v. of Illinois, 1993
Eric Robinson, ESIE; Ithaca College, 1994

1993

Lloyd Douglas, DMS
Tina Straley, DUE; Kennesaw State Univ., 1995
James Sandefur, ESIE, Georgetown Univ., 1994

1994

Richard Lesh, REC; Educational Testing Service

1995

Lee Zia, DUE; Univ. of New Hampshire, 1997

PART B:
Evidence of Impact of NSF Calculus Program

From the perspective of the undergraduate mathematics community in 1996, there is substantial evidence that there have been major changes in lower division collegiate mathematics instruction. Significant alterations have been made in the stated mathematical content and pedagogical goals of the three semester college level Calculus course as well as on the Advanced Placement (AP) Calculus course taught in the schools. Large numbers of mathematics faculty are professionally involved in efforts to improve Calculus instruction and identify themselves with the reform effort. The mathematical community believes that this change in undergraduate mathematics instruction focused on the semester Calculus sequence would not have occurred without support of the NSF Calculus Program. The following information describes the scope of the dramatic changes that have occurred.

I. Major changes have been made in the stated mathematical content and pedagogical goals of the three semester college level Calculus course.

	1990-91	1991-92 1992-93	1993-94	1994-95	1995-96
Academic Year S...	0	at test sites only	86,100	180,000	225,000

Table 1. *Student Use of NSF Supported "Reform" Calculus Texts*

These changes include: (a) extensive recognition of technology, both as a tool for learning and as the context in which mathematics is currently used, (b) substantially more applications of Calculus, both as a way of understanding a variety of everyday phenomena and as a tool in other academic disciplines, (c) more explicit expectations that students work and study as members of teams with other students, and that they work on long-term, demanding projects and problems in addition to short exercises, and (d) restructured courses of study that are designed to assure that students achieve a deeper understanding of Calculus from a geometric and numerical as well as analytical point of view. While different instructors and approaches incorporate these themes in various ways, they have been widely adopted.

With direct support from NSF a number of completely revised courses, with accompanying curricular materials, have been developed and widely adopted. Each of these full courses has been developed under the leadership of large numbers of different individuals at multiple institutions. These courses reflect the themes stated above (see Table 1).

Many Calculus courses are making use of new supplemental materials which variously support more applications, the use of computers or calculators, student projects, cooperative learning groups, and student writing. A survey conducted in preparation for the MAA report *Assessing Calculus Reform Efforts* [13] indicates that at the time of the survey "about half of the reform efforts are using traditional texts supplemented with materials and activities developed locally or nationally". NSF provided support for the development of many of these materials including New Mexico State's *Student Research Projects in Mathematics* (MAA; 6000 copies sold); the five volume *Resourcesfor Calculus Collection* (MAA; 2400 copies distributed free, 4000 sold); and Ithaca College's *Calculus: An Active Approach with Projects* (Wiley; 3,800 copies distributed).

Virtually all traditional college level texts have been written or revised to reflect the change in content and pedagogical goals described above. For example, see Table 2.

Text	Overall philosophy; rule of three: geometric, numeric and analytic approach	Use of technology	Use of longer term projects	Written and oral communication
Stuart; Brooks Cole	*"spirit* of reform pervades the book, but within the context of a traditional curriculum" "stronger focus on numerical and graphical points of view than other traditional books"	"appropriate use of technology ... characterized as involving the interaction between technology and Calculus"	"designated exercises are... Calculus laboratories and require considerable time for their completion"; "particularly value problems, in which a student has to combine methods from two or three different chapters"	Text does not assume that instructor will increase student communication expectation
Finney, Thomas, Demana, Waits; Addison Wesley	"A function ... can be represented numerically as a table of input-output pairs, graphically as a plot of output versus input, and symbolically as an algebraic representations". Text strives to use Graphical Numerical, and Algebraic (GNA) representations.	"Use of graphing utility is required with this textbook"	"GNA Explorations make the student an active partner in the learning process, ... explore and "do" mathematics ... Exploration bits ... provide an opportunity for active experimentation"	Explorations require written responses
Bradley and Smith; Prentice Hall	"Text was written to blend many of the best aspects of Calculus reform with the reasonable goals and methodologies of traditional Calculus ... text is attempt to *Reform witn Reason."*	"included over 60 pages devoted to use of technology". In addition, "platform specific" technology manuals are also available as supplements to the text.	Most Chapters include a group research project that "is loosely tied to the material contained in the chapter in which it is presented ... the complexity of the projects increase as we progress through the book."	Students should be encouraged to communicate mathematically ... students are encouraged to provide verbal explanations ... and are asked to summarize procedures and processes or to describe a mathematical result in everyday terms."

