Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004

A summary of the report by the
Committee on the Undergraduate Program in Mathematics of
The Mathematical Association of America

The full report is available at `www.maa.org/cupm/`.

CUPM Guide 2004 was prepared with support from the National Science Foundation
and from the Calculus Consortium for Higher Education.

Copyright ©2004 by
The Mathematical Association of America (Incorporated)

Library of Congress Catalog Card Number 2004100651

ISBN 0-88385-814-2

Printed in the United States of America

Current printing (last digit):
10 9 8 7 6 5 4 3 2 1

Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004

A summary of the report by the
Committee on the Undergraduate Program in Mathematics of
The Mathematical Association of America

CUPM Writing Team

William Barker
David Bressoud
Susanna Epp
Susan Ganter
Bill Haver
Harriet Pollatsek (chair)

Published and Distributed by
The Mathematical Association of America

Committee on the Undergraduate Program in Mathematics (CUPM)

Dora Ahmadi, Morehead State University
Thomas Banchoff, Brown University
*William Barker, Bowdoin College
*Lynne Bauer, Carleton College
*Thomas Berger (Chair 1994–2000), Colby College
David Bressoud, Macalester College
*Amy Cohen, Rutgers University
Lynda Danielson, Albertson College
*Susanna Epp, DePaul University
*Naomi Fisher, University of Illinois at Chicago
*Joseph Gallian, University of Minnesota-Duluth
Ramesh Gangolli, University of Washington
*Frank Giordano, US Military Academy (ret) and COMAP
Jose Giraldo, Texas A&M University, Corpus Christi
*Bill Haver, Virginia Commonwealth University
Dianne Hermann, University of Chicago
*Peter Hinman, University of Michigan
*Herbert Kasube, Bradley University
Daniel Maki, Indiana University
Joseph Malkevitch, York College CUNY
Mercedes McGowen, Harper Community College
Harriet Pollatsek (Chair 2000–2003), Mount Holyoke College
Marilyn Repsher, Jacksonville University
Allan Rossman, California Polytechnic State University, San Luis Obispo
Kathleen Snook, US Military Academy (ret)
*Olaf Stackelberg, Kent State University
Michael Starbird, University of Texas

* Term on CUPM ended before completion of *CUPM Guide2004*

Curriculum Project Steering Committee

William Barker
Thomas Berger
David Bressoud
Susanna Epp
Susan Ganter
Bill Haver
Herbert Kasube
Harriet Pollatsek (chair)

Susan Ganter chairs the CUPM subcommittee on Curriculum Renewal Across the First Two Years (CRAFTY); William Barker is former chair of CRAFTY.

The writing team acknowledges with gratitude the assistance of Barry Cipra (preliminary drafting), Kathleen Snook (compiling and editing Illustrative Resources during the winter and spring of 2003), and Tom Rishel and Michael Pearson (providing MAA staff support).

Contents

Executive Summary

The Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM) is charged with making recommendations to guide mathematics departments in designing curricula for their undergraduate students. CUPM began issuing reports in 1953, updating them at roughly 10-year intervals. *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004* is based on four years of work,[1] including extensive consultation with mathematicians and members of partner disciplines.[2] Available at `www.maa.org/cupm/`, *CUPM Guide 2004* contains the recommendations unanimously approved by CUPM in January 2003.

Many recommendations in *CUPM Guide 2004* echo those in previous CUPM reports, but some are new. In particular, previous reports focused on the undergraduate program for mathematics majors, although with a steadily broadening definition of the major. *CUPM Guide 2004* addresses the *entire* college-level mathematics curriculum, for *all* students, even those who take just one course. *CUPM Guide 2004* is based on six fundamental recommendations for departments, programs and all courses in the mathematical sciences. The MAA Board of Governors approved these six recommendations at their Mathfest 2003 meeting.

Recommendation 1: *Mathematical sciences departments should*

- *Understand the strengths, weaknesses, career plans, fields of study, and aspirations of the students enrolled in mathematics courses;*
- *Determine the extent to which the goals of courses and programs offered are aligned with the needs of students as well as the extent to which these goals are achieved;*
- *Continually strengthen courses and programs to better align with student needs, and assess the effectiveness of such efforts.*

Recommendation 2: *Every course should incorporate activities that will help all students progress in developing analytical, critical reasoning, problem-solving, and communication skills and acquiring mathematical habits of mind. More specifically, these activities should be designed to advance and measure students' progress in learning to*

- *State problems carefully, modify problems when necessary to make them tractable, articulate assumptions, appreciate the value of precise definition, reason logically to conclusions, and interpret results intelligently;*
- *Approach problem solving with a willingness to try multiple approaches, persist in the face of difficulties, assess the correctness of solutions, explore examples, pose questions, and devise and test conjectures;*

[1] Supported by the National Science Foundation and the Calculus Consortium for Higher Education.

