Curriculum Burst 58: Parking Probability

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> A parking lot has 16 spaces in a row. Twelve cars arrive, each of which requires one parking space, and their drivers choose their spaces at random from among the available spaces. Auntie Em then arrives in her SUV, which requires 2 adjacent spaces. What is the probability that she is able to park?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the 12th grade level.

YouTube
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MATHEMATICAL TOPICS

Probability, Permutations and Combinations

COMMON CORE STATE STANDARDS

S-CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 4: **DRAW A PICTURE**

SOURCE: This is question # 22 from the 2008 MAA AMC 12B Competition.

THE PROBLEM-SOLVING PROCESS:

As always, start with …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I understand this question, but it feels hard! There are sixteen "slots" in a row in which twelve will be occupied and four will be left vacant. We need to count all the possible arrangements of vacant spaces that allow Auntie Em to park.

I have a picture already in my mind, so let me put it down on paper.

O	V	O	V	O	O	O	V	O	O	V	O

Each picture is just a "word" composed of 12 Os and 4 Vs. (And the word I've drawn here is not good for Auntie Em.)

I know how to count words. (http://www.jamestanton.com/?p=659.) There are

$$\frac{16!}{12!4!}=\frac{16\times15\times14\times13}{24}$$
$$=2\times5\times14\times13=1820$$

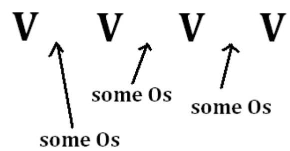
words with this many Os and Vs. Thus Auntie Em will arrive and see one of 1820 possible configurations of parked cars.

Now we need to count how many of these have two neighboring Vs. (Then the probability we seek is the ratio of these two counts.)

A classic technique in probability theory is to count the opposite of what we want. How many arrangements of 12 Os and 4 Vs don't have two consecutive Vs? Is that easier?

So what does it mean for the Vs to be non-adjacent?

Between any two Vs there must be at least one O.



(Plus there could be some Os at the beginning and at the end.) This seems hard to count.

What's another way to think of Vs being non-adjacent? Well, each V has to sit between two Os (or it could sit between an O and the void at the end of the line). Is this a better way to think of matters?

We have 13 "spaces" around the Os and if we insert four Vs in these spaces, then we have an arrangement on non-adjacent Vs!

| O | O | O | O | O | O | O | O | O | O | O | O |

Chose any of the four red slots to be Vs

So of 13 slots we need to label four of them as Vs and nine of them as "not selected" and there are

$$\frac{13!}{9!4!}=\frac{13\times12\times11\times10}{24}=143\times5=715$$

ways to do this.

So the number of configurations that will allow Auntie Em to part is $1820-715=1105$, and the probability that she can park is:

$$\frac{1105}{1820}=\frac{221}{364}=\frac{17\times13}{28\times13}=\frac{17}{28}.$$

Extension 1: How many solutions are there to the equation $a+b+c=12$ with a,b and c each a positive integer or zero. (Answer this question in the style of this essay solution!)

Extension 2: Mega Bowl Special: A Sunday parlor is having a special: 12 scoops of ice-cream in a bowl from a selection of 12 flavors. How many different megabowls are there?

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