

Curriculum Inspirations

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MAA American Mathematics Competitions



Curriculum Burst 62: Counting Wins

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A singles tournament had six players. Each player played every other player only once, with no ties. If Helen won 4 games, Ines won 3 games, Janet won 2 games, Kendra won 2 games and Lara won 2 games, how many games did Monica win?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the 8th grade level.

MATHEMATICAL TOPICS

Combinatorics

COMMON CORE STATE STANDARDS

7-SP.8b Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

MATHEMATICAL PRACTICE STANDARDS

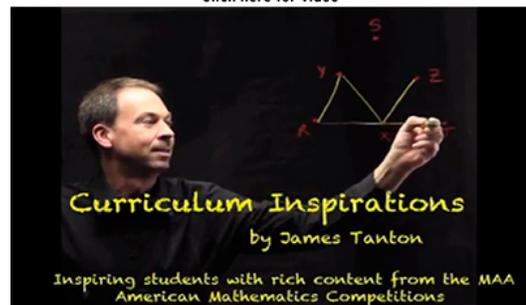
- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.

PROBLEM SOLVING STRATEGY

ESSAY 2: [DO SOMETHING](#)

SOURCE: This is question # 20 from the 2006 MAA AMC 8 Competition.

[Click here for video](#)



THE PROBLEM-SOLVING PROCESS:

As always, start with ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Okay. I understand the question: Six people played in matches and we know how many matches each person won – except for Monica. We are meant to figure out how many games she won. Hmm. That seems hard! How am I meant to figure that out?

I really do feel at a loss with this question. It is not a typical school classroom question and I haven't seen questions like this before.

In order to just do something, let me add the counts of wins we are told:

$$4 + 3 + 2 + 2 + 2 = 13.$$

That's something, but I don't know what it means!

Do I know how many wins there should be? Hmm.

Well each game results in a winner and a loser, so there are just as many wins as there are games. So ... How many games are played?

Helen plays five games (one each against Ines, Janet, Kendra, Lara and Monica), and Ines plays five games, and so on. There are six people each playing five games, so that's a total of $6 \times 5 = 30$ games.

Okay, there are 30 games, so there are 30 wins and we have 13 so far. This means Monica won 17 matches.

Ouch! Something is wrong. Monica only played five matches. She didn't have seventeen wins!

Were there really 30 matches? Let me list them:

HI
HJ IJ
HK IK JK
HL IL JL KL
HM IM JM KM LM

There were only 15 matches!

[Question: Where did I go wrong in my reasoning that there were 30 matches?]

We know about 13 of the wins, so this means Monica won 2 games!

Extension 1: One hundred and one people shook hands – each person shaking hands exactly once with each and every other person. How many handshakes, in total, occurred?

Extension 2: One hundred one people play matches of “tri-bridge,” a game for three people that produces one winner and two losers. Rounds of this game are played, with all losers leaving the premises as soon as they lose a match, until a single player remains. How many rounds of tri-bridge will be played?

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