[2] Reports from a series of workshops on the mathematics curriculum with members of partner disciplines are contained in *The Curriculum Foundations Project: Voices of the Partner Disciplines,* edited and with an introduction by Susan Ganter and William Barker (MAA, 2004).

- *Read mathematics with understanding and communicate mathematical ideas with clarity and coherence through writing and speaking.*

Recommendation 3: *Every course should strive to*

- *Present key ideas and concepts from a variety of perspectives;*
- *Employ a broad range of examples and applications to motivate and illustrate the material;*
- *Promote awareness of connections to other subjects (both in and out of the mathematical sciences) and strengthen each student's ability to apply the course material to these subjects;*
- *Introduce contemporary topics from the mathematical sciences and their applications, and enhance student perceptions of the vitality and importance of mathematics in the modern world.*

Recommendation 4: *Mathematical sciences departments should encourage and support faculty collaboration with colleagues from other departments to modify and develop mathematics courses, create joint or cooperative majors, devise undergraduate research projects, and possibly team-teach courses or units within courses.*

Recommendation 5: *At every level of the curriculum, some courses should incorporate activities that will help all students progress in learning to use technology*

- *Appropriately and effectively as a tool for solving problems;*
- *As an aid to understanding mathematical ideas.*

Recommendation 6: *Mathematical sciences departments and institutional administrators should encourage, support and reward faculty efforts to improve the efficacy of teaching and strengthen curricula.*

Part I of *CUPM* G*uide 2004* elaborates on these recommendations and suggests ways that a department can evaluate its progress in meeting them. Part II contains supplementary recommendations concerning particular student audiences:

A. Students taking general education or introductory courses in the mathematical sciences;

B. Students majoring in partner disciplines, including those preparing to teach mathematics in elementary or middle school;

C. Students majoring in the mathematical sciences;

D. Mathematical sciences majors with specific career goals: secondary school teaching, entering the non-academic workforce, and preparing for post-baccalaureate study in the mathematical sciences and allied disciplines.

Specific methods for implementation are not prescribed, but the online document *Illustrative Resources for CUPM Guide 2004* at `www.maa.org/cupm/` describes a variety of experiences and resources associated with these recommendations. These illustrative examples are not endorsed by CUPM, but they may serve as a starting point for departments considering enhancement of their programs. Pointers to additional resources, such as websites (with active links) and publications, are also given.

Introduction

Mathematics is universal: it underlies modern technology, informs public policy, plays an essential role in many disciplines, and enchants the mind. At the start of the twenty-first century, the undergraduate study of mathematics can and should be a vital and engaging part of preparation for many careers and for well-informed citizenship. In the *CUPM Guide 2004*, the term ‘mathematics’ is generally synonymous with ‘mathematical sciences’ and refers to a collection of mathematics-related disciplines, including, but not necessarily limited to, pure and applied mathematics, mathematics education, computational mathematics, operations research, and statistics. Departments of mathematical sciences can and should play a central role in their institutions’ undergraduate programs. The *CUPM Guide 2004*[3] calls on mathematicians and mathematics departments to rethink the full range of their undergraduate curriculum and co-curriculum to ensure the best possible mathematical education for all their students, from liberal arts students taking just one course to students majoring in the mathematical sciences.

The need for action

Over the past one hundred years mathematics has become more important to more disciplines than ever before. At the same time both the number and diversity of students in post-secondary education and the variety of their mathematical backgrounds have increased dramatically. Additionally, computer technology has forever altered the way mathematics is used in the workplace, from retail store registers to financial institutions to laboratories doing advanced scientific research.

These developments present unprecedented curricular challenges to departments of mathematical sciences—challenges many departments and individual faculty members are engaged in meeting. During the past twenty years there has been an explosive increase in the number of presentations and publications on issues and innovations in the teaching of post-secondary mathematics.[4] This activity reflects a growing movement to address the undergraduate mathematics curriculum conscientiously and creatively.

