Curriculum Burst 94: Hundreds Digit of a Power

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What is the hundreds digit of \(2011^{2011}\)?

QUICK STATS:

**MAA AMC GRADE LEVEL**
This question is appropriate for the lower high-school grade levels.

**MATHEMATICAL TOPICS**
The binomial theorem.

**COMMON CORE STATE STANDARDS**
A-APR.C5 (+) Know and apply the Binomial Theorem for the expansion of \((x+y)^n\) in powers of \(x\) and \(y\) for a positive integer \(n\), where \(x\) and \(y\) are any numbers, with coefficients determined for example by Pascal's Triangle

**MATHEMATICAL PRACTICE STANDARDS**
MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

**PROBLEM SOLVING STRATEGY**

ESSAY 1: ENGAGE IN SUCCESSFUL FLAILING

**SOURCE:** This is question # 23 from the 2011 MAA AMC 10B Competition.
THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

**STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I have mixed feelings about this problem. It is comprehensible – I completely understand what the question is and what I am looking for – but at the same time I don’t have a clue how to go about what I am being asked to do! 2011^{2011} is some huge number. I don’t want to work out what that number is.

Okay. Deep breath. All I need is the hundreds digit of 2011^{2011}.

Hmm. What does that mean? What is the hundreds digit of a number?

Consider 348712, for instance. The third-to-last digit, 7, is the hundreds digit. It is the multiple of 100 we need to write it in base ten.

\[
348712 = 3 \times 100000 + 4 \times 10000 + 8 \times 1000 + 7 \times 100 + 1 \times 10 + 2 \times 1
\]

So all I need to do is to work out how many multiples of 100 there are in 2011^{2011}.

Okay then! We can ignore all the terms that involve 2000. We also don’t care about y^b with b > 2 as that gets me into the thousands as well. So a = 0 and b = 0,1,2 is all I need write out. Okay then!

\[
\binom{N}{a b c} = \binom{2011}{000} + \binom{2011}{001} + \binom{2011}{100} + \binom{2011}{101} + \binom{2011}{200} + \binom{2011}{201} + \binom{2011}{202}
\]

= thousands stuff + 2011\times1005\times100 + 2011\times10 + 1

= thousands stuff + (2011000 + 10055)\times100 + 20110 + 1

= thousands stuff + thousands stuff + 1005500 + 20111

So 2011^{2011} ends with 611 and has hundreds digit 6!

**Extension:** What is the hundreds digit of 211^{211}?

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