

Curriculum Inspirations

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MAA American Mathematics Competitions



Curriculum Burst 66: Probably a Multiple of Three

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A bag contains four pieces of paper, each labeled with one of the digits 1, 2, 3, or 4, with no repeats. Three of these pieces are drawn, one at a time without replacement, to construct a three-digit number. What is the probability that the three-digit number is a multiple of three?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grades.

MATHEMATICAL TOPICS

Probability, Number Sense

COMMON CORE STATE STANDARDS

6EE.3 Apply the properties of operations to generate equivalent expressions.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.

MP2 Reason abstractly and quantitatively.

MP3 Construct viable arguments and critique the reasoning of others.

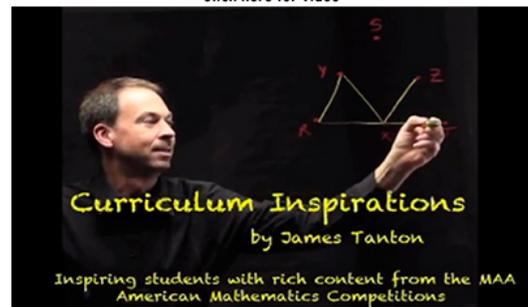
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 2: [DO SOMETHING](#)

SOURCE: This is question # 24 from the 2007 MAA AMC 8 Competition.

[Click here for video](#)



THE PROBLEM-SOLVING PROCESS:

As always, the best start is ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I don't feel like I really understand the question! Let me see if I can make sense of what it is saying by acting it out.

There are four cards in a bag, one labeled "1," and one labeled "2," and so on. I pull out a card. Maybe I get

3

Then I pull out a second card, say:

3 2

And then a third card:

3 2 4

And this represents a three-digit number. We're asking: *What are the chances that the three-digit number we obtain is a multiple of three?* In my example here, $324 = 300 + 24 = 3 \times (108)$ is a multiple of three.

Is it possible not to get a multiple of three? Sure! With 134 or 241, for example.

Alright. How am I meant to think about this? I could list all the possible three-digit numbers we could get:

123 213
132 231
124 214
142 241
134 234
143 243 *etc*

(Actually, that's not too bad a job!) But I still need to be able to tell quickly if a number is divisible by three.

I do remember being told a rule:

A number is divisible by three if its digits sum to a multiple of three.

Am I remembering this correctly? Even if I am, do I believe it? If I have a number " abc ," the rule is saying that it's a multiple of three if $a + b + c$ is. This works for 324: it's a multiple of 3 and $3 + 2 + 4 = 9$ is too. And it works for 241: this is not a multiple of 3 and nor is $2 + 4 + 1 = 7$.

Hmm. I am running out of time so I am just going to go with this and mull on thinking about whether this divisibility rule is true as I walk home tonight. (That will be fun!)

So any number composed of the digits 1, 2, and 3, in any order, will be a multiple of three (since its digits will add to 6). Numbers composed of the digits 1, 2, and 4 (leaving 3 in the bag) won't be multiples of three. Numbers composed of 1, 3, and 4 (leaving 2 in the bag) won't be multiples of three; and numbers composed of the digits 2, 3, and 4 will be.

SO ... leaving the 1 card or the 4 card behind in the bag produces a multiple of three, and leaving the 2 card or the 3 card behind won't. That's it! There's a 50% chance we'll see a multiple of three then.

CHECK: Complete the table at the bottom of the left column and verify that, indeed, half the numbers in that list are multiples of three.

Extension: So ... Is the divisibility rule for the number 3 described here correct?

Do you know a divisibility rule for the number 9? For 11? For 5? 4? 7? 19? 47?

See <http://www.jamestanton.com/?p=1287> for the mathematics of all sorts of divisibility rules.

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