Table 2. *Self Description of Revised "Standard" Texts*

Highest Degree Granted by Institution	1992	1994
Ph.D.	60%	76%
M.S.	62%	74%
B.S.	60%	74%
Associate	45%	54%

Table 3. *Percentage of Institutions Participating in Calculus Reform*

According to *Assessing Calculus Reform Efforts* [13], a large percentage of mathematics departments nationally describe themselves as participating in Calculus reform activities to a modest or major extent (1048 of2439 departments responded to the 1994 survey). See Table 3.

Major changes have been made in the assessment of student learning. A collection of Calculus final exams was assembled for *Calculus for a New Century*, published in 1987 [12]. A similar collection of 24 final exams given in 1994- 95 was assembled by the MAA Calculus and the First Two Years Committee and published in *Calculus: The Dynamics of Change* [11] (these exams were selected from "reform courses", not randomly from courses across the country). While a full comparison can only be made by directly reading the two sets of exams, the comparisons in Table 4 below give an idea of the difference in flavor.

The changes in the Calculus course have resulted in a new Graduate Record Exam on mathematical reasoning to replace the current quantitative test for students planning graduate study in engineering, the physical sciences, the mathematical sciences, computer science, economics, and some areas of biology. According to the MAA report *Assessing Calculus Reform Efforts* [13, p. 40], "the most significant impact of the Calculus reform to date outside of the mathematics community is the recent decision of the Graduate Record Examination program to develop ... [this] test which will contain questions in the spirit of Calculus reform."

Equally dramatic and related changes are beginning to take place in business Calculus, linear algebra and differential equations courses.

II. The NSF Calculus Program has had a major impact on Advanced Placement (AP) Calculus course taught in the schools.

Each year approximately 130,000 high school students take an Advanced Placement (AP) exam following this college level AP Calculus course (taken by 250,000 students).

Beginning with the 1994-95 academic year students studying AP Calculus nationwide have made use of technologically advanced graphing calculators on a regular basis throughout their course. In addition, the AP exams were designed with the expectation that all students taking the exam would make use of their graphing calculators during the exam.

A much more significant change will take place beginning in the Fall of 1997. This dramatic change will occur since the goals of the AP course have been significantly revised and clearly reflect both the National Council of Teachers of Mathematics *Standards* and the changes that are occurring in Calculus as taught in colleges and universities. These new goals apply to courses offered in 1997- 98 and beyond they determined the content of the May 1998 APexam:

> Students should be able to work with functions in a variety of ways: graphical, numerical, analytical, or verbal.
>
> Students should be able to use technology to help solve problems, experiment, interpret results, and verify conclusions.
>
> Students should be able to communicate mathematics both orally and in well-written sentences.
>
> Students should understand the meaning of derivative "' and definite integral ... and should be able to

	1985 EXAMS	1995 EXAMS
use of technology	paper and pencil	use of calculators and/or computers expected/required on at least a portion of almost all exams
computations by hand of derivatives and integrals	significant portion of almost all exams	omitted from or minor portion of most exams
students required to analyze information given graphically or numerically	rarely	required on almost every exam
expectation for student writing	writing not expected on most exams	typical exams: require student "essay"; ask students to "describe how to find (I want words here not drawing)"; ask students to "write in complete sentences and include pictures and/or formulas if you think it helps the explanation"; or otherwise require student explanation in writing.

Table 4. *Comparison of* 1985 *and* 1995 *Course Exams*

model a written description of a physical situation with a function, a differential equation, or an integral. For a more detailed summary of these changes see *Reform and AP Calculus* by Ray Conner and Anita Solow in [11].

III. Large numbers of mathematics faculty are professionally involved in efforts to improve Calculus instruction and identify themselves with the reform effort.

Victory finds a hundred fathers but defeat is *an orphan-Ciano.* The ranks of those who identify themselves with Calculus reform extends far beyond those who have ever received any financial support from the NSF program, and surely beyond the very few who currently receive such support.

