

# Curriculum Inspirations

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MAA American Mathematics Competitions



## Curriculum Burst 23: Union of Two Sets

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Set  $A$  has 20 elements and set  $B$  has 15 elements. What is the smallest possible number of elements in  $A \cup B$ , the union of  $A$  and  $B$ ?

**SOURCE:** This is question # 6 from the 2011 MAA AMC 10a Competition.

### QUICK STATS:

#### MAA AMC GRADE LEVEL

This question is appropriate for the 10<sup>th</sup> grade level.

#### MATHEMATICAL TOPICS

Probability and Statistics: Counting sizes of sample spaces

#### COMMON CORE STATE STANDARDS

**S-CP.1:** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events.

#### MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

#### PROBLEM SOLVING STRATEGY

ESSAY 10: [GO TO EXTREMES](#)



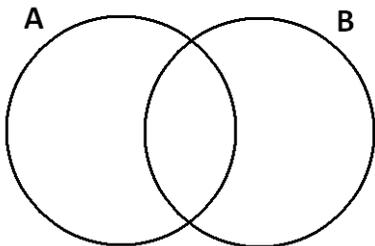
Click here for video

## THE PROBLEM-SOLVING PROCESS:

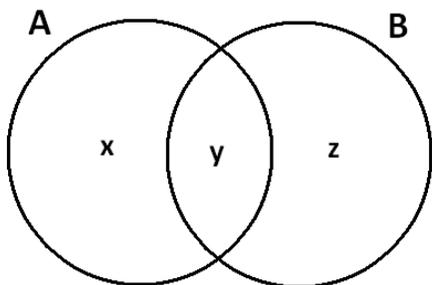
This is always the right first step:

**STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

As I read the question I feel compelled to draw a Venn diagram to accompany it. (This is a problem-solving strategy in, and of, itself – ESSAY 4: DRAW A PICTURE.) We have two sets,  $A$  and  $B$ , most likely to intersect (but perhaps not!).



The set  $A$  contains 20 objects, and the set  $B$  15 objects. Give the count of objects in each of the three regions of the diagram names  $x$ ,  $y$  and  $z$ , as shown:



Then we have:

$$x + y = 20$$

$$y + z = 15.$$

Is this helpful? What's the question?

*What is the smallest possible number of elements in the union?*

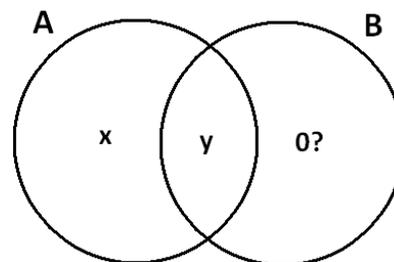
The union  $A \cup B$ , in our picture, has  $x + y + z$  elements. We want the value of this sum to be as small as possible. Well, since  $x + y = 20$  we can rewrite this sum as:

$$20 + z$$

and, again, we want this sum to be as small as possible.

**GO TO EXTREMES!**

Could  $z$  be zero? That would be the best extreme case!



Well ...  $y$  would have to be 15 (since  $B$  has fifteen elements) and so  $x$  would need to be 5. And this is possible!! The smallest possible number of elements in the union is  $20 + 0 = 20$ .

**Question:** What might be a better depiction of the Venn diagram for this case with  $z = 0$ ?

**Extension:** Three sets  $A$ ,  $B$  and  $C$  have 407, 833 and 1001 elements respectively. What is the smallest possible count of elements in their union? What can you say about smallest possible number of elements in the union of any finite number of sets? (Of an infinite number of sets?)