Curriculum Inspirations

Inspiring students with rich content from the MAA American Mathematics Competitions

MAA

Curriculum Burst 72: A Curious Surface Area

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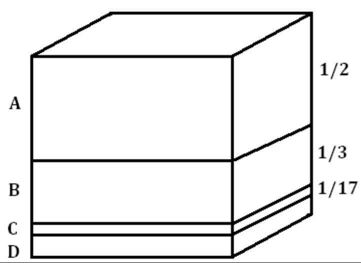
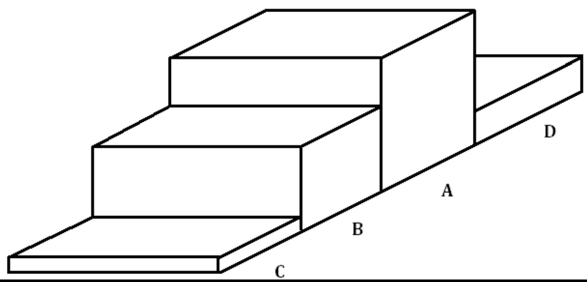
A one-cubic-foot cube is cut into four pieces by three cuts parallel to the top face of the cube. The first cut is $1/2$ foot from the top face. The second cut is $1/3$ foot below the first cut, and the third cut is $1/17$ foot below the second cut. From the top to the bottom the pieces are labeled A, B, C and D. The pieces are then glued together end to end in the order C, B, A, D to make a long solid as shown below. What is the total surface area of this solid in square feet?





QUICK STATS:

Click here for video

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Geometry

COMMON CORE STATE STANDARDS

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.

PROBLEM SOLVING STRATEGY

ESSAY 9: **AVOID HARD WORK**

SOURCE: This is question # 25 from the 2009 MAA AMC 8 Competition.

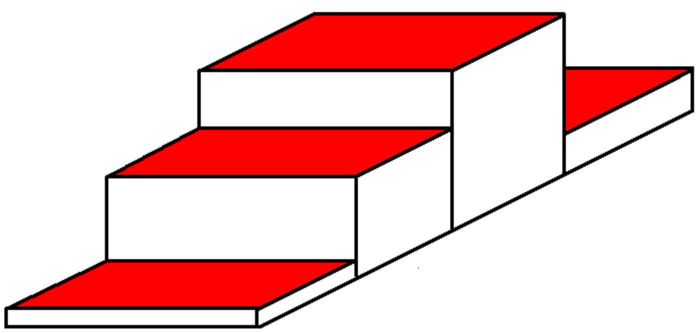
THE PROBLEM-SOLVING PROCESS:

As always, one starts the problem-solving process with …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

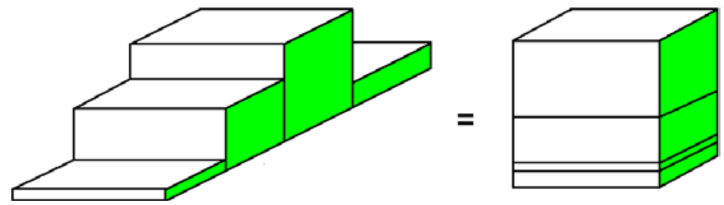
This question looks positively frightening! For starters, we don't know the thickness of the final piece. It is, in principle, straightforward to work out – it is just $1-\frac{1}{2}-\frac{1}{3}-\frac{1}{17}$ - but the numbers are horrible and I don't want to do this arithmetic! And then we have to work out the areas of all the rectangles of surface area we see and sum them up. Again, in principle, this is straightforward to do, but it is going to be a lot of tedious arithmetic. Surely the author of the question didn't want us to waste hours grinding through these calculations. There's got to be a way to avoid this hard work.

Okay then. Are there any parts of the surface whose areas are "obvious" and don't need work?



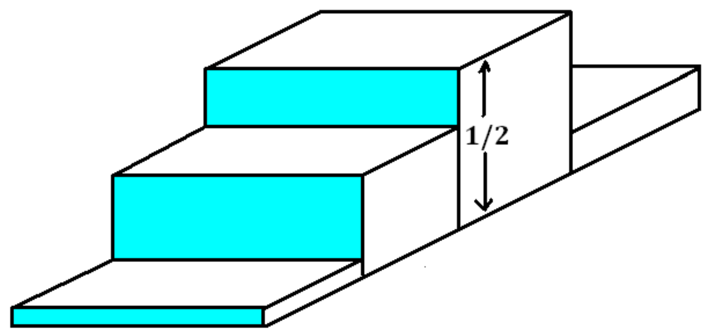
The four tops are just 1×1 square so they make 4 four square units of area. Oh .. And the bottom surface of this shapes is another four 1×1 squares. That's a total of 8 square units of area so far.

Actually, looking the figure from these different directions is cool! For example, looking at the right face we see that its surface area matches that that of one face of the original cube!



The left face is the same. So we have another 2 two square units of area making a total of 10 square units so far.

What about the front surfaces?



Well, these are like drawers in a chest of drawers. If I push them back in, they'd make a $1\times\frac{1}{2}$ flat surface. Their surface areas thus add to $\frac{1}{2}$ a square unit. Wow!

And the same is true for the two back faces we can't see in the picture.

This means that the total surface are we seek is $4+4+1+1+\frac{1}{2}+\frac{1}{2}=11$ square units!

Extension: Is the answer to this problem sure to be the same if we change the numbers $1/3$ and $1/17$ to different values (and keep piece A half a unit thick)?

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