Substantially larger numbers of mathematicians are reporting on their scholarly work related to curriculum development and lower level collegiate instruction at national meetings. The data provided in Table 5 was obtained from the January 1988 and 1996 *Notices of the American Mathematical Society* [5,7]. The 1988 issue also contained the first official announcement of the NSF Calculus Program.

There is substantial evidence that large numbers of faculty are actively engaged in long-term efforts to improve Calculus instruction at their home institutions. At the University of Michigan, where Calculus I has a Fall enrollment of approximately 1900 in 60 sections, all new Calculus instructors (TAs, new junior faculty) participate in a seven day beginning-of-term training program and all instructors meet weekly in small groups. In the state of Washington, faculty from 20 different institutions have been actively engaged in a four year effort to improve Calculus instruction. For the past three years all faculty who teach Calculus at Virginia Commonwealth University have received training in workshops and then participated in regular meetings to share experiences. Faculty at Rio Hondo, a community college in Whittier, California, committed itself through a formal vote: "only people who attend a workshop and agree to the new techniques (new materials, graphing calculators, collaborative learning) will be scheduled to teach any of the Calculus classes." [13 p. 42]; Curtis Chipman of Oakland University reports "Our colleagues in Student Affairs now regard us as the academic unit most concerned with actual student performance." [10 p. 49].

Large numbers of mathematics faculty identify themselves with the Calculus reform efforts in many ways: In June 1996, over 500 mathematics faculty participated in the Fifth Annual Conference on the Teaching of Mathematics (formerly Calculus) sponsored by John Wiley, publishers of the Calculus Consortium based at Harvard reform Calculus text. Reporting on the 1995 conference [3], George Rublein stated, "Having won the main battle, Calculus reformers are now focusing their sights on preparation and aftermath". More than 70 mathematics faculty helped prepare the MAA report, *Calculus; The Dynamics of Change* by [11] providing examinations, being interviewed for articles, participating in panel discussion or writing articles. In *Crossroads in Mathematics* [1] the American Mathematical Association of Two-Year Colleges standards call for: appropriate technology; interactive leaming through student writing, reading, speaking and collaborative activities; connections between mathematics and other disciplines; and

	January 1988 National "Joint Mathematics Meetings"[5]	January 1996 National "Joint Mathematics Meetings"[7]
Total Registered Attendance	2882	3379
Lead Workshops on Lower Level Instruction	14 (8 different workshops)	22 (10 different workshops)
Presented Posters at Poster Sessions	0	150 (75 different poster presentations)
Served on Panels Discussing Introductory Level Mathematics	13 (3 different panels)	33 (7 different panels)
Presented Papers on Introductory Level Mathematics	31	338 (233 different presentations)
Total Individuals in Leadership Positions (Some Individuals Counted in More than one Category)	58	543

Table 5. *Presentations at National Meetings: 1988-1996*

the use of multiple approaches: numeric, graphical, symbolic, and verbal. More than 100 individuals participated in developing these standards. More than 75 individuals participated in writing the 1995 Special Issue of UME Trends devoted to Calculus Reform [14].

IV. The mathematical community believes that this major change in undergraduate mathematics instruction focused on the semester Calculus sequence would not have occurred without support of the NSF Calculus Program.

This belief is shared by those actively involved in the curriculum development, by those who lead change at their own institution with or without direct NSF support, by textbook publishers, by those who participated in assessment of Calculus reform efforts, as well as by critics of the reform movement.

Describing the initiation of the Calculus reform efforts, with its attendant slogan "Calculus should be a pump, not a filter", Wayne Roberts [11] reports "What is clear is that the National Science Foundation became a spigot, and the mathematical community became lively."

Those who place new Ph.D. 's report that applicants with experience with NSF supported Calculus reform efforts have an edge in many settings.

According to respondents to the MAA assessment survey [13], those involved in changing Calculus believe that the NSF Calculus Initiative was critical: "There is absolutely no doubt that the NSF Calculus Incentive was fundamental to the changes ... "; "it was essential at our institution"; "the NSF initiative was a very positive source at our institution;" "there has been awareness of the need for change for about 20 years, but the materials weren't there .. the quantum leap has been made, thanks to NSF support."