But there are indicators that all is not well. National data provide clear evidence that undergraduate mathematics programs are under serious pressure, with decreasing numbers of mathematics majors and declining enrollment in advanced mathematics courses.[5] From 1985 to 2000 the total number of bachelor’s degrees awarded annually in the U.S. rose 25% and the number of science and technology degrees grew

---

[3] The MAA publication *Guidelines for Programs and Departments in Undergraduate Mathematical Sciences*, MAA, 2001, available at `www.maa.org/guidelines/guidelines.html`, complements the *CUPM Guide 2004* and other curricular reports by presenting a set of recommendations that deal with a broad range of structural issues that face mathematical sciences departments and their institutional administrations.

[4] For example, at the January 2003 Joint Mathematics Meetings, more than one third of the talks concerned mathematics education.

[5] Appendix 3 of the full *Guide* contains further analysis of data on numbers of majors and the supply of secondary teachers of mathematics, and Appendix 4 contains data on enrollment in and availability of advanced courses.

20%. However, data collected by the Conference Board of the Mathematical Sciences (CBMS)[6] show that the total number of degrees awarded annually by mathematics and statistics departments, including those in secondary mathematics education, stayed essentially flat during this 15-year period. In fact, the annual total in these departments fell 4% between 1995 and 2000, and the number of annual degrees in mathematics fell 19% in the 1990s. The drop in mathematics degrees occurred at the same time as the need for new teachers of secondary mathematics grew more acute.

One might expect the increase in science and technology degrees to translate into higher enrollment in advanced mathematics courses as allied subjects. In fact, the opposite has occurred: enrollment in advanced courses taught in mathematics departments has fallen, dropping 25% from 1985 to 2000; an increase from 1995 to 2000, while encouraging, has not returned enrollment to 1990 levels. Further, CBMS data show that even the *availability* of advanced courses has declined in the past five years, as the percentages of departments offering several typical courses[7] has decreased, in some cases by more than 20%. This trend is unfortunate, not only for the health of the mathematical sciences major but also because the health of disciplines that use mathematics—and by extension the health of society—is enhanced when a significant number of students are knowledgeable about the advanced mathematics that is relevant to their fields.

Striking successes at a number of colleges and universities demonstrate that these trends can be reversed. For instance, the MAA volume *Models That Work: Case Studies in Effective Undergraduate Mathematics Programs*[8], summarizes effective practices at a set of mathematics departments that have excelled in (i) attracting and training large numbers of mathematics majors, or (ii) preparing students to pursue advanced study in mathematics, or (iii) preparing future school mathematics teachers, or (iv) attracting and training underrepresented groups in mathematics. Site-visits to ten departments and information on a number of others revealed "no single key to a successful undergraduate program in mathematics." However, there were common features. "What was a bit unexpected was the common attitude in effective programs that the faculty are not satisfied with the current program. They are constantly trying innovations and looking for improvement."

Areas for attention and action

Mathematics departments need to serve *all* students well—not only those who major in the mathematical or physical sciences. The following steps will help departments reach this goal.

- Design undergraduate programs to address the broad array of problems in the diverse disciplines that are making increasing use of mathematics.
- Guide students to learn mathematics in a way that helps them to better understand its place in society: its meaning, its history, and its uses. Such understanding is often lacking even among students who major in mathematics.
- Employ a broad range of instructional techniques, and require students to confront, explore, and communicate important ideas of modern mathematics and the uses of mathematics in society. Students need more classroom experiences in which they learn to think, to do, to analyze—not just to memorize and reproduce theories or algorithms.

[6] *CBMS 2000: Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States*, D. Lutzer, J. Maxwell and S. Rodi, AMS, 2002.

[7] Including algebra, analysis, geometry, mathematical modeling and applied mathematics; see Table 4-3 in Appendix 4 of the full *Guide*.

[8] MAA Notes **38** (1995), Alan C. Tucker, editor.

- Understand and respond to the impact of computer technology on course content and instructional techniques.
- Encourage and support faculty in this work—a task both for departments and for administrations.

The *CUPM Guide 2004* presents six general recommendations to assist mathematics departments in the design and teaching of all of their courses and programs. Later in the *Guide* these recommendations are elaborated and made specific for particular student audiences.

Using the *CUPM Guide 2004*

Part I of the *CUPM Guide 2004* elaborates on and specifies the meaning of the six general recommendations as well as suggesting ways that a department can evaluate progress in meeting them. Part II contains supplementary recommendations for particular student audiences.

Some students major in fields that do not require specific mathematical preparation. They may take one course in mathematics, perhaps to satisfy a general education requirement of their institution or major program. Section A of Part II addresses the needs of these students, many of whom—especially among the hundreds of thousands enrolled each semester in courses called College Algebra—are not optimally served by the mathematics courses they take.

