Curriculum Burst 1: Two Trees
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The top of one tree is 16 feet higher than the top of another tree. The heights of the two trees are in the ratio 3 : 4.
In feet, how tall is the taller tree?

SOURCE: This is question # 11 from the 2010 MAA AMC 8 Competition.

QUICK STATS:

MAA AMC GRADE LEVEL
This question is appropriate for the 8th grade level.

MATHEMATICAL TOPICS
Ratio and Proportion

COMMON CORE STATE STANDARDS

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
7.RP.2 Recognize and represent proportional relationships between quantities.
7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.

PROBLEM SOLVING STRATEGY
ESSAY 4: DRAW A PICTURE
THE PROBLEM-SOLVING PROCESS:

As always ...

**STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I don’t feel I can really “take in” this question unless I draw a picture. Here are two trees, one 16 feet taller than the other.

![Tree Diagram]

Okay, the question now feels manageable. Except, as I read it again, the sentence:

_The heights of the two trees are in the ratio 3:4._

is cryptic. Does it mean one tree is three feet tall and the other is four feet tall? No.

Hmm. What does the “ratio 3:4” mean?

To get a feel for it, let’s ask: What are some possible tree-heights that would be in a 3:4 ratio? Well, 30 feet and 40 feet are in a 3:4 ratio. So are 60 feet and 80 feet. And 120 feet and 160 feet.

Okay. Heights being in a 3:4 ratio means one tree is of height $3x$ and the other of height $4x$ for some number $x$. Let’s mark this on the diagram.

![Tree Diagram]

Oh. Look at the picture! It is now obvious that $x$ is 16!

So the tallest tree is $4x = 4 \times 16 = 64$ feet tall!

**Extension:** Would the question be significantly harder if were told that one tree is still 16 feet taller than the other but that their heights now come in a 31:32 ratio? How about a 117:201 ratio? (These might lead to very tall trees!)

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