Curriculum Burst 70: A Random Sequence

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> Jacob uses the following procedure to write down a sequence of numbers. First he chooses the first term to be 6. To generate each succeeding term, he flips a fair coin. If it comes up heads, he doubles the previous term and subtracts 1. If it comes up tails, he takes half of the previous term and subtracts 1. What is the probability that the fourth term in Jacob's sequence is an integer?

QUICK STATS:

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MAA AMC GRADE LEVEL

This question is appropriate for the junior high-school grade levels.

MATHEMATICAL TOPICS

Probability

COMMON CORE STATE STANDARDS

S.CP.1 (Tangentially) Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

7.SP.8b Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.

PROBLEM SOLVING STRATEGY

ESSAY 7: **PERSEVERANCE IS KEY**

SOURCE: This is question # 22 from the 2008 MAA AMC 10A Competition.

THE PROBLEM-SOLVING PROCESS:

As always, one starts the problem-solving process with …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

My first reaction to this question is: *Why is Jacob doing this?* It seems like an odd activity.

My second reaction is: Figuring out this probability seems hard! There are many possible outcomes, all quite complicated to analyze.

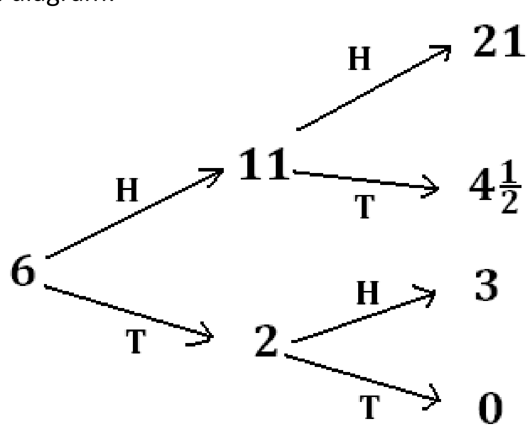
Let's just take it slowly.

In rereading the questions I see that Jacob starts with the number 6, and then flips a coin. If it lands HEADS, he then writes down $2\times 6-1=11$. If it lands tails, he writes: $\frac{1}{2}\times 6-1=2$. Both are integers so far.

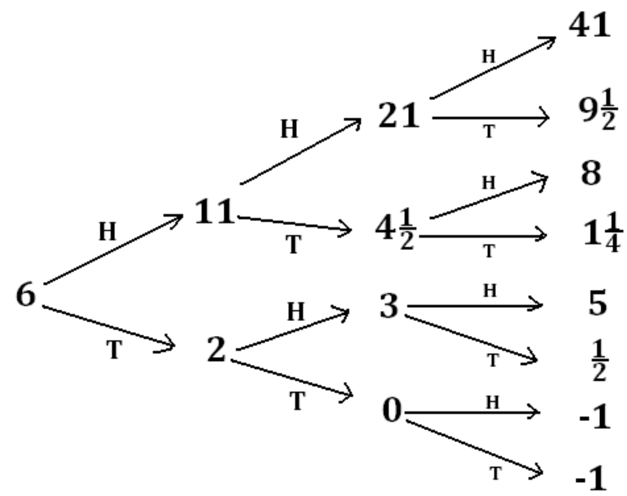
He's now got two terms of his sequence:

$6\ 11$ or $6\ 2$.

I don't know what else to do but to go through all the possibilities of what could happen next. We can organize this is a diagram:



Well, we've only got one more step to do (the question wants four terms) so we might as well do it!



What is the probability that the fourth term in Jacob's sequence is an integer?

Well there are eight paths in this tree diagram, with four paths yielding an integer for the fourth final value. The probability we seek is $\frac{4}{8}=50\%$.

Ooh! Hang on! The integer "-1" appears twice as a final value in the tree above. So maybe the answer is actually "four integer values out of seven different values." This gives the probability $4/7\approx 57\%$.

Oh dear, which is the correct answer?

I guess the underlying action in this problem is Jacob tossing coins. There are eight possible results for tossing a coin three times. Just that two of them happen to yield the same final value doesn't matter: four of the triples of coin tosses give an integer and four of the triples don't. (And we don't really care what those integer values are.) Okay, I am confident: the answer to this question is 50%.

Extension: Could Jacob toss his coin an infinite number of times and get an integer value for each and every term of his sequence?

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