Partner disciplines are those whose majors are required to take one or more specific mathematics courses. These disciplines vary by institution but usually include the physical sciences, the life sciences, computer science, engineering, economics, business, education, and often several social sciences. Recommendations concerning these students are in Section B of Part II, including those for pre-service K–8 teachers.

Section C of Part II contains recommendations concerning students majoring in the mathematical sciences. The recommendations urge departments to learn the probable career paths and needs of their majors and offer them a flexible program that provides appropriate breadth and depth. Section D contains further recommendations for mathematical sciences majors preparing to teach secondary school mathematics, planning for non-academic employment, or intending post-baccalaureate study.

There are many ways to carry out each recommendation, and different choices will be appropriate in different institutional settings. Consequently, these recommendations rarely specify particular courses or syllabi. That doesn't mean "anything goes." Indeed, each recommendation is accompanied by measures to help a department gauge its effectiveness.[9] As stated in the *MAA Guidelines for Programs*, "These measures will, of necessity, be multi-dimensional since no single statistic can adequately represent departmental performance with respect to most departmental goals. Measures of student learning and other student outcomes should be included."[10] Course syllabi and sample assignments, along with their contribution to students' grades, are other valuable measures.

Although no specific methods for implementation are outlined, the section entitled Illustrative Resources is designed to help departments implement and improve practices to satisfy the recommendations. It is organized and numbered the same way as the recommendations in Parts I and II. A variety of examples, including assignments, courses (with suggested syllabi and texts), and programs are provided for each recommendation. The examples range along a continuum, from modest first steps and small changes that can be easily effected to more ambitious efforts. Pointers to additional resources, such as websites and publications, are also given.

These recommendations have been reduced to a core judged essential for building and supporting department strength and effectively meeting department obligations. They are not a wish list for an ideal

[9] Appendix 6 of the full *Guide* contains sample questions for department self-study.

[10] MAA, 2001, available at `www.maa.org/guidelines/guidelines.html`.

future department. Indeed, the reality is that departments at many institutions are coping with diminished human and financial resources and conflicting and escalating demands on faculty time. Moreover, meaningful change is never easy. Nonetheless, the use of the word "should" in a recommendation means that departments are expected to make a conscientious effort to achieve steady improvement until they are able to satisfy it.

Background for the recommendations

The Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM) is charged with making recommendations to guide mathematics departments in designing curricula for their undergraduate students. CUPM began issuing reports in 1953, updating them at roughly 10-year intervals. In 1999 work began on the current recommendations. CUPM solicited position papers from prominent mathematicians and conducted panel discussions and focus groups at national meetings to obtain reactions to preliminary drafts of these recommendations. There has been extensive consultation with other professional societies in the mathematical sciences. From 1999 to 2002 CUPM's subcommittee on Curriculum Renewal Across the First Two Years (CRAFTY) conducted a series of workshops on the mathematics curriculum with participants from a broad range of partner disciplines.[11]

The six fundamental recommendation of this *Guide* were approved by the Board of Governors of the MAA in July 2003. All of the recommendations were unanimously approved by CUPM in January 2003. Many of the current recommendations echo those in previous CUPM reports, but some are new. In particular, previous reports focused on the undergraduate program for mathematics majors, although with a steadily broadening definition of the major in the 1981 and 1991 reports. The *CUPM Guide 2004*, in contrast, addresses the *entire* college-level mathematics curriculum, for *all* students, even those who take just one course.[12]

[11] Appendices 1 and 2 of the full *Guide* contain detailed accounts of CUPM's activities and of the CRAFTY workshops collectively known as the Curriculum Foundations project. The results of the project are contained in the MAA publication *The Curriculum Foundations Project: Voices of the Partner Disciplines,* edited and with an introduction "A Collective Vision: Voices of the Partner Disciplines" by Susan Ganter and William Barker.

[12] While attempting to address the college-level curriculum in mathematics more comprehensively, the *CUPM Guide 2004* does not discuss a number of important issues, including non-credit or developmental courses and articulation between institutions.

