Curriculum Burst 45: A Sneaky Area

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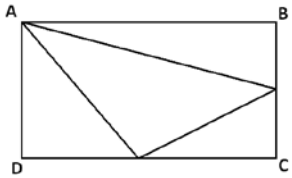
The area of rectangle $ABCD$ is 72.



If point A and the mid-points of $\overline{BC}$ and $\overline{CD}$ are joined to form a triangle, the area of that triangle is … ?

THE QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the 8th grade level.

MATHEMATICAL TOPIC

Geometry

YouTube
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COMMON CORE STATE STANDARDS

7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 1: ENGAGE IN SUCCESSFUL FLAILING

SOURCE

This is question # 25 from the 2000 MAA AMC 8 Competition.

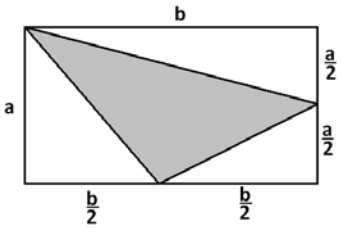
THE PROBLEM-SOLVING PROCESS:

As always, the best start is …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I have the feeling that this question is hard! It seems innocent … *given the area of a rectangle, find the area of a nice triangle in it*. And we are even told the area of the rectangle: it's 72 square units. But that is all we are told. What are the dimensions of the rectangle? (Is it a 8×9 rectangle? Probably not!)

I suppose I could call lengths of the sides of the rectangle a and b and then label all the distances in the picture. (We have midpoints.)



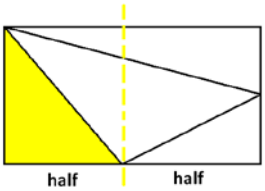
Noting the right triangles in the corners I could use the Pythagorean Theorem three times and find formulas for the sides of the middle triangle. But those formulas would be horrible! (And even if I had them, how would I work out the area of that triangle?)

Actually … maybe I could work out the areas of the three right triangles instead (those formulas are easier) and subtract those three areas from 72. That would leave the area of the middle triangle, which is what we want.

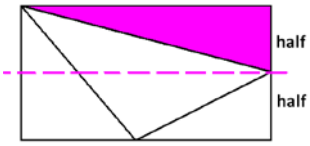
But I am nervous about that. I still don't know what the numbers a and b actually are!

Is there a way to work out the areas of each of the three right triangles without relying on the numbers a and b?

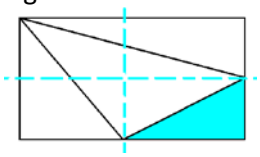
Look at the bottom left triangle:



It's half of half the rectangle. That does it! Its area is half of 36 which is 18. And this right triangle has area 18 too:



So that leaves the third one. It looks like it is half of one quarter of the rectangle:

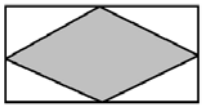


This has area 9. So the area of the middle triangle is:

$$72-18-18-9=27 \text{ square units!}$$

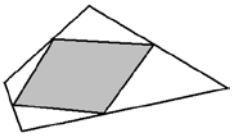
Exercise: Going back to the labels a and b, do find a formula for the areas of each of the three right triangles in terms of a and b, and hence a formula for the areas of the middle triangle. Can you see why this formula must evaluate to the answer 27?

Extension: A quadrilateral is drawn inside a rectangle by connecting the midpoints of its sides.



Can you see why the area of this new quadrilateral is half the area of the original rectangle?

Is it always true that the "midpoint quadrilateral" of any four-sided shape is half the original area?



Comment: If our goal is to simply race through a competition problem, we could employ:

Strategy 8: **Second-guess the author.**

Since this question is meant to have a definitive answer, why not assume that it is an 8×9 rectangle and get a number? That number must be the solution!

Curriculum Inspirations is brought to you by the Mathematical Association of America and the MAA American Mathematics Competitions.