Within the community of textbook publishers, there is the feeling that a company has "missed the boat" if it does not have a text developed with NSF support. Sales personnel report that the NSF support imprimatur is a strong selling point with those faculty interested in reform.

Mathematicians routinely link the Calculus reform with NSF. For example, in expressing concern with plans of the University of Rochester to "relegate the teaching of Calculus" to non-mathematicians, Kenneth A. Ross wrote as President of the MAA and on behalf of the MAA Executive Committee [8, p. 3 I I]. He stated "Recent initiates *by the National Science Foundation* have resulted in major changes in the way that Calculus is taught [emphasis added]."

The view of the impact of the NSF support is shared by those who question the priorities for funding. For example, in a signed editorial in the October 1995 *Notices of the American Mathematical Society* [6], Steven Kranz, states:

"If you don't think that government funding can influence the direction that mathematical activity will take, then look around you ... projects to develop mathematical instruction are proliferating ... In one case, a person who was formerly the world's greatest expert in a fairly narrowly defined branch of analysis suddenly became the world's greatest expert in teaching Calculus using Mathematica. I'm not passing judgment; that's just how it is."

The MAA Assessment report states [13], "From the start of serious efforts to develop reform Calculus courses, there has been universal agreement that the Calculus reform movement, as we now know it, would have been impossible without the NSF Calculus Initiative."

Calculus Reform Timeline

January 1986. Tulane Conference. Organized by Ronald Douglas; 25 participants; Conference proceedings *"Toward a Lean and Lively Calculus"* [2], published by MAA.

March 1986. Publication of NSF Neal Report *"Undergraduate Science, Mathematics and Engineering Education,"* [15] urging renewed NSF support for undergraduate education.

June 1986. NSF Mathematics Disciplinary Workshop [10]. In response to the Neal Report the Foundation conducted workshops in a number of different scientific disciplines. The mathematics workshop recommended that the Foundation focus mathematics curriculum development activities on Calculus.

October 1987. National Research Council national colloquium; held with Sloan Foundation support; 700 participants. Conference Proceedings *"Calculus for a New Century"* [12] published by MAA.

October 1987. NSF Calculus Program announced by Division of Mathematical Sciences (DMS), in cooperation with Office of Undergraduate Education. Management responsibility placed with Undergraduate Education (now DUE) with understanding that program would continue as joint DUEIDMS effort.

Spring 1988. First proposals for support under Calculus Program received.

Fall 1988. First grants awarded as part of NSF Calculus Program. Many of the awards were for planning grants to enable groups of faculty to develop visions of what was desirable and achievable in the teaching of Calculus. Startup funds provided for dissemination publication *UME Trends* (Undergraduate Mathematics Education)

October 1988. First annual meeting of Calculus project directors.

September 1989. First large-scale awards made under Calculus Program. The Foundation received a substantial number of carefully designed long-term proposals in Spring

1989. A number of proposals for large-scale projects received very high reviews. Funding of some of these was made possible by severely cutting budgets and committing as much future year NSF Calculus funds as regulations permitted. Others received smaller awards and were encouraged to apply again in future years.

October 1989. Browder-Barrett report published. Browder and Barrett, Presidents respectively ofthe American Mathematical Society and the Mathematical Association of America, reviewed the beginning of the NSF Calculus Program and endorsed the initial directions of the Program.

December 1990. Calculus Program Announcement broadened to include courses following first year Calculus and to support local adaptation and large-scale implementation of newly developed approaches.

August 1991. Funded project to train Calculus AP teachers nationwide to prepare for new course.

December 1991. Program announcement revised to increase emphasis on courses leading up to Calculus. Division of Elementary and Secondary Education formally commits to full participation in program.

June 1992. NSF sponsors Assessment of Calculus Workshop. Subsequent to the Workshop a Working Group on Assessment was formed in November 1992 to guide assessment activities in Calculus. The report of the group was published in 1997.

December 1993. NSF announces new program, "Mathematical Sciences and their Applications throughout the Curriculum" .

September 1994. Last projects supported by separate NSF Calculus Program are funded and these proposers begin their work.

Summer 1995. MAA report *Assessing Calculus Reform Efforts* [13] published.

March 1999. Last projects funded by the separate Calculus Program complete their funded work. NSF continues to support curriculum development in Calculus through its Course, Curriculum and Laboratory Improvement Program.

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