Part I: Recommendations for Departments, Programs, and all Courses in the Mathematical Sciences

1. Understand the student population and evaluate courses and programs

In summarizing the common features of the programs described in *Models That Work,* the authors wrote that one of the "states of mind that underlie faculty attitudes in effective programs" is "teaching for the students one has, not the students one wished one had." *Towards Excellence: Leading a Doctoral Mathematics Department in the 21st Century* echoes this theme: "Mathematics departments should position themselves to receive new or reallocated resources by meeting the needs of their institutions. That does not mean sacrificing the intellectual integrity of an academic program, nor does it mean relegating mathematics to a mere service role. It *does* mean fulfilling a bargain with the institution in which one lives, and for most departments a major part of that bargain involves instruction."[13]

Recommendation 1: Mathematical sciences departments should

- Understand the strengths, weaknesses, career plans, fields of study, and aspirations of the students enrolled in mathematics courses;
- Determine the extent to which the goals of courses and programs offered are aligned with the needs of students as well as the extent to which these goals are achieved;
- Continually strengthen courses and programs to better align with student needs, and assess the effectiveness of such efforts.

2. Develop mathematical thinking and communication skills

The power of mathematical thinking — pattern recognition, generalization, abstraction, problem solving, careful analysis, rigorous argument — is important for every citizen. It is highly valued by employers and by other disciplines but widely misunderstood and undervalued by students.

Communication is integral to learning and using mathematics, and skill in communicating is commonly listed as the most important quality employers seek in a prospective employee.[14] However, many students expect mathematics classes to be wordless islands where they won't be asked to read, write, or discuss ideas.

Appropriate instructional approaches to reasoning and proof have been passionately debated among mathematicians for decades, but with a greater sense of urgency during the last twenty years. While much remains to be learned about how best to teach reasoning and proof skills—as well as how best to improve

---

[13] American Mathematical Society Task Force on Excellence, J. Ewing editor, AMS, 1999, p. xiii.

[14] See, for instance, surveys by the National Association of Colleges and Employers, `www.naceweb.org`.

communication skills—a variety of strategies can help students progress. Mathematics faculty should deliver an unambiguous message concerning the importance of mathematical reasoning and communication skills and adopt instructional methods and curriculum content that develop these skills. Designing a curriculum that develops these skills effectively and at appropriate levels for all students is one of the biggest and most important challenges for mathematics departments.

Recommendation 2: Every course should incorporate activities that will help all students progress in developing analytical, critical reasoning, problem-solving, and communication skills and acquiring mathematical habits of mind. More specifically, these activities should be designed to advance and measure students' progress in learning to

- State problems carefully, modify problems when necessary to make them tractable, articulate assumptions, appreciate the value of precise definition, reason logically to conclusions, and interpret results intelligently;
- Approach problem solving with a willingness to try multiple approaches, persist in the face of difficulties, assess the correctness of solutions, explore examples, pose questions, and devise and test conjectures;
- Read mathematics with understanding and communicate mathematical ideas with clarity and coherence through writing and speaking.

3. Communicate the breadth and interconnections of the mathematical sciences

Many students do not see the connections between mathematics and other disciplines or between mathematics and the world in which they live. Too often they leave mathematics courses with a superficial mastery of skills that they are unable to apply in non-routine settings and whose importance to their future careers is unrecognized. Conceptual understanding of mathematical ideas and facility in mathematical thinking are essential for both applications and further study of mathematics, yet they are often lost in a long list of required topics and computational techniques. Even when students successfully apply mathematical techniques to problems, they are often unable to interpret their results effectively or communicate them with clarity.

The beauty, creativity, and intellectual power of mathematics and its contemporary challenges and discoveries, are often unknown and unappreciated. The interplay between differing perspectives—continuous and discrete, deterministic and stochastic, algebraic and geometric, exact and approximate—is appreciated by very few students, even though flexible use of these varying perspectives is critical for applications and for learning new mathematics.

Recommendation 3: Every course should strive to

- Present key ideas and concepts from a variety of perspectives;
- Employ a broad range of examples and applications to motivate and illustrate the material;
- Promote awareness of connections to other subjects (both in and out of the mathematical sciences) and strengthen each student's ability to apply the course material to these subjects;
- Introduce contemporary topics from the mathematical sciences and their applications, and enhance student perceptions of the vitality and importance of mathematics in the modern world.

4. Promote interdisciplinary cooperation

Mathematics programs have traditionally drawn heavily from the physical sciences for applications. In recent years, mathematics has come to play a significant role in far more disciplines, but many mathematics programs have not adjusted to this new reality.

Mathematics departments should seize the opportunity to harness the growing awareness in other disciplines of the power and importance of mathematical methods. A curriculum developed in consultation with other disciplines that includes a variety of courses and degree options can attract more students, help them learn important mathematical ideas, retain more students for intermediate and advanced coursework, strengthen their ability to apply mathematics to other areas, and improve the quantity and quality of the mathematics majors and minors.

Recommendation 4: Mathematical sciences departments should encourage and support faculty collaboration with colleagues from other departments to modify and develop mathematics courses, create joint or cooperative majors, devise undergraduate research projects, and possibly team-teach courses or units within courses.

5. Use computer technology to support problem solving and to promote understanding

Recent advances in desktop and handheld computer technology can be used to improve the pedagogy and content of mathematics courses at all levels. Some mathematical ideas and procedures have become less important because of these emerging technological tools; others have gained importance. The 2001 *MAA Guidelines for Programs and Departments in Undergraduate Mathematical Sciences* recommended that departments "should employ technology in ways that foster teaching and learning, increase the students' understanding of mathematical concepts, and prepare students for the use of technology in their careers or in their graduate study."[15] However, few mathematics departments have effectively met the challenges posed by the growth of technology, and many are only beginning to address seriously the issues it raises.

Recommendation 5: At every level of the curriculum, some courses should incorporate activities that will help all students progress in learning to use technology

- Appropriately and effectively as a tool for solving problems;
- As an aid to understanding mathematical ideas.

6. Provide faculty support for curricular and instructional improvement

Many of the recommendations in this *Guide*, including collaborating with colleagues in other disciplines, adapting material from other parts of mathematics or from other disciplines for use in teaching, evaluating student writing, and making effective use of technology, require time and effort from faculty beyond what they might ordinarily devote to the revision and creation of courses. Departments and administrators need to acknowledge that meeting these recommendations makes substantial demands on faculty (and, in some cases, on graduate teaching assistants and other temporary or part-time instructors).

Recommendation 6: Mathematical sciences departments and institutional administrators should encourage, support and reward faculty efforts to improve the efficacy of teaching and strengthen curricula.

[15] *Guidelines for Programs and Departments in Undergraduate Mathematical Sciences*, MAA, 2001, available at `www.maa.org/guidelines/guidelines.html`.

Part II: Additional Recommendations Concerning Specific Student Audiences

The recommendations in Part II of the *Guide* are supplementary to those in Part I addressing all students.

A. Students taking general education or introductory collegiate courses in the mathematical sciences

General education and introductory courses enroll almost twice as many students as all other mathematics courses combined.[16] They are especially challenging to teach because they serve students with varying preparation and abilities who often come to the courses with a history of negative experiences with mathematics. Perhaps most critical is the fact that these courses affect life-long perceptions of and attitudes toward mathematics for many students—and hence many future workers and citizens. For all these reasons these courses should be viewed as an important part of the instructional program in the mathematical sciences.

This section concerns the student audience for these entry-level courses that carry college credit. A large percentage of these students are enrolled in college algebra. Traditional college algebra courses, with a primary emphasis on developing skills in algebraic computation, have a long history at many institutions.

Students enrolled in college algebra courses generally fall into three categories:

1. Students taking mathematics to satisfy a requirement but not specifically required to take a course called college algebra;
2. Students majoring in areas or studying within states or university systems that specifically require a course called college algebra;
3. Students intending to take courses such as statistics, calculus, discrete mathematics, or mathematics for prospective elementary or middle school teachers and who need additional preparation for these courses.

Unfortunately, there is often a serious mismatch between the original rationale for a college algebra requirement and the actual needs of the students who take the course. A critically important task for mathematical sciences departments at institutions with college algebra requirements is to clarify the rationale for the requirements, determine the needs of the students who take college algebra, and ensure that the department's courses are aligned with these findings (see Recommendation A.2).

---

[16]According to the CBMS study in the Fall of 2000, a total of 1,979,000 students were enrolled in courses it classified as "remedial" or "introductory" with course titles such as elementary algebra, college algebra, pre-calculus, algebra and trigonometry, finite mathematics, contemporary mathematics, quantitative reasoning. The number of students enrolled in these courses is much greater than the 676,000 enrolled in calculus I, II or III, the 264,000 enrolled in elementary statistics, or the 287,000 enrolled in all other undergraduate courses in mathematics or statistics. At some institutions, calculus courses satisfy general education requirements. Although calculus courses can and should meet the goals of Recommendation A.1, such courses are not the focus of this section.

Because many students taking introductory mathematics decide not to continue to higher level courses, general education and introductory courses often serve as students' last exposure to college mathematics. It is important, therefore, that these courses be designed to serve the future mathematical needs of such students as well as to provide a basis for further study for students who do continue in mathematics. All students, those for whom the course is terminal and those for whom it serves as a springboard, need to learn to think effectively, quantitatively and logically. Carefully-conceived courses—described variously as quantitative literacy, liberal arts mathematics, finite mathematics, college algebra with modeling, and introductory statistics—have the potential to provide all the students who take them with the mathematical experiences called for in this section.

A common feature of many effective courses and programs that have been developed for these students is the leadership provided by key faculty members. It requires committed and talented faculty to understand the needs of these students and the opportunities inherent in these courses. Continuing leadership is needed and special training must be provided for instructors—including graduate assistants and part-time faculty—to offer courses that will meet the needs of these students.

A.1: Offer suitable courses

All students meeting general education or introductory requirements in the mathematical sciences should be enrolled in courses designed to

- Engage students in a meaningful and positive intellectual experience;
- Increase quantitative and logical reasoning abilities needed for informed citizenship and in the workplace;
- Strengthen quantitative and mathematical abilities that will be useful to students in other disciplines;
- Improve every student's ability to communicate quantitative ideas orally and in writing;
- Encourage students to take at least one additional course in the mathematical sciences.

A.2: Examine the effectiveness of college algebra

Mathematical sciences departments at institutions with a college algebra requirement should

- Clarify the rationale for the requirement and consult with colleagues in disciplines requiring college algebra to determine whether this course—as currently taught— meets the needs of their students;
- Determine the aspirations and subsequent course registration patterns of students who take college algebra;
- Ensure that the course the department offers to satisfy this requirement is aligned with these findings and meets the criteria described in A.1.

A.3: Ensure the effectiveness of introductory courses

General education and introductory courses in the mathematical sciences should be designed to provide appropriate preparation for students taking subsequent courses, such as calculus, statistics, discrete mathematics, or mathematics for elementary school teachers. In particular, departments should

- Determine whether students that enroll in subsequent mathematics courses succeed in those courses and, if success rates are low, revise introductory courses to articulate more effectively with subsequent courses;
- Use advising, placement tests, or changes in general education requirements to encourage students to choose a course appropriate to their academic and career goals.

B. Students majoring in partner disciplines

Partner disciplines vary by institution but usually include the physical sciences, the life sciences, computer science, engineering, economics, business, education, and often several social sciences.[17] It is especially important that departments offer appropriate programs of study for students preparing to teach elementary and middle school mathematics. Recommendation B.4 is specifically for these prospective teachers.

B.1 Promote interdisciplinary collaboration

Mathematical sciences departments should establish ongoing collaborations with disciplines that require their majors to take one or more courses in the mathematical sciences. These collaborations should be used to

- Ensure that mathematical sciences faculty cooperate actively with faculty in partner disciplines to strengthen courses that primarily serve the needs of those disciplines;
- Determine which computational techniques should be included in courses for students in partner disciplines;
- Develop new courses to support student understanding of recent developments in partner disciplines;
- Determine appropriate uses of technology in courses for students in partner disciplines;
- Develop applications for mathematics classes and undergraduate research projects to help students transfer to their own disciplines the skills learned in mathematics courses;
- Explore the creation of joint and interdisciplinary majors.

B.2: Develop mathematical thinking and communication

Courses that primarily serve students in partner disciplines should incorporate activities designed to advance students' progress in

- Creating, solving, and interpreting basic mathematical models;
- Making sound arguments based on mathematical reasoning and/or careful analysis of data;
- Effectively communicating the substance and meaning of mathematical problems and solutions.

B.3: Critically examine course prerequisites

Mathematical topics and courses should be offered with as few prerequisites as feasible so that they are accessible to students majoring in other disciplines or who have not yet chosen majors. This may require modifying existing courses or creating new ones. In particular,

- Some courses in statistics and discrete mathematics should be offered without a calculus prerequisite;
- Three-dimensional topics should be included in first-year courses;
- Prerequisites other than calculus should be considered for intermediate and advanced non-calculus-based mathematics courses.

B.4: Pre-service elementary (K–4) and middle school (5–8) teachers

Mathematical sciences departments should create programs of study for pre-service elementary and middle school teachers that help students develop

---

[17] Appendix 2 of the full *CUPM Guide 2004* contains a list of the disciplines represented at the Curriculum Foundations workshops.

- A solid knowledge—at a level above the highest grade certified—of the following mathematical topics: number and operations, algebra and functions, geometry and measurement, data analysis and statistics and probability;
- Mathematical thinking and communication skills, including knowledge of a broad range of explanations and examples, good logical and quantitative reasoning skills, and facility in separating and reconnecting the component parts of concepts and methods;
- An understanding of and extensive experience with the uses of mathematics in a variety of areas;
- The knowledge, confidence, and motivation to pursue career-long professional mathematical growth.

C. Students majoring in the mathematical sciences

The recommendations in this section refer to all major programs in the mathematical sciences, including programs in mathematics, applied mathematics, and various tracks within the mathematical sciences such as operations research or statistics. Also included are programs designed for prospective mathematics teachers, whether they are "mathematics" or "mathematics education" programs, although requirements in education are not specified in this section.

Although these recommendations do not specifically address minors in the mathematical sciences, departments should be alert to opportunities to meet student needs by creating minor programs—for example, for students preparing to teach mathematics in the middle grades.

These recommendations also provide a basis for discussion with colleagues in other departments about possible joint majors with any of the physical, life, social or applied sciences.

C.1: Develop mathematical thinking and communication skills

Courses designed for mathematical sciences majors should ensure that students

- Progress from a procedural/computational understanding of mathematics to a broad understanding encompassing logical reasoning, generalization, abstraction and formal proof;
- Gain experience in careful analysis of data;
- Become skilled at conveying their mathematical knowledge in a variety of settings, both orally and in writing.

C.2: Develop skill with a variety of technological tools

All majors should have experiences with a variety of technological tools, such as computer algebra systems, visualization software, statistical packages, and computer programming languages.

C.3: Provide a broad view of the mathematical sciences

All majors should have significant experience working with ideas representing the breadth of the mathematical sciences. In particular, students should see a number of contrasting but complementary points of view:

- Continuous and discrete,
- Algebraic and geometric,
- Deterministic and stochastic,
- Theoretical and applied.

Majors should understand that mathematics is an engaging field, rich in beauty, with powerful applications to other subjects, and contemporary open questions.

C.4: Require study in depth

All majors should be required to

- Study a single area in depth, drawing on ideas and tools from previous coursework and making connections, by completing two related courses or a year-long sequence at the upper level;
- Work on a senior-level project that requires them to analyze and create mathematical arguments and leads to a written and an oral report.

C.5: Create interdisciplinary majors

Mathematicians should collaborate with colleagues in other disciplines to create tracks within the major or joint majors that cross disciplinary lines.

C.6: Encourage and nurture mathematical sciences majors

In order to recruit and retain majors and minors, mathematical sciences departments should

- Put a high priority on effective and engaging teaching in introductory courses;
- Seek out prospective majors and encourage them to consider majoring in the mathematical sciences;
- Inform students about the careers open to mathematical sciences majors;
- Set up mentoring programs for current and potential majors, and offer training and support for any undergraduates working as tutors or graders;
- Assign every major a faculty advisor and ensure that advisors take an active role in meeting regularly with their advisees;
- Create a welcoming atmosphere and offer a co-curricular program of activities to encourage and support student interest in mathematics, including providing an informal space for majors to gather.

D. Mathematical sciences majors with specific career goals

D.1: Majors preparing to be secondary school (9–12) teachers

In addition to acquiring the skills developed in programs for K–8 teachers, mathematical sciences majors preparing to teach secondary mathematics should

- Learn to make appropriate connections between the advanced mathematics they are learning and the secondary mathematics they will be teaching. They should be helped to reach this understanding in courses throughout the curriculum and through a senior-level experience that makes these connections explicit;
- Fulfill the requirements for a mathematics major by including topics from abstract algebra and number theory, analysis (advanced calculus or real analysis), discrete mathematics, geometry, and statistics and probability with an emphasis on data analysis;
- Learn about the history of mathematics and its applications, including recent work;
- Experience many forms of mathematical modeling and a variety of technological tools, including graphing calculators and geometry software.

D.2: Majors preparing for the nonacademic workforce

In addition to the general recommendations for majors, programs for students preparing to enter the nonacademic workforce should include

- A programming course, at least one data-oriented statistics course past the introductory level, and coursework in an appropriate cognate area; and
- A project involving contemporary applications of mathematics or an internship in a related work area.

D.3: Majors preparing for post-baccalaureate study in the mathematical sciences and allied disciplines

Mathematical sciences departments should ensure that

- A core set of faculty members are familiar with the master's, doctoral and professional programs open to mathematical sciences majors, the employment opportunities to which they can lead, and the realities of preparing for them;
- Majors intending to pursue doctoral work in the mathematical sciences are aware of the advanced mathematics courses and the degree of mastery of this mathematics that will be required for admission to universities to which they might apply. Departments that cannot provide this coursework or prepare their students for this degree of mastery should direct students to programs that can supplement